Mixed Integer Model Predictive Control for Renewable Energy Systems
– Formulations and Applications –

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Outline of talk

1. Introduction to MIMPC
   - Motivation
   - Formulations and solution methods
   - MIP Modelling techniques
   - MI-NMPC Applications in RES

2. MPC for district heating networks with multiple decentral heat producers
   - Primer on control in district heating networks
   - Problem formulation
   - Real world application: Heating network in Weil am Rhein

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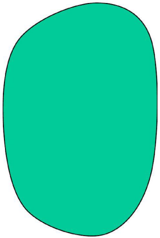
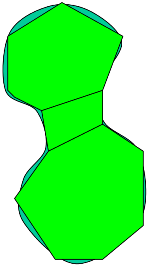
Motivation

- What is Mixed-integer Programming?
  - MIP is an optimization method that combines continuous and discrete variables
- Why is it useful?
  - MIP can model complex planning and control problems involving both continuous and discrete decisions
- Why now? Is MIP new?
  - MIP is not a new concept, BUT online use ($\rightarrow$ MPC) has only arrived with fast computers and software

Classes of optimization problems

(nonlinear) convex	non-convex	integer
		see table below
QP/NLP	Can approx. by MIP	MIP

	T	F
T	✗	✓
F	✓	✓

Adopted from Mixed-integer Programming for Control notes by Arthur Richards and Jonathan How `https://seis.bristol.ac.uk/~aeagr/acc05_tut_mip.pdf`

Modelling with integer variables

If the variable is associated with a physical entity that is indivisible, then it must be integer: number of wind turbines to be placed, to take or not to take an (expensive) measurement, ...

We can use integer or 0–1 (binary) variables to

- logic: model decisions (yes/no), disjunctions (either-or), implications (if-then)
- plant dynamics: on-off, minimum on-time/off-time, number of on-times, minimum power,...
- model discontinuous dynamics (to some extend): phase change material (PCM) storage, change in flow direction

And, in the context of general modeling techniques:

- to model piecewise linear or affine (continuous) functions
- to convexify/linearize nonlinear/nonconvex dynamics

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MIOCP formulation

$$\begin{array}{rrcll}
\min\limits_{x,u,v} & & & J(x,u,v) & & \\
\text{s.t.} & \dot{x}(t) & = & f(x(t),u(t),v(t)), & t \in [0,T], & \text{(ODE model)} \\
& x(0) & = & x_0, & & \text{(fixed initial value)} \\
& 0 & \geq & h(x(t),u(t),v(t)), & t \in [0,T], & \text{(path constraints)} \\
& v(t) & \in & \mathbb{Z}^{n_v} & & \text{(integer controls)}
\end{array}$$

- Usual assumption: J, f, h sufficiently smooth
- Can equivalently be formulated with binary controls $b(t) \in \{0,1\}^{n_b}$
- LMPC or special structure makes problem easier to solve, e.g.: switched nonlinear system

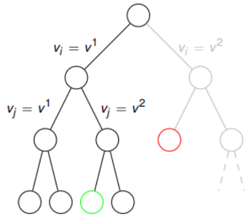
$$\dot{x}(t) = f_0(x(t),u(t)) + \sum_{i=1}^{n_b} b_i f_i(x(t),u(t)) \quad \text{where } b(t) \in \{0,1\}^{n_b}$$

with special ordered set type 1(SOS1) constraint $\sum_{i=1}^{n_b} b_i = 1$

Optimal control methods for solving MIOCP

- Numerical approaches
  - Dynamic Programming
    → "Curse of dimensionality"
  - Indirect Methods
    → BVP difficult to solve
  - Direct Methods
    → (MINLP)

- (MINLP) generally *NP*-hard
- Branch-and-bound with NLP subproblems:



MIPs

MINLP

$$\begin{aligned} \min_{x\in\mathcal{R}^n, z\in\mathcal{Z}^m} \quad & f\left(x,z\right) \\ \text{s.t.} \quad & 0 = g(x,z), \\ & 0 \geq h(x,z) \end{aligned}$$

MILP

$$\begin{aligned} \min_{x\in\mathcal{R}^n, z\in\mathcal{Z}^m} \quad & f_1^\top\left(x\right) + f_2^\top\left(z\right) \\ \text{s.t.} \quad & A_1 x + A_2 z \leq b \end{aligned}$$

Solution methods (MILP & MINLP):

