Wind Energy Systems – Exam

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Mark: Exam inspected on: Signature of examiner:

Surname: First name: Matriculation number:

Subject: Programme: Bachelor ☐ Master ☐ Lehramt ☐ others ☐ Signature:

1. Approximately, what is the power density through a circular area of radius $R = 1$m, when the constant air density $\rho = 1$kg/m^3 at a constant and uniform wind speed of $u_\infty = 10$m/s. Please neglect induction effects.

(a) ☐ 160 W/m^2	(b) ☐ 500 W/m^2
(c) ☐ 610 W/m^2	(d) ☐ 1570 W/m^2

2 ☐

2. Consider that there is a site at which a developer wants to put a GE 1.5MW turbine. There is a known probability distribution of wind speeds at that site, and the turbine's power curve is known. Both can be found in Figure 1.

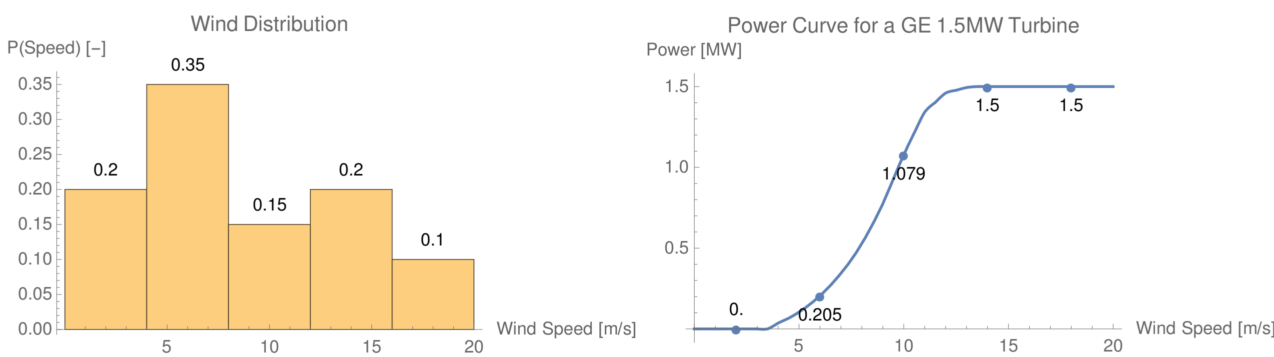

Figure 1: Wind distribution (left) and power curve (right) for problem 2

Which of the following values is closest to the expected capacity factor for this turbine?

(a) ☐ 0.20	(b) ☐ 0.33
(c) ☐ 0.46	(d) ☐ 0.59

2 ☐

points on page: 4 ☐

3. All else being equal, at which latitude would you expect a wind farm (of turbines of height less than 100m) to produce the MOST power?

(a) ☐ 0 degrees North	(b) ☐ 15 degrees North
(c) ☐ 30 degrees North	(d) ☐ 60 degrees North

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4. Considering the logarithmic wind profile: $u(z) = u(z_1)\frac{\log \frac{z}{z_0}}{\log \frac{z_1}{z_0}}$. Which of the following statements is FALSE?

(a) ☐ The logarithmic wind profile represents the wind speed distribution at any specific instant.	(b) ☐ This wind profile is not valid at altitudes above (approximately) 500m.
(c) ☐ This wind profile is not valid at altitudes below the roughness length.	(d) ☐ The logarithmic wind profile describes the atmospheric boundary layer.

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5. Consider the flow travelling through the actuator disk (AD) of a turbine. Which of the following statements is FALSE?

(a) ☐ The axial-direction flow is slower at the AD than it is upstream of the AD.	(b) ☐ The axial-direction flow is faster at the AD than it is downstream of the AD.
(c) ☐ The cross-section of the streamtube is wider downstream of the AD than it is upstream of the AD.	(d) ☐ Air can cross the boundaries of the streamtube.

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6. Let's consider the wake rotation behind an rotor disk (RD), as described by rotor disk theory. Which of the following statements is FALSE?

(a) ☐ Upstream of the RD, there is no wake rotation.	(b) ☐ Half of the rotation is added exactly at the RD; half is added immediately downstream of the RD.
(c) ☐ For horizontal axis wind turbines, who generate power primarily through torque, wake rotation is significant.	(d) ☐ For airborne wind energy systems, who generate power primarily through torque, wake rotation is significant.

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7. For which of the following simplified wake models can you find the tangential momentum analytically? (That is, not numerically).

(a) ☐ classic actuator disk model	(b) ☐ rotor disk model
(c) ☐ blade element momentum model	(d) ☐ it cannot be found analytically in any of these models

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points on page: 10	

8. Consider the wind turbine tower as a vertically cantilevered beam, that is only loaded by a normal point-load F that is at three-quarters of the tower height. The second moment of area of the tower cross-section I is constant over the whole tower, which has constant outer radius r. The tower height is L. Which formula will best describe the maximum bending stress on the tower?

