Systems and Control II (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 12: Full state feedback control

(Thursday 28.01.2016 at 15:00 in Room SR 00 014)

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1. The dynamics of a certain continuous-time SISO system can be described by a state space model in Kalman-decomposed form:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & -0.5 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}(t) + u(t)\ .$$

   (a) Identify the uncontrollable and unobservable states of the system

   (b) If the system is not controllable, is it stabilizable? If the system is not observable, is it detectable?

   (c) Compute the transfer function $G(s)$ of the system, based on this state space model.

2. Consider a continuous-time system with a single input signal, with the following state equation:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 2 & -3 \\ 0.5 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ -3 \end{bmatrix} u(t)$$

   Design via pole placement a static linear state feedback controller with feedback matrix $\mathbf{K} = \begin{bmatrix} k_0 & k_1 \end{bmatrix}$, so that the poles of the closed-loop system correspond to the desired pole locations $\bar{\lambda}_{\text{cl},i} = -1 \pm j\frac{1}{3}$.

   (a) Compute the eigenvalues of the uncontrolled system. Is this system Lyapunov stable?

   (b) Is this system controllable?

   (c) Choose the controller constants k_0 and k_1 in order to meet the pole location requirements, by equating the coefficients of the closed-loop characteristic polynomial with the coefficients of the desired characteristic polynomial.

   (d) (`MATLAB`) Check the resulting controller constants with the command `acker`. Plot the natural response of the closed-loop system and evaluate.

3. Consider the continuous-time MIMO-system, described by the state space model:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 3 \\ 5 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{u}(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \mathbf{x}(t)$$

   Using cyclic control, design a static full state feedback controller for this system so that the eigenvalues of the closed-loop system are $\bar{\lambda}_{\text{cl,i}} = \begin{bmatrix} -1 \pm j0.5 & -3 & -4 \end{bmatrix}$.

   (a) Choose a random vector $\mathbf{q}$ that computes the system input signal $\mathbf{u}(t)$ from the control signal $\bar{u}(t)$ (with $\mathbf{u}(t) = \mathbf{q}\bar{u}(t)$). Is the modified system controllable? If not, iterate.

   (b) Write down the expression for the controller matrix $\mathbf{K}$ that ensures that the eigenvalues of the closed-loop system lie at the desired pole locations $\bar{\lambda}_{\text{cl,i}}$.
   Hint: Use Ackermann's formula for controller design.

   (c) (`MATLAB`) Compute $\mathbf{K}$ with the function `acker` and simulate the natural response of the closed-loop system with the function `lsim`. Iteratively tune the vector $\mathbf{q}$ in order to optimize the dynamic behaviour of the closed-loop system.

   (d) (`MATLAB`) Calculate the controller matrix $\mathbf{K}$ for the static full state feedback controller by eigenstructure assignment, using `place` command of `MATLAB`. Compare the dynamic behaviour of the obtained closed-loop system with that from the closed-loop system designed in (c).