- Branch & Bound (B&B): divide and conquer approach
- Other methods (e.g. Cutting Plane) and extensions (e.g. Branch & Cut) are used to accelerate solution.
- For MINLP: convex relaxations and other decomposition methods
- Software: HiGHS (open-source, MILP/MIQP), SCIP (open-source), BONMIN (Basic Open-source Nonlinear Mixed Integer), CPLEX, Gurobi, MOSEK; most are mature only for MILP, convex MINLP

Branch & Bound (B&B)

- Systematically explore search space by branching on variables and pruning sub-optimal branches using previously found solution bounds + techniques to handle nonlinearities (interval arithmetic, convex hull relaxations, LP relaxations)
- Algorithm: search the enumeration tree, but at each node:
  1. Solve the relaxed problem at the node (e.g. with LP or conxev optimization solver)
  2. Eliminate the subtree (fathom it) if
     - The solution is integer (there is no need to go further), or,
     - the best solution in the subtree cannot be as good as the best available solution (the incumbent), or,
     - there is no feasible solution.
- Challenge non-convex MINLP: computationally intensive + absence of efficient global optimization methods (no good bounds for pruning)

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Piecewise affine functions

- Any continuous nonlinear function can be arbitrarily well approximated by a Piecewise Affine (PWA) function
- Any PWA function can be encoded using MILP
- Advantage of the MILP approach over NLP: handling of non-convexity

Minimization of a continuous PWA function $f(x), X_1 \leq x \leq X_N$ can be written as MILP:

$$\min_x \quad \sum_{i=1}^{N} y_i F_i$$

$$\text{s.t.} \quad \sum_{i=1}^{N} y_i = 1$$

$$y_i \geq 0 \quad \forall i \in \{1 \dots N\}$$

$$y_1 \leq z_1$$

$$y_i \leq z_{i-1} + z_i \quad \forall i \in \{2 \dots N-1\}$$

$$y_N \leq z_{N-1}$$

$$\sum_{i=1}^{N-1} z_i = 1, z_i \in \{0,1\}$$

where

- values at intermediate points $f\left(X_i\right) = F_i$
- binary variable $z_i = 1$ if $X_i \leq x \leq X_{i+1}$ (choosing which interval the solution lies in)
- "multipliers" y_i and y_{i+1} of the function values at the ends of that interval
- $f(x) = y_i F_i + y_{i+1} F_{i+1}$ corresponding to $x = y_i X_i + y_{i+1} X_{i+1}$

Disjunctions and Non-convex Sets

One of two constraints must be satisfied, e.g., either $a_1^T x \leq b_1$ or $a_2^T x \leq b_2$

MILP formulation:

$$
\begin{aligned}
&a_1^T x \leq b_1 + M z_1 \\
&a_2^T x \leq b_2 + M z_2 \\
&z_1 + z_2 \leq 1
\end{aligned}
$$

where

- z_1 and z_2 are additional binary variables
- M is a very large positive number ("big- M ")
- if M was chosen sufficiently large, this constraint is effectively relaxed
- the binary variables z_1 and z_2 encode a choice of which constraint to apply, and the logical constraint ensures that at least one of them is applied.

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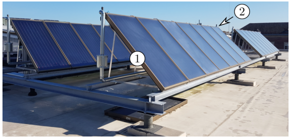
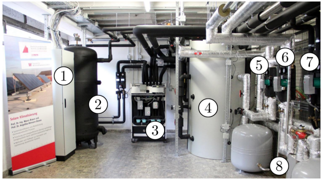
MIP in Optimal Control

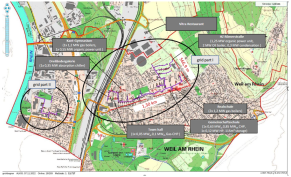
Modeling of practical applications often leads to nonlinear differential equations with integer controls/dynamics.

Typical examples in RES:

- Valve/plant is either open or closed
- Minimum thermal power if plant is on
- Minimum up/down times
- Different operating modes, e.g. electric battery mode (in)active, ice-storage mode vs. water storage mode, free cooling mode is on vs. mechanical cooling
- One out of multiple alternative producers is active







