Wind Energy Systems – PRACTICE Exam

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Mark: Exam inspected on: Signature of examiner:

Surname: First name: Matriculation number:

Subject: Programme: Bachelor ☐ Master ☐ Lehramt ☐ others ☐ Signature:

1. Approximately, what is the power density through a circular area of radius $R = 1\text{m}$, when the constant air density $\rho = 1\text{kg/m}^3$ at a constant and uniform wind speed of $u_\infty = 10\text{m/s}$. Please neglect induction effects.

(a) ☐ 160 W/m^2	(b) [x] 500 W/m^2
(c) ☐ 610 W/m^2	(d) ☐ 1570 W/m^2

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2. Consider that there is a site at which a developer wants to put a GE 1.5MW turbine. There is a known probability distribution of wind speeds at that site, and the turbine's power curve is known. Both can be found in Figure 1.

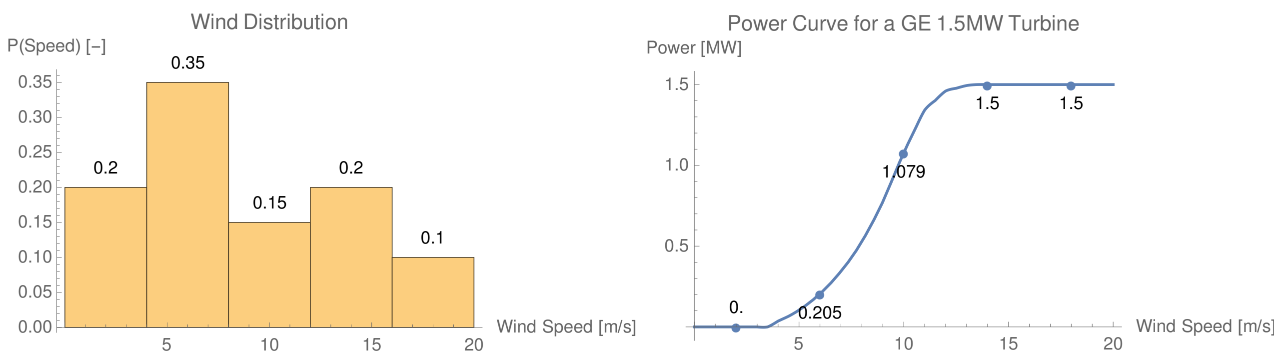

Figure 1: Wind distribution (left) and power curve (right) for problem 2

Which of the following values is closest to the expected capacity factor for this turbine?

(a) ☐ 0.20	(b) ☐ 0.33
(c) [x] 0.46	(d) ☐ 0.59

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3. All else being equal, at which latitude would you expect a wind farm (of turbines of height less than 100m) to produce the MOST power?

(a) ☐ 0 degrees North	(b) ☒ 15 degrees North
(c) ☐ 30 degrees North	(d) ☐ 60 degrees North

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4. Considering the logarithmic wind profile: $u(z) = u(z_1)\frac{\log \frac{z}{z_0}}{\log \frac{z_1}{z_0}}$. Which of the following statements is FALSE?

(a) ☒ The logarithmic wind profile represents the wind speed distribution at any specific instant.	(b) ☐ This wind profile is not valid at altitudes above (approximately) 500m.
(c) ☐ This wind profile is not valid at altitudes below the roughness length.	(d) ☐ The logarithmic wind profile describes the atmospheric boundary layer.

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5. Consider the flow travelling through the actuator disk (AD) of a turbine. Which of the following statements is FALSE?

(a) ☐ The axial-direction flow is slower at the AD than it is upstream of the AD.	(b) ☐ The axial-direction flow is faster at the AD than it is downstream of the AD.
(c) ☐ The cross-section of the streamtube is wider downstream of the AD than it is upstream of the AD.	(d) ☒ Air can cross the boundaries of the stream-tube.

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6. Let's consider the wake rotation behind an rotor disk (RD), as described by rotor disk theory. Which of the following statements is FALSE?

(a) ☐ Upstream of the RD, there is no wake rotation.	(b) ☐ Half of the rotation is added exactly at the RD; half is added immediately downstream of the RD.
(c) ☐ For horizontal axis wind turbines, who generate power primarily through torque, wake rotation is significant.	(d) ☒ For airborne wind energy systems, who generate power primarily through torque, wake rotation is significant.

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7. For which of the following simplified wake models can you find the tangential momentum analytically? (That is, not numerically).

(a) ☐ classic actuator disk model	(b) ☒ rotor disk model
(c) ☐ blade element momentum model	(d) ☐ it cannot be found analytically in any of these models

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8. Consider the wind turbine tower as a vertically cantilevered beam, that is only loaded by a normal point-load F that is at three-quarters of the tower height. The second moment of area of the tower cross-section I is constant over the whole tower, which has constant outer radius r. The tower height is L. Which formula will best describe the maximum bending stress on the tower?

