Exercises for Lecture Course on Modeling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2017

Exercise 6: Maximum Likelihood Estimation

Prof. Dr. Moritz Diehl, Robin Verschueren, Alex Resch

In this exercise you deepen your knowledge of maximum likelihood estimation.

Exercise Tasks

1. Assume an experiment where the scalar measurements $y(k) = x + \epsilon(k), k = 1, \ldots, N$ have zero-mean and i.i.d. Gaussian noise $\epsilon \sim \mathcal{N}(0, \sigma^2)$. Show that the maximum likelihood (ML) estimator of x is equal to the linear least squares (LLS) estimate.

2. **Computer exercise with MATLAB - Estimation on wind distribution** (10 points)

   Imagine the municipial energy supplier of Freiburg is planning to set up a wind farm on Feldberg. Hence, they want to check whether a wind farm at Feldberg would be profitable or not. In wind energy, one is usually interested in the average power that can be produced at a specific location with a specific wind turbine. A very important quantity for estimating the average power of a wind turbine is the wind speed that varies according to the weather condition and location.

   Due to the high complexity, a deterministic wind model would be very difficult to obtain. However, it has been observed that the magnitude of the wind speed throughout a year follows a distribution given by the following conditional probability density function:

$$p(v|\lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{v}{\lambda}\right)^{k-1} e^{-(v/\lambda)^k} & v \geq 0 \\ 0 & v \leq 0. \end{cases} \tag{1}$$

   This distribution is known as the Weibull distribution, where $\lambda > 0$ and $k > 0$ are the parameters of the distribution and v is the magnitude of the wind speed at the turbine location. The parameters λ and k depend on the location. The task of this sheet will be to estimate the parameters λ and k using wind speed data taken at Feldberg.

   (a) Formulate the negative log-likelihood function $-L(\lambda, k)$ given N measurements of wind speed $v_1, \ldots, v_N$. Simplify this function as much as possible. (3 points)

   (b) State the minimisation problem to estimate λ and k and simplify again the objective function. *Hint: constant terms in the objective function do not alter the solution of a minimisation problem.* (1 point)

   (c) Import the wind speed data of the German Weather Service taken on Feldberg (dataset with index 01346) from the following link:

   ftp://ftp-cdc.dwd.de/pub/CDC/observations_germany/climate/hourly/wind/historical/

   Only use the measurements taken in 2015. Have a look at the dataset description to get a better understanding of the dataset. Plot the probability density function of the wind speed data using the `histogram` function. (1 point)

(d) Solve the minimisation problem obtained in Task (b) to estimate λ and k. To do that you should use the function `fmincon` of MATLAB. Take a look to the following website to learn how to use fmincon: http://www.mathworks.com/help/optim/ug/fmincon.html Keep in mind that the minimisation has the objective function that you derived in Task (b) but also a constraints on the parameters λ and k. Plot the fitted Weibull distribution into your histogram. (3 points)

The main motivation on estimating these parameters is the study of the average power that a certain turbine will produce in a certain location. In order to do that, the expected value $\mathbb{E}\{P_{\text{Power}}(v)\}$ of the power profile $P_{\text{Power}}(v)$ of the turbine is calculated using the wind distribution (eq. (1)). As an illustration, a typical power profile of a wind turbine is given in Figure 1.

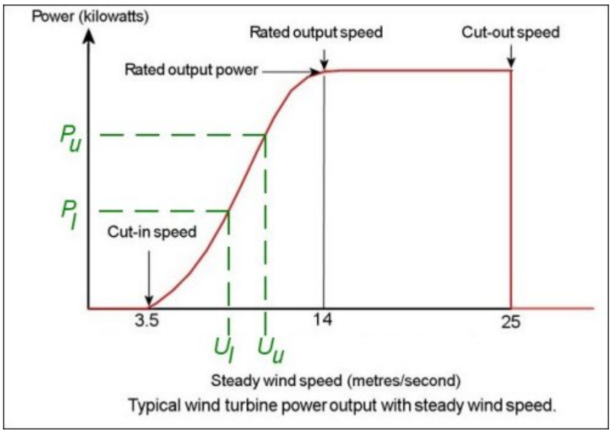

Figure 1: Wind turbine power profile[1]

(e) Using the data of the power curve of a specific wind turbine given in table (1), compute the expected value of the turbine power on the studied location. Use the trapezoidal rule for integration to compute the expected value of the power. *Hint: First compute the power distribution.* (2 points)

Table 1: Power curve data

Wind speed (m/s)	0	1	2	3	4	5	6	7	8
Output power (kW)	0	0	3	25	82	174	321	532	815
Wind speed (m/s)	9	10	11	12	13	14	15	16	17
Output power (kW)	1180	1580	1900	2200	2480	2700	2850	2950	3020
Wind speed (m/s)	18	19	20	21	22	23	24	25	> 25
Output power (kW)	3020	3020	3020	3020	3020	3020	3020	3020	0

[1] Source: http://www.wind-power-program.com/Images/wind_statistics.htm/power%20profile%20graph.jpg