Example 1: (at most) One of multiple producers is active

Prosumer-based solar district heating networks

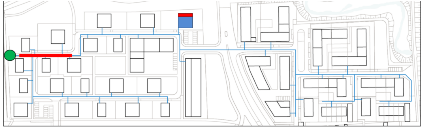
- optimal heat distribution between prosumers in the network
- binary controls: $b = b_{\mathrm{dhn}} \cup \left(\{b_{i,j}\}_{1 \le i,j \le M, i \ne j}\right)$
  $b_{\mathrm{dhn}} = 1$: DHN mode is activated,
  $b_{i,j} = 1$: heat transfer from building i to j on
- at most one operation at each time point: either central heat provider (DHN) or building-level heat transfer

$$b_{\mathrm{dhn}} + \sum_{1 \le i,j \le M, i \ne j} b_{i,j} \le 1$$

- bilinear dependence (nonlinear), e.g.:

$$\dot{Q}_{i,j}(t) = b_{i,j}(t)\dot{m}(t)c_{\mathrm{w}}\left(T_i(t) - T_j(t)\right)$$

Heat transfer between two buildings: $b_{\mathrm{dhn}} = 0$



Central heat provider mode: $b_{\mathrm{dhn}} = 1$

Example 2: Pressure loss in heating networks

The mass flow in a pipe is determined by the nodal pressure of the interconnection nodes

$$\text{sign}(f_{i,j})f_{i,j}^2 = \frac{1}{K}(p_i - p_j)$$

where $f_{i,j} \equiv \dot{m}_{i,j}$ denotes the mass flow of pipe $i - j$, p_i is the nodal pressure of node i in the heating network and K is the resistance coefficients of the pipe (depends on roughness and diameter of pipe) [1]

- $\text{sign}(f_{i,j})f_{i,j}^2$ is non-convex depending on flow direction $f_{i,j} \geq 0$ or $f_{i,j} \leq 0$

[1] This holds under certain simplifications: horizontal pipelines, laminar flow,...

Replacing the signum function: the NLP way

- Assume $|f_{i,j}| \leq F$ for all i, j
- Binary variable $z_{i,j}$ to indicate the flow direction

$$z_{ij} = \begin{cases} 1 & f_{ij} \geq 0 \quad f_{ij} \geq -F\left(1 - z_{ij}\right) \\ 0 & f_{ij} \leq 0 \quad f_{ij} \leq F z_{ij} \end{cases}$$

- Since

$$\operatorname{sign}\left(f_{ij}\right) = 2z_{ij} - 1$$

we can replace the original non-convex function with

$$\left(2z_{ij} - 1\right) f_{ij}^2 - \frac{1}{K}\left(p_i - p_j\right) = 0.$$

Replacing the signum function: the MIP way

Summarized:

- $z_{ij} \in \{0,1\}$: Indicator for positive flow $f_{ij} > 0$
- $y_{ij} \in \{0,1\}$: Indicator for negative flow $f_{ij} < 0$

Constraints:

$$f_{ij} > 0 \Rightarrow \begin{cases} f_{ij}^2 \leq \frac{1}{K}(p_i - p_j) \\ f_{ij}^2 \geq \frac{1}{K}(p_i - p_j) \end{cases}$$

$$f_{ij} < 0 \Rightarrow \begin{cases} f_{ij}^2 \leq \frac{1}{K}(p_j - p_i) \\ f_{ij}^2 \geq \frac{1}{K}(p_j - p_i) \end{cases}$$

Turn on/off using big-M/small-m formulation.

$$\begin{aligned} f_{ij} &\leq F z_{ij} \\ f_{ij} &\geq -F y_{ij} \\ z_{ij} + y_{ij} &= 1 \\ f_{ij}^2 + M z_{ij} &\leq M + \frac{1}{K}(p_i - p_j) \\ f_{ij}^2 + m z_{ij} &\geq m + \frac{1}{K}(p_i - p_j) \\ f_{ij}^2 + L y_{ij} &\leq L + \frac{1}{K}(p_j - p_i) \\ f_{ij}^2 + \ell y_{ij} &\geq \ell + \frac{1}{K}(p_j - p_i) \end{aligned}$$

We obtain a convex function. Such reformulations are neither obvious nor always possible.

Example 3: Unit commitment problem in heating networks

The unit commitment problem in thermal power production deals with the cost optimal scheduling with forecasted demands of on/off decisions and output levels for generating units in a thermal power system over a certain time horizon.

Hybrid ($\equiv$ mixed-integer/discrete) constraints:

- Min./Max. power
- Ramp-down and shutdown ramp limits
- Discrete output levels
- Minimum up- and down-time: min. up-time $\equiv$ minimum number of time steps the unit has to stay committed for once it is switched on

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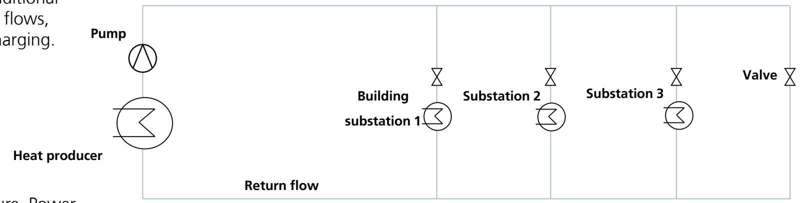
What can be controlled in DH?