(a) ☐ $\frac{4F}{3Lr}$	(b) ☐ $\frac{4^4 FI}{(3^4)\pi L^4 r^2}$
(c) ☐ $\frac{3^3 FL^3}{4^3 \pi r I}$	(d) ☒ $\frac{3FLr}{4I}$

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9. A cantilevered beam under a concentrated end load P has a deflection $y(x,t)$. This function reads as:

$$y(x,t) = \frac{Px^2}{6EI}(3L - x)\cos(\omega t),$$

with a beam of length L, Young's modulus E and second moment of area I. The distance from from the fixed end of the beam is x, and ω is the vibration frequency. The strain energy $V(t)$ and kinetic energy $T(t)$ in the bending beam reads as:

$$V(t) = \frac{EI}{2}\int_0^L \left(\frac{\partial^2 y}{\partial x^2}\right)^2 \mathrm{d}x = \frac{P^2}{2k}\cos^2(\omega t) \qquad T(t) = \frac{m}{2L}\int_0^L \left(\frac{\partial y}{\partial t}\right)^2 \mathrm{d}x = \frac{33}{280}\frac{m\omega^2 P^2}{k^2}\sin^2(\omega t),$$

where $k = 3EI/L^3$ and m is the beam's mass.

Using the Rayleigh method, which of the following values most closely approximates the beam's natural frequency?

(a) ☐ $1.41\sqrt{\frac{k}{m}}$	(b) ☐ $2.01\sqrt{\frac{k}{m}}$
(c) ☒ $2.06\sqrt{\frac{k}{m}}$	(d) ☐ $4.24\sqrt{\frac{k}{m}}$

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10. Consider different wind energy conversion systems in normal operation at their rated wind speeds. For which system are heavy components mounted at the highest altitude?

(a) ☒ horizontal axis wind turbine	(b) ☐ darrieus wind turbine
(c) ☐ savonius wind turbine	(d) ☐ lift-mode airborne wind energy system

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11. Consider Loyd's limit for the maximum power-harvesting factor of an airborne wind energy system. Which of the following assumptions was NOT made during its derivation?

(a) ☐ steady crosswind flight	(b) ☒ lift force parallel to the tether
(c) ☐ reel-out direction parallel to freestream wind	(d) ☐ known lift and drag coefficients

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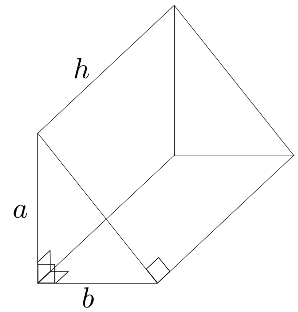
12. Consider a control volume of air, through which wind is flowing. This control volume has the shape of a right triangular prism. This prism has height h and triangle dimensions, as given by the the triangular face's altitude a perpendicular to a base of length b. The height is parallel to the uniform and steady wind velocity $\boldsymbol{u}$.



(a) Given a uniform and constant air density ρ, how much kinetic energy is present in the air within the control volume? The kinetic energy in the air is:

$$K = \frac{1}{2}(\rho V) \|\boldsymbol{u}\|_2^2 = \frac{1}{4}\rho abh \|\boldsymbol{u}\|_2^2$$

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(b) Please use the above expression to derive the power density passing through the triangular cross-section.

The power density is the kinetic energy per surface area per unit time. That is:

$$D = \frac{K}{\frac{1}{2}ab\Delta t} = \frac{1}{2}\rho \frac{h}{\Delta t} \|\boldsymbol{u}\|_2^2$$

Since the time it takes for the flow to cover the distance h is by definition the average of the (constant) wind speed, we get:

$$D = \frac{1}{2}\rho \|\boldsymbol{u}\|_2^3$$

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13. Consider the power harvesting factor ζ and the power coefficient C_{P}.

(a) Which value is likely to be more useful for horizontal axis wind turbines? Briefly, why?

Consider for horizontal axis wind turbines: the rotor radius is well known, and easy to compare between different models. Comparatively, the blade surface area is not generally public information. This would make normalizing the power by the product of the power density and the rotor area (ie, C_{P}) an 'easier' value to compare turbine performance.

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(b) Which value is likely to be more useful for an airborne wind energy system? Briefly, why?

In comparison, AWE systems fly variable radius trajectories. This means that the normalization of the C_{P} will change over time for the same system, even at constant wind speed. This makes for an impractical comparison value. The wing area, by contrast is well known, and is a specification that is frequently published by AWE groups about their systems. As such, ζ is likely to be more useful for AWE systems.

