Systems and Control II (SC2)
Albert-Ludwigs-Universität Freiburg – Wintersemester 2015/2016

Exercises 11: Description and analysis of MIMO-systems
(Thursday 21.01.2016 at 15:00 in Room SR 00 014)

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1. Consider the continuous-time MIMO system shown in Fig. 1.

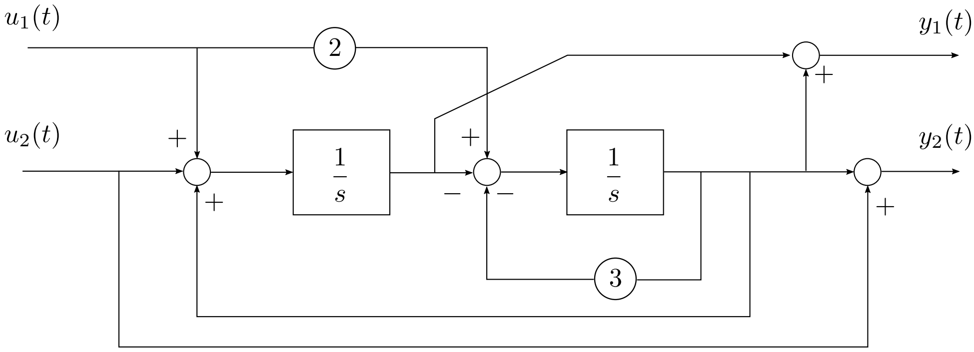

Figure 1: Diagram of a continuous-time MIMO system

   (a) Find a state space model that describes the dynamics of this system.
   (b) Compute the eigenvalues and -vectors of the state matrix and transform the state space model into the Jordan form.
   (c) Is this system controllable?
   (d) Is this system observable?
   (e) Compute the transfer function matrix for this system. Use the state space model from (a) or (b) for this, depending on which one is the most convenient.

2. Consider the digital SISO system shown in Fig. 2.

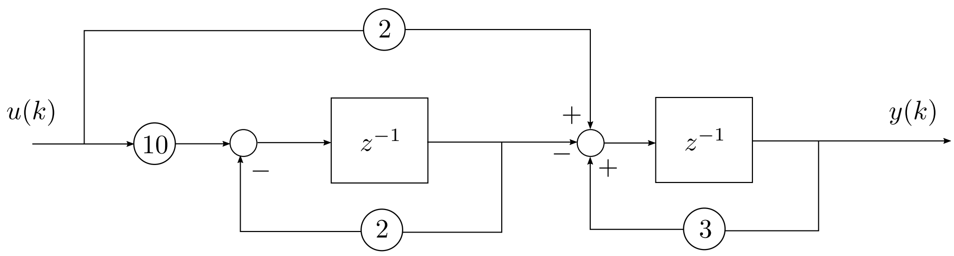

Figure 2: Diagram of a digital SISO system

   (a) Find a state space model that describes the dynamics of this system.
   (b) Compute the eigenvalues and -vectors of the state matrix and transform the state space model into the Jordan form.
   (c) Is this system controllable?
   (d) Is this system observable?
   (e) Compute the transfer function for this system.

3. Consider the MIMO feedback system shown in Fig. 3. The transfer function matrices of the partial systems are respectively

$$\mathbf{G}_{\mathrm{R}}(s) = \begin{bmatrix} \frac{1}{s+5} & \frac{1}{s+3} & 0 \\ 0 & \frac{0.5}{s+2} & \frac{0.5}{s+1} \end{bmatrix}, \ \mathbf{G}_{\mathrm{Y}}(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & \frac{1}{s^2+1} \\ 3 & 2 \end{bmatrix},$$

$$\mathbf{G}_{\mathrm{S}}(s) = \begin{bmatrix} [0 & 1 & \frac{1}{s+2} & 1 \\ 1 & 0 & 0 & \frac{4}{s+1} \end{bmatrix}, \ \mathbf{G}_{\mathrm{M}}(s) = \begin{bmatrix} 1 & 0 \\ 3 & 1 \\ 0 & 1 \end{bmatrix}.$$

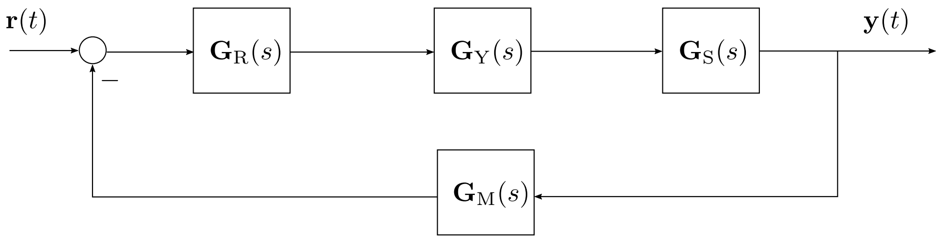

Figure 3: Diagram of a continuous-time MIMO feedback system

(a) Derive an expression for the closed-loop transfer function $\mathbf{G}_{\mathbf{r}}(s)$ for this set-up.

(b) (`MATLAB`) Compute $\mathbf{G}_{\mathbf{r}}(s)$ for this system and determine its poles. Is the closed-loop system BIBO-stable?
Hint: Use the the function `feedback` to compute $\mathbf{G}_{\mathbf{r}}(s)$ and compare it with the result you get when using the expression derived in (a).

(c) (`MATLAB`) Plot the impulse and step response of the closed-loop system.