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Exercise Sheet 5: Mechanics of Wind Turbines

In this exercise sheet we'll explore the role of deflections and vibrations in wind turbine design, focusing on the blades and the tower. To accomplish this exploration, we will play briefly with simple Euler-Bernoulli beam theory, the Rayleigh energy method, and the Campbell diagram.

Blade deflection

1. In this problem, we would like to explore the blade deflection. Let's assume that the blade is approximately straight so that it lays more-or-less in the rotor tip plane, even when deflected.

   We will use the same three-bladed demonstration turbine. Turbine A is defined by the following parameters: the rotor radius $R = 50$m, and blades of constant chord $c = 5$m and constant profile shape. Turbine A is running in a free-stream wind of $u_\infty = 12$m/s with air density $\rho = 1.225$kg/m^3. For a solid symmetric airfoil, we can approximate the blade's second moment of area as $I_x \approx K_\mathrm{I} c^4 \tau^3$, where K_I approx 0.036 and $\tau = t_\mathrm{max}/c$ is the maximum airfoil thickness to chord length ratio. Assume that the airfoil non-dimensional thickness τ =24 percent.

   (a) We might make the assumption that the blade behaves as a slender beam. In that case, the downwind-direction blade deflection x could be found with Euler-Bernoulli beam theory from the distributed load q, the Young's modulus E and the cross-section second moment of area I_x:

   $$\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(E I_x \frac{\mathrm{d}^2 x}{\mathrm{d}r^2}\right) = q$$

   There are some boundary conditions to this integral:

   $$x(0) = 0; \quad x'(0) = 0; \quad x''(R) = 0; \quad x'''(R) = 0$$

   Briefly, what do these boundary conditions mean?

   (b) What is the relationship between the downwind-direction blade deflection at the tip and the rotor radius R?

   (c) For "Turbine A", what is the ratio between the tip blade deflection and the rotor radius, if the blade is made of the following materials?

      i. carbon-fiber composite ($E \approx 150$GPa),
      ii. fiberglass aka. glass-reinforced plastic ($E \approx 17$GPa),
      iii. polystyrene ($E \approx 3$GPa)?

   (d) What trade-offs might be relevant when selecting blade material?

   (e) The fact that the blades are rotating will likley lead to a smaller deflection than predicted here. Briefly, why would that be? What is this phenomenon called?

   (f) Qualitatively, what happens to the blade loading under the following conditions?

      i. yawed flow
      ii. shaft tilt
      iii. wind shear
      iv. tower shadow

Preliminary tower design

2. We would like to make a preliminary design of a wind turbine tower. This tower should support an un-yawed and un-tilted three-bladed wind turbine ('Turbine B'), with the following dimensions:

Table 1: wind turbine dimensions and properties for Turbine B

property	symbol	value
tower height	L	84 m
nacelle + hub mass	m_{nac}	143 tonnes
rotor radius	R	12 m
design tip speed ratio	λ_{rated}	5
cut-in wind speed	$u_{\text{cut-in}}$	3 m/s
rated wind speed	u_{rated}	12 m/s
cut-out wind speed	$u_{\text{cut-out}}$	25 m/s

Some other information that you might find useful is as follows:

Table 2: other potentially useful information

property	symbol	value
density of A36 structural steel	ρ_{steel}	$7.8{\cdot}10^3$ kg/m^3
Young's modulus of A36 structural steel	E_{steel}	200 GPa
yield stress of A36 structural steel	U_{steel}	250 MPa
air density	ρ_{air}	1.225 kg/m^3
typical wind turbine structural safety factor	f_{safety}	1.35

(a) **rotor thrust**

i. What is the design angular velocity Ω_{rated} of the wind turbine?

ii. What is the magnitude of the thrust force F on the rotor as a function of u_{rated}?

(b) **tower bending stress**

Let's consider the tower as a simple cantilevered beam, where the rotor thrust is acting at the top of the tower.

