Systems and Control II (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 10: Discrete controller design

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1. Compute the discrete equivalents of the controller
$$K(s) = \frac{a}{s+a} ,$$
where a is a given parameter, using a) the forward rectangular rule, b) the backward rectangular rule, and c) the trapezoid rule (Tustins method). Consider the sample time T_p also as a given parameter.

2. Use the zero-pole matching method to compute the discrete equivalent of the controller
$$K(s) = \frac{a}{s+a} ,$$
where a is a given parameter. Consider the sample time T_p also as a given parameter. It is desired that the discrete controller shows no delay in its discrete time response.

3. Use the ZOH-method to compute the discrete equivalent of the controller
$$K(s) = \frac{a}{s+a} ,$$
where a is a given parameter. Consider the sample time T_p also as a given parameter.

4. Design a discrete controller for a DC-motor that is preceeded by a zero-order hold (see 1), so that the closed-loop system has an overshoot of no more than 20%, a rise time $T_\mathrm{r} < 0.3$ s, and a settling time of not more than 2s. Use the discrete root locus method to evaluate different controller types and to tune the parameters of the appropriate controller. The sampling time is $T_\mathrm{P} = 0.1$ ms . The DC-motor can be approximately described in continuous-time by
$$G(s) = \frac{1}{s(s+1)} \quad .$$

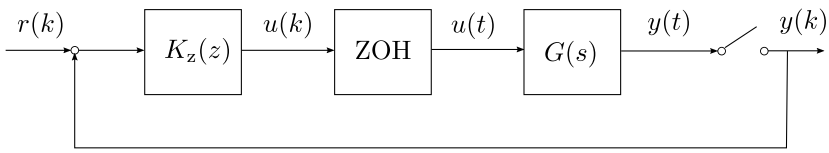

Figure 1: Diagram of a discrete time system

   (a) Assume that the closed-loop continuous system can be approximated by a dominant pole pair. Translate the specifications of the continuous closed-loop system into corresponding requirements on the step response, using the dynamic behavior heuristics in Table 1.

   (b) Discretize the DC-motor preceeded by the ZOH.

   (c) (MATLAB) Consider a proportional controller $K_\mathrm{z}(z) = k_\mathrm{p}$. Plot the root locus of the closed-loop system with respect to k_p. Which value of k_p allows us to meet the design objectives?

   (d) (MATLAB) Consider a lead compensator $K_\mathrm{z}(z) = k_\mathrm{p}\frac{z-z_{01}}{z-z_1}$. Choose the parameters z_{01}, z_1, and k_p so that the design objectives are met.
   Hint: Use the MATLAB-function `rltool` and activate the grid over the context menu.

   (e) What is the steady state error for a step input of the closed-loop system using the lead-compensator from d).

Peak time T_{m}	$\frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$
Rise time T_{r}	$\frac{1.8}{\omega_0}$
Settling time $T_{5\%}$	$\frac{3}{\zeta\omega_0}$
Settling time $T_{2\%}$	$\frac{4.5}{\zeta\omega_0}$

ϕ_ζ	ζ	Δh
66°	0.4	25%
54°	0.58	10%
45°	0.7	5%
37°	0.8	2%

Table 1: Dynamic behavior heuristics of a second order system with complex conjugate poles $\zeta\omega_0 \pm j\omega_0\sqrt{1-\zeta^2}$ for $\zeta < 0.8$.

Nr.	function $f(t)$ with f(t) = for t¡0	$F(s) = \mathcal{L}\{f(t)\}$	$F(z) = \mathcal{Z}\{f(k)\}$ with $f(k) = f(kT_\mathrm{p})$	
1	$\sigma(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	
2	t	$\frac{1}{s^2}$	$\frac{T_\mathrm{p} z}{(z-1)^2}$	
3	t^2	$\frac{2}{s^3}$	$\frac{T_\mathrm{p}^2 z(z+1)}{(z-1)^3}$	
4	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT_\mathrm{p}}}$	
5	te^{-at}	$\frac{1}{(s+a)^2}$	$\frac{\alpha T_\mathrm{p} z}{(z-\alpha)^2}$	$\alpha = e^{-aT_\mathrm{p}}$
6	$1 - e^{at}$	$\frac{a}{s(s+a)}$	$\frac{(1-\alpha)z}{(z-1)(z-\alpha)}$	$\alpha = e^{-aT_\mathrm{p}}$
7	$\frac{1}{a}\left(at - 1 - e^{-at}\right)$	$\frac{a}{s^2(s+a)}$	$\frac{z[(aT_\mathrm{p}-1+\alpha)z+(1-\alpha-aT_\mathrm{p}\alpha)]}{a(z-1)^2(z-\alpha)}$	$\alpha = e^{-aT_\mathrm{p}}$
8	$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$\frac{z[z-\alpha(1+aT_\mathrm{p})]}{(z-\alpha)^2}$	$\alpha = e^{-aT_\mathrm{p}}$
9	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{\beta z}{z^2-2\gamma z+1}$	$\beta = \sin(\omega T_\mathrm{p})$ $\gamma = \cos(\omega T_\mathrm{p})$
10	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z^2-\gamma z}{z^2-2\gamma z+1}$	$\gamma = \cos(\omega T_\mathrm{p})$
11	$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{\alpha\beta z}{z^2-2\alpha\gamma z+\alpha^2}$	$\alpha = e^{-aT_\mathrm{p}}$ $\beta = \sin(\omega T_\mathrm{p})$ $\gamma = \cos(\omega T_\mathrm{p})$

Table 2: Table of Laplace- and z-Transforms