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Exercise Sheet 1

Power Harvesting Factor

Consider a symmetrical, three-bladed ($B = 3$) wind turbine with rotor radius R. Assume a constant angular velocity Ω of the rotor and a uniform wind field with velocity $\boldsymbol{u}_\infty$ so that the dominant wind direction $\hat{\boldsymbol{x}}$ is along the turbine axis of rotation. We will also use a nondimensional spanwise position $\mu = r/R$ that is 0 at the blade root/rotor hub, and 1 at the blade tips.

1. What is the tip speed ratio λ of the turbine?

2. What is the local speed ratio λ_r at some spanwise location μ?

3. What is the effective wind (also called apparent velocity) $\boldsymbol{u}_\mathrm{a}$ at the position μ?

4. Sketch the velocity triangles for the following positions:

   (a) $\mu = 0.1$
   (b) $\mu = 0.9$

5. You've heard that the lift c_l and drag c_d coefficients are related to the angle of attack α. Assume that the blades are uniformly pitched with an angle ϕ, but have a 'perfect' twist distribution $\theta(\mu)$ so that α always takes its design value of 6 degrees if $\phi = 0$. What is $\theta(\mu)$?

6. For arbitrary lift c_l and drag c_d coefficients, what is the aerodynamic force $\mathrm{d}\boldsymbol{F}_\mathrm{aero}$ for an infinitesimal segment of area $\mathrm{d}A$ around a position μ? Assume that the blades point straight, radially outwards.

7. What is the mechanical power production $\mathrm{d}P(\mu)$ of that segment around position μ?

8. If the lift c_l and drag c_d coefficients can be found with the following relations, what is the power harvested by the blade segment around position μ?

$$c_\mathrm{l}(\mu) = 1.2\mu, \qquad \frac{c_\mathrm{l}}{c_\mathrm{d}}(\mu) = 100\mu$$

9. What is the relationship between the power harvesting factor ζ and μ?

10. How would you go about finding the total power P harvested by the entire turbine? (*Hint: just give the procedure; don't follow it yet.*)

11. How would you go about finding the power coefficient C_P of the entire turbine? Use the following definition: $\mathrm{d}A = c(\mu)\mathrm{d}\mu R$, where $c(\mu)$ is a chord length as a function of μ.

12. If we use the above model that we've described to this point, for some given parameter values ($\lambda = 7$, $c_0 = 0.15R$, $c_1 = 0.05R$, $u_\infty = 10$ m/s, $\rho = 1.225$ kg/m^3, $R = 50$ m and $B = 3$), can you find how much power the full turbine will extract?

    *here assume that the chord is a linear interpolation between the chord c_1 at the tip and the chord c_0 at the root: $c(\mu) = c_0 + (c_1 - c_0)\mu$, what gives us $\mathrm{d}A = (c_0 + (c_1 - c_0)\mu)\,\mathrm{d}\mu R$

    (a) plot the power harvesting factor ζ vs. μ
    (b) find how much power the full turbine will extract