Let's assume that the tower is a thin walled tube with a constant cross-section along its length. This constant cross-section is an annulus, with an outer radius of r and a thickness τ.

i. Make a contour plot of the total mass of steel in the tower, based on $r \in [1\text{m}, 6\text{m}]$ and $\tau \in [0\text{m}, 0.15\text{m}]$.

ii. You happen to know that the second moment of area of a filled circular area with radius a is $\pi a^4/4$. What is the second moment of area I_x of the tower cross-section?

iii. What is the distance d between the beam's neutral axis and the outer radius?

iv. What is the bending moment of the tower at the ground due only to the thrust on the rotor M_{T}?

v. What is the maximum stress $\sigma_{\max}$ due to bending on the tower?

vi. Considering the safety factor f_{safety}, please devise a ratio ϕ which indicates whether the tower can safely support the maximum bending stress. Let's define $\phi < 1$ as safe, and $\phi > 1$ as unsafe.

vii. For the following proposed tower outer diameters r, what thickness τ would you propose? Please motivate your choices. Also, please round thicknesses to the nearest 5mm.

A. $r = 5.5$ m

B. $r = 3.0$ m

C. $r = 1.5$ m

(c) **tower natural frequency**

Let's use Rayleigh's energy method to estimate the natural frequency of the tower. In this method, we assume that the strain energy from bending perfectly trades off with the kinetic energy of the tower's displacement x. We will again approximate the tower as a cantilevered beam.

Let's assume that the tower's displacement is sinusoidal in time:

$$x(t) = x_0 \sin(\omega t)$$

and that the tower remains approximately straight during its displacement.

Futher, we know that the strain energy from bending can be found as:

$$E_{\text{pot}} = \frac{1}{2} k x^2, \text{ where } k = 3 \frac{E_{\text{steel}} I_x}{L^3}.$$

i. What is $\dot{x}(t)$?

ii. What is the kinetic energy due to the nacelle displacement T_{nac}?

iii. What is the kinetic energy due to the displacement of the tower T_{t}? (*Hint: the tower is not massless...*) (*Hint: also, you might assume that the deflection of the tower is roughly proportional to the distance to the fixed point.*)

iv. What is the total kinetic energy T of the swaying cantilevered beam?

v. What equation can you formulate, that would implicitly define the vibration frequency ω?

vi. Please find ω.

vii. What is the natural frequency f_{nat} of the cantilevered tower?

viii. What is the natural frequency of each of the three potential tower designs (defined by r and τ) that you determined in Exercise Sheet 3, Problem 2b)? (*Hint: If you do not have this solution, you can use the following combinations of* (r, τ)*:* $(1.5m, 0.05m)$, $(3.0m, 0.015m)$, $(5.5m, 0.005m)$.)

A. $r = 5.5$ m

B. $r = 3.0$ m

C. $r = 1.5$ m

(d) **Campbell diagram**

i. With what frequency (1P, 2P, 3P, ...) would you expect the tower to experience the following effects? What is this frequency (in Hertz), as a function of the wind turbine's rotor speed (in RPM)?

A. 'rotor-rotation' effects, such as having unequally dirty blades?

B. 'blade-passing' effects, such as tower shadow?

ii. Make a plot of frequency [Hz] vs rotor speed [RPM] that we will call the Campbell plot. Add the 'rotor-rotation' and 'blade-passing' frequencies into the Campbell plot. Include a 15 percent safety margin to either side of each curve.

iii. What is the design rotor speed (in RPM) of the wind turbine?

iv. Please show the design rotor speed in the Campbell plot.

v. Please add the tower natural frequencies corresponding to your three possible tower designs (one for each of the outer diameters 5.5m, 3.0m and 1.5m) into the Campbell plot.

vi. Which of the three investigated tower designs (outer diameters 5.5m, 3.0m, 1.5m) can be classified as the following? Please explain briefly.

   A. soft-soft

   B. soft-stiff

   C. stiff-stiff

vii. Suggest some considerations you might have when chosing between your three proposed tower designs?