Systems and Control II (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 8: Description of Digital Systems and Digitization

(Thursday 17.12.2015 at 15:00 in Room SR 00 014)

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1. Consider the discrete time system, shown in Fig. 1, that maps a discrete input signal $u(k)$ on the discrete output signal $y(k)$.

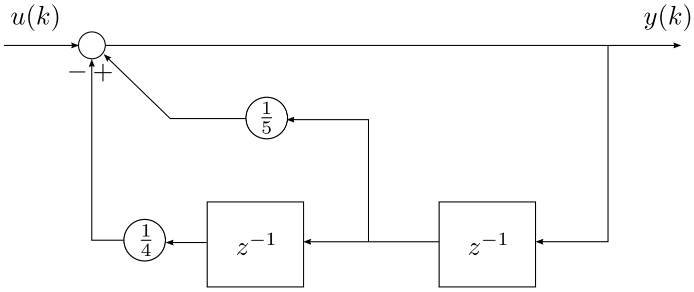

Figure 1: Diagram of a discrete time system

(a) Formulate the difference equation of this system.

(b) Determine the characteristic equation for this system and calculate the solutions of this equation.

(c) Compute the homogeneous solution $y_{\mathrm{nat}}(k)$ of the difference equation for $y(-1) = 0$ and $y(0) = 1$.

(d) Sketch the impulse response $g(k)$ of this system for $k = 1$ to 10.

(e) The discrete time step response $h(k)$ is the system response to the discrete step input $\sigma_{\mathrm{d}}(k)$, defined by

$$\sigma_{\mathrm{d}}(k) = \begin{cases} 0 & \text{if } k < 0 \\ 1 & \text{if } k \geq 0 \; . \end{cases}$$

Derive the relation between $h(k)$ and the impulse response $g(k)$.
Hint: Use the convolution sum as a starting point.

2. The speed of a micro-servo motor $G(s)$ is controlled by a PID-controller $K(s)$. The transfer functions are determined by

$$G(s) = \frac{360000}{(s+60)(s+600)} \text{ and } K(s) = k_{\mathrm{p}}(1 + \frac{1}{T_{\mathrm{i}}s} + T_{\mathrm{D}}s) \; .$$

The parameters of the PID-controller are tuned as $k_{\mathrm{p}} = 5$, $T_{\mathrm{D}} = 0.0008$ sec and $T_{\mathrm{i}} = 0.003$ sec, and give satisfactory continuous-time closed-loop dynamics. Our goal now is to transform the continuous controller to a digital controller.

(a) Sketch the Bode diagram of the open-loop system and estimate the bandwidth of the closed-loop system.

(b) Choose a suitable sampling time constant T_{s} for this system.

(c) Derive a digital control law, i.e., a difference equation for the digitized controller, using the backwards Euler method for the approximation of derivatives:

$$\dot{u}(kT_{\mathrm{s}}) \approx \frac{u(kT_{\mathrm{s}}) - u((k-1)T_{\mathrm{s}})}{T_{\mathrm{s}}} \; .$$

(d) Draw a diagram that realizes the digitized linear controller by using only multiplier and shift operators.

(e) (**Extra:**) Compare the closed-loop step response for the continuous and the digitized controller with `MATLAB` and evaluate.