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Exercise Sheet *4*: Blade Element Momentum Method

In this problem, we want to see how the Blade Element Momentum (BEM) method gives the total thrust on a wind turbine.

To do this, consider an infinitesimally thin annulus (with radius r) sliced from a three-bladed ($B = 3$) wind turbine rotor of radius R. Aassume for the following problem that tip losses can be neglected. We will also again use $\mu = r/R$ the normalized radial position of the annulus.

The effective velocity at the rotor annulus is called $\boldsymbol{W}(r) = W(\sin\phi\hat{\boldsymbol{x}} + \cos\phi\hat{\boldsymbol{t}})$, where $\hat{\boldsymbol{x}}$ points along the axis of rotation in the downwind direction, and $\hat{\boldsymbol{t}}$ points tangentially in the direction of rotation. Assume that the problem is axially symmetric so that all the blades behave identically.

We will use a demonstration turbine called 'Turbine A.' Turbine A is defined by the following parameters: tip speed ratio $\lambda = 7$, the local chord solidity $\sigma(r) = 8/(441\mu)$, the rotor radius $R = 50\text{m}$, the effective velocity angle $\phi = 5\text{deg}$, and the 2D lift and drag coefficients $c_\text{l} = 1$ and $c_\text{d} = 0.01$. Turbine A is running in a freestream wind of $u_\infty = 12\text{m/s}$ with air density $\rho = 1.225\text{kg/m}^3$.

1. **Geometry**

   (a) What is the area $\mathrm{d}A$ of the annulus, if the annulus has a thickness of $\mathrm{d}r$?

   (b) Assume that that the rotor is in a uniform flow field with a freestream wind $\boldsymbol{u}_\infty$ that is aligned with the rotor axis. What is the freestream dynamic pressure q_∞?

   (c) Find the magnitude of the effective velocity W in terms of some parameters of the wind turbine system: the freestream velocity $u_\infty = \|\boldsymbol{u}_\infty\|_2$, the tip speed ratio λ, the annulus radius r, rotor radius R, and the induction factors.

   (d) What is the effective dynamic pressure $q_\text{e}(r)$ based on the magnitude of the effective wind velocity?

   (e) Let's define the chord solidity $\sigma(r)$ as:

   $$\sigma(r) = \frac{B}{2\pi\mu}\frac{c}{R}.$$

   If $c(r)$ is the chord length of the blade at the annulus, what is the area $\mathrm{d}S$ of the blade section at the annulus?

2. **Momentum expressions**

   (a) What is $\mathrm{d}T(r)$ ($\mathrm{d}F_\text{A}$ in the lecture), the change in axial momentum in the flow due to that annulus, in terms of axial a and tangential a' induction factors?

   (b) What is $\mathrm{d}Q(r)$, the change in angular momentum in the flow due to that annulus, in terms of axial a and tangential a' induction factors?

3. **Blade element expressions**

   (a) If you know that the blade section experiences lift ($\mathrm{d}\boldsymbol{L}$) and drag ($\mathrm{d}\boldsymbol{D}$) forces, what is the thrust $\mathrm{d}T(r)$ on the blade section (for one blade)?

   (b) Under the same conditions, what is the the torque $\mathrm{d}Q(r)$ on the blade element?

   (c) Use the 2D lift and drag coefficients c_l and c_d to write your blade element thrust and torque expressions in terms of the defining parameters: B, u_∞, λ, r, R, a, a', ϕ, and σ.

4. **The full rotor thrust**

(a) You happen to learn that the induction factors can be approximated as:

$$a(\mu) \approx (0.8 + 28\mu) \cdot 10^{-2}, \qquad a'(\mu) \approx (0.3 + 0.6/\mu + 2.9\mu) \cdot 10^{-3}$$

What is the thrust distribution over μ on one blade?

(b) For Turbine A, what is the thrust on the whole rotor?

(c) For Turbine A, What is the thrust coefficient C_{T} for the full rotor?