Exercises for Lecture Course on Numerical Optimization (NUMOPT)
Albert-Ludwigs-Universität Freiburg – Winter Term 2023/2024

Exercise 5: Exam Type Question

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Exercise Tasks

1. **A sample exam question.**

   Regard the following minimization problem:

$$\min_{x \in \mathbb{R}^2} \quad x_2^4 + (x_1+2)^4 \quad \text{s.t.} \quad \begin{cases} x_1^2 + x_2^2 & \leq & 8 \\ x_1 - x_2 & = & 0. \end{cases}$$

   (a) How many scalar decision variables, how many equality, and how many inequality constraints does this problem have?

2	

   (b) Sketch the feasible set $\Omega \in \mathbb{R}^2$ of this problem.

3	

   (c) Bring this problem into the NLP standard form

$$\min_{x \in \mathbb{R}^n} f(x) \quad \text{s.t.} \quad \begin{cases} g(x) & = & 0 \\ h(x) & \geq & 0 \end{cases}$$

   by defining the functions f, g, h appropriately.

3	

FROM NOW ON UNTIL THE END TREAT THE PROBLEM IN THIS STANDARD FORM.

(d) Is this optimization problem convex? Justify.

2

(e) Write down the Lagrangian function of this optimization problem.

2

(f) A feasible solution of the problem is $\bar{x} = (2, 2)^T$. What is the active set $\mathcal{A}(\bar{x})$ at this point?

2

(g) Is the *linear independence constraint qualification (LICQ)* satisfied at $\bar{x}$? Justify.

3

(h) An optimal solution of the problem is $x^* = (-1, -1)^T$. What is the active set $\mathcal{A}(x^*)$ at this point?

1

(i) Is the *linear independence constraint qualification (LICQ)* satisfied at x^* ? Justify.

2

(j) Describe the tangent cone $T_\Omega(x^*)$ (the set of feasible directions) to the feasible set at this point x^*, by a set definition formula with explicitly computed numbers.

2

(k) Compute the Lagrange gradient and find the multiplier vectors λ^*, μ^* so that the above point x^* satisfies the KKT conditions.

4	

(l) Describe the critical cone $C(x^*, \mu^*)$ at the point (x^*, λ^*, μ^*) in a set definition using explicitly computed numbers

3	

(m) Formulate the second order necessary conditions for optimality (SONC) for this problem and test if they are satisfied at (x^*, λ^*, μ^*). Can you prove whether x^* is a local or even global minimizer?

4	