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14. Regard a high-pressure region in the northern hemisphere at a latitude of $\phi = 50\,\deg$, with an air density of approximately 1kg/m^3. We have learnt that the geostrophic wind - as well as its refinement, the gradient wind - is parallel to the isobars, and grows with the gradient of the pressure.

(a) In what direction (as seen from above) does the air flow around the high pressure region described: clockwise or counterclockwise?

An anticyclone will rotate clockwise in the northern hemisphere.

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(b) The pressure gradient at a specific location A on the boundary of the high-pressure region is 5 Pa/km. What would be the geostrophic wind at this location?

The geostrophic wind lies parallel to the isobars, with a magnitude proportional to the pressure gradient:

$$\nu = -\frac{\partial p}{\partial x} \frac{1}{2\rho \sin \phi \omega_0}$$

Here, $\rho = 1\text{kg/m}^3$ is the density of the air, $\phi = 0.87$ rad is the latitude, and $\omega_0 = 2\pi/((24)(3600s)) = 7.3 \cdot 10^{-5}$ rad/s is the Earth's rotation frequency.

Notice that $\frac{\partial p}{\partial x} = 5 \cdot 10^{-3}$ Pa/m. Then: $\nu \approx -45$m/s.

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(c) Would the gradient wind be faster or slower than the geostrophic wind at this location?

The gradient wind occurs when isobars are straight; the geostrophic wind, when isobars are curved. Since the centrifugal force has magnitude mv^2/r, and the radius of curvature of a straight line is infinite, the centrifugal force for the gradient wind will be zero, and positive for the geostrophic wind.

For the case of a high pressure system, the centrifugal force will push in the same direction as the pressure gradient force, and oppose the Coriolis force. That is, as the centrifugal force increases, the pressure gradient contributes less (relatively speaking) to the force balance. That means, that the more centrifugal force is present (about a high-pressure system) the less the pressure gradient can 'push' the fluid element.

The gradient wind will be faster than the geostrophic wind, for a high-pressure system.

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15. Please describe briefly, in words and without equations, what the difference is between "supervisory control" and "dynamic control".

'Supervisory control' controls the high-level operation of the turbine (when to start, when to brake, etc.); and 'dynamic control' controls the subsystems that implement the commands of the supervisory control system.

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16. Consider actuator disk theory, with an induction factor a, a freestream wind velocity u_∞, and a constant air density ρ.

(a) What is the axial-direction wind speed at the actuator disk u_1, based on the definition of the induction factor?

By definition: $u_1 = u_\infty(1-a)$.

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(b) Please describe (briefly, in words, and without equations) how to go about finding the relationship between u_2, u_∞ and a, where u_2 is the axial-direction wind speed far downstream of the actuator disk.

First, we would make two Bernoulli expressions (upstream of the rotor and downstream of the rotor) that can be subtracted from one another to give the pressure drop over the rotor. The thrust is then equal to this pressure drop times the area. However, the thrust is also equal to the mass flow rate times the change in velocity from far upstream to far downstream. When we simplify, we find the relationship between u_2, u_∞ and a.

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(c) What is the power per unit area (P/A) that an actuator disk can extract?

This can be found as:

$$\frac{P}{A} = \frac{1}{2}\dot{m}(u_\infty^2 - u_2^2)$$

Since we know that the mass flow rate is: $\dot{m} = \rho u_\infty(1-a)A$, and that the far downstream speed is $u_2 = (1-2a)u_\infty$, we can simplify:

$$\frac{P}{A} = \frac{1}{2}\rho u_\infty^3(1-a)(1-(1-2a)^2) = \frac{1}{2}\rho u_\infty^3(4a)(1-a)^2)$$

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(d) What optimal induction factor a^* will maximize the power per unit area P/A?

We can find the optimal a^* as:

$$a^* = \operatorname*{argmax}_a \frac{P}{A}$$

We solve this by solving the equation $\frac{\partial P/A}{\partial a}(a^*) = 0$. That is: $\frac{\partial P/A}{\partial a} = 2\rho u_\infty^3\left((1-a)^2 - 2a(1-a)\right) = 2\rho u_\infty^3\left(1-4a+3a^2\right)$. That is: $3a^{*2} - 4a^* + 1 = 0 \rightarrow a^* = \frac{4\pm\sqrt{16-12}}{6} = \frac{4\pm 2}{6}$. So: $a^* = \frac{1}{3}$ or $a^* = 1$. Since $0 \le a \le \frac{1}{2}$, we know that $a^* = \frac{1}{3}$.

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(e) What is the maximum power per unit area $(P/A)^*$ that corresponds to this optimal induction factor?

With $a^* = \frac{1}{3}$:

$$\frac{P}{A}(a^*) = \frac{1}{2}\rho u_\infty^3(\frac{4}{3})(\frac{2}{3})^2 = \frac{1}{2}\rho u_\infty^3\frac{16}{27}$$

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