- PUMPS: Differential Pressure Control
  - Manipulated Variable: Pump Speed
  - Controlled Variable: Differential Pressure
  - Disturbance Variable: changing pressure losses in the network due to opening/closing of valves (substations) Possibly additional constraints due to min/max mass flows, min/max temperature, storage charging.

- BUILDING SUBSTATIONS: Mass Flow Control for Heat Transfer
  - Manipulated Variable: Valve Position at each transfer station
  - Controlled Variable: Mass flow on the network side into the transfer station / Heat output to the transfer station
  - Disturbance Variable: changing heat demand on the building side.



- **HEAT GENERATORS**: Flow Temperature, Power
  - Manipulated Variable: Generator Output
  - Controlled Variable: Flow Temperature (usually based on heating curve)
  - Disturbance Variable: changes in mass flow / return temperature.

- HEATING/DHW: Room Temperature Control in the Building
  - Manipulated Variable: Thermostatic Valve Position / Mixing Valve Faucets
  - Controlled Variable: Room Temperature / Domestic Hot Water Temperature
  - Disturbance Variable: Outside Temperature / Solar Gains / Individual Behavior of the Occupants.

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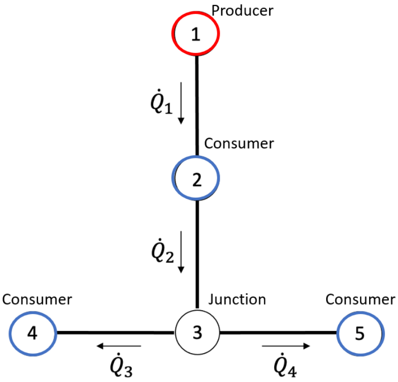
Network topology

- $\mathcal{N}$: set of nodes (producers, consumers or junctions)
- $\mathcal{P}$: set of pipes (edges) with $\mathcal{P}_n \subseteq \mathcal{P}$ set of pipes adjacent to node n
- $\mathcal{N}_{TU} \subseteq \mathcal{N} = \{i_1, \dots, i_{N_{TU}}\}$ indices of thermal power generation units
- $\mathcal{C} \subseteq \mathcal{N}$ indices of consumer nodes



- $\mathcal{N} = \{1, 2, 3, 4, 5\}$
- $\mathcal{P} = \{1, 2, 3, 4\}$
- $\mathcal{P}_1 = \{1\}$,
  $\mathcal{P}_2 = \{1, 2\}$,
  $\mathcal{P}_3 = \{2, 3, 4\}$,
  $\mathcal{P}_4 = \{3\}$,
  $\mathcal{P}_5 = \{4\}$
- $\mathcal{N}_{TU} = \{1\}$
- $\mathcal{C} = \{2, 4, 5\}$

Variables

Thermal power generation units:

$\dot{Q}_j(t) \geq 0$: Thermal power generated by unit j at time t [MW] **(control output)**

$y_j(t) \in \{0, 1\}$: Binary variable indicating on (1) or off (0) for unit j at time t

$S_j^{\mathrm{do}}(t) \in \{0, 1\}$: Binary variable indicating switch down for unit j at time t

$S_j^{\mathrm{up}}(t) \in \{0, 1\}$: Binary variable indicating switch up for unit j at time t

Network:

$\dot{Q}_p(t) \in \mathbb{R}$: Power flow through pipe p [MW] (can be $\geq 0 / \leq 0$ depending on direction)

Objective function and parameters

Goal: Minimize operational costs

Formulation:

$$J(\dot{Q}) = \sum_{t=0}^{N_T} \sum_{j=1}^{N_{TU}} \left(\dot{Q}_j(t) c_j(t) + S_j^{\mathrm{up}}(t) c_j^{\mathrm{p}} \right)$$

N_T: Number of time steps

N_{TU}: Number of thermal power generation units

Parameters:

$c_j(t)$: Costs of producing one unit of heat at plant j at time t $\left[\frac{\text{MW}}{\text{EUR}}\right]$

c_j^{p}: Start-up cost [EUR]