(a) ☐ $\frac{4F}{3Lr}$	(b) ☐ $\frac{4^4 FI}{(3^4)\pi L^4 r^2}$
(c) ☐ $\frac{3^3 FL^3}{4^3 \pi r I}$	(d) ☐ $\frac{3FLr}{4I}$

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9. A cantilevered beam under a concentrated end load P has a deflection $y(x,t)$. This function reads as:

$$y(x,t) = \frac{Px^2}{6EI}(3L - x)\cos(\omega t),$$

with a beam of length L, Young's modulus E and second moment of area I. The distance from from the fixed end of the beam is x, and ω is the vibration frequency. The strain energy $V(t)$ and kinetic energy $T(t)$ in the bending beam reads as:

$$V(t) = \frac{EI}{2}\int_0^L \left(\frac{\partial^2 y}{\partial x^2}\right)^2 \mathrm{d}x = \frac{P^2}{2k}\cos^2(\omega t) \qquad T(t) = \frac{m}{2L}\int_0^L \left(\frac{\partial y}{\partial t}\right)^2 \mathrm{d}x = \frac{33}{280}\frac{m\omega^2 P^2}{k^2}\sin^2(\omega t),$$

where $k = 3EI/L^3$ and m is the beam's mass.

Using the Rayleigh method, which of the following values most closely approximates the beam's natural frequency?

(a) ☐ $1.41\sqrt{\frac{k}{m}}$	(b) ☐ $2.01\sqrt{\frac{k}{m}}$
(c) ☐ $2.06\sqrt{\frac{k}{m}}$	(d) ☐ $4.24\sqrt{\frac{k}{m}}$

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10. Consider different wind energy conversion systems in normal operation at their rated wind speeds. For which system are heavy components mounted at the highest altitude?

(a) ☐ horizontal axis wind turbine	(b) ☐ darrieus wind turbine
(c) ☐ savonius wind turbine	(d) ☐ lift-mode airborne wind energy system

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11. Consider Loyd's limit for the maximum power-harvesting factor of an airborne wind energy system. Which of the following assumptions was NOT made during its derivation?

(a) ☐ steady crosswind flight	(b) ☐ lift force parallel to the tether
(c) ☐ reel-out direction parallel to freestream wind	(d) ☐ known lift and drag coefficients

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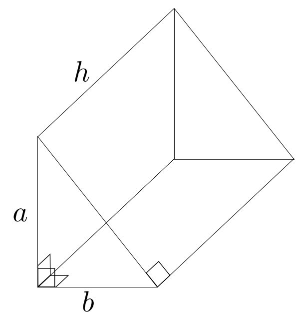
points on page: 8	

12. Consider a control volume of air, through which wind is flowing. This control volume has the shape of a right triangular prism. This prism has height h and triangle dimensions, as given by the the triangular face's altitude a perpendicular to a base of length b. The height is parallel to the uniform and steady wind velocity $\boldsymbol{u}$.



(a) Given a uniform and constant air density ρ, how much kinetic energy is present in the air within the control volume?

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(b) Please use the above expression to derive the power density passing through the triangular cross-section.

2	

13. Consider the power harvesting factor ζ and the power coefficient C_{P}.

(a) Which value is likely to be more useful for horizontal axis wind turbines? Briefly, why?

2	

(b) Which value is likely to be more useful for an airborne wind energy system? Briefly, why?

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points on page: 8	

14. Regard a high-pressure region in the northern hemisphere at a latitude of $\phi = 50\,\mathrm{deg}$, with an air density of approximately 1kg/m^3. We have learnt that the geostrophic wind - as well as its refinement, the gradient wind - is parallel to the isobars, and grows with the gradient of the pressure.

    (a) In what direction (as seen from above) does the air flow around the high pressure region described: clockwise or counterclockwise?

1	

    (b) The pressure gradient at a specific location A on the boundary of the high-pressure region is 5 Pa/km. What would be the geostrophic wind at this location?

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    (c) Would the gradient wind be faster or slower than the geostrophic wind at this location?

2	

15. Please describe briefly, in words and without equations, what the difference is between ”supervisory control” and ”dynamic control”.

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points on page: 7	

16. Consider actuator disk theory, with an induction factor a, a freestream wind velocity u_∞, and a constant air density ρ.

(a) What is the axial-direction wind speed at the actuator disk u_1, based on the definition of the induction factor?

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(b) Please describe (briefly, in words, and without equations) how to go about finding the relationship between u_2, u_∞ and a, where u_2 is the axial-direction wind speed far downstream of the actuator disk.

2	

(c) What is the power per unit area (P/A) that an actuator disk can extract?

2	

(d) What optimal induction factor a^* will maximize the power per unit area P/A?

2	

(e) What is the maximum power per unit area $(P/A)^*$ that corresponds to this optimal induction factor?

points on page: 8	