$d_c(t) \in \mathbb{R}$: Heat demand of consumer c in the set of consumers $\mathcal{C}$

Generation types: Boiler (Gas/Oil), Biomass, CHP, (Electricity)

Constraints

1. Network

2. Thermal power generation units
   - **Define switches**
   - Maximum power
     - Maximum capacity
     - Ramp-up and start-up ramp limits
   - Minimum power
     - Minimum capacity
     - Ramp-down and shutdown ramp limits
   - Discrete output levels
   - **Minimum up- and down-time**

Constraints - Network

Energy conservation:

- Nodes are connected to each other with pipes and can exchange a heat flow through the pipes $\dot{Q}_{p,n}$.
- Sum of the incoming and outgoing heat flows at the specific node $n \in \mathcal{N}$ for each connected producer $j \in \mathcal{N}_{TU}$, consumer $c \in \mathcal{C}$ or pipe $p \in \mathcal{P}_n$.

$$\sum_{j \in \mathcal{N}_{TU}, n=j} \dot{Q}_j(t) - \sum_{c \in \mathcal{C}, n=c} d_c(t) + \sum_{p \in \mathcal{P}_n} \dot{Q}_p(t) = 0 \quad t = 0, .., N_T, \; n \in \mathcal{N}$$

Bounds:

$$\dot{Q}_p^{\min} \leq \dot{Q}_p(t) \leq \dot{Q}_p^{\max} \quad \text{for all } p \in \mathcal{P}$$

Constraints - Thermal power generation units

Detect switches:

$$y_j(t) - y_j(t-1) = S_j^{\mathrm{up}}(t) - S_j^{\mathrm{do}}(t),$$
$$S_j^{\mathrm{up}}(t) + S_j^{\mathrm{do}}(t) \leq 1.$$

Minimum up-time (down-time similar)[2]: for each thermal unit $j \in \mathcal{N}_{TU}$ with min. up-time UT_j

Unit is already on, must remain committed for the first L_j^{up} intervals:

$$\sum_{k=0}^{L_j^{\mathrm{up}}-1} (1 - y_j(k)) = 0.$$

Unit continues generating power for the remaining intervals if it is switched on in the control horizon:

$$\sum_{k=t-\mathrm{UT}_j+1}^{t} S_j^{\mathrm{up}}(k) \leq y_j(t), \qquad \text{for all } t = \{L_j^{\mathrm{up}}, \dots, N_T\},$$

$L_j^{\mathrm{up}} = \mathsf{UT}_j - \underbrace{\mathsf{UT0}_j}_{\text{no. timesteps unit has been on}}$: no. timesteps unit has to stay on (memory of its operational history before the first time step).

[2]Reference: `https://dominoweb.draco.res.ibm.com/cdcb02a7c809d89e8525702300502ac0.html`

MILP formulation summary

$$\min_{\dot{Q},y,S^{\mathrm{up}}} \quad \sum_{t=0}^{N_T} \sum_{j=1}^{N_{TU}} \left(\dot{Q}_j(t) c_j(t) + S_j^{\mathrm{up}}(t) c_j^{\mathrm{p}} \right)$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{N}_{TU}, j=n} \dot{Q}_j(t) - \sum_{c \in \mathcal{C}, c=n} d_c(t) + \sum_{p \in \mathcal{P}_n} \dot{Q}_p(t) = 0 \qquad t = 0, .., N_T,\ n \in \mathcal{N}$$

$$\sum_{k=0}^{L_j^{\mathrm{up}}-1} (1 - y_j(k)) = 0, \qquad \text{for all } j \in \mathcal{N}_{TU}$$

$$\sum_{k=t-\mathrm{UT}_j}^{t} S_j^{\mathrm{up}}(k) \leq y_j(t), \qquad \text{for all } t = \{L_j^{\mathrm{up}} + 1, \dots, N_T\}, j \in \mathcal{N}_{TU}$$

$$\dot{Q}_p^{\min} \leq \dot{Q}_p(t) \leq \dot{Q}_p^{\max} \qquad \text{for all } p \in \mathcal{P}$$

$$\dot{Q}_j(t) \geq 0 \qquad \text{for all } j \in \mathcal{N}_{TU}$$

$$y(t) \in \{0,1\}^{\mathcal{N}_{TU}},\ S^{\mathrm{up}}(t) \in \{0,1\}^{\mathcal{N}_{TU}} \qquad t = 0, .., N_T$$

(without maximum/minimum capacity, ramp up/down, discrete output level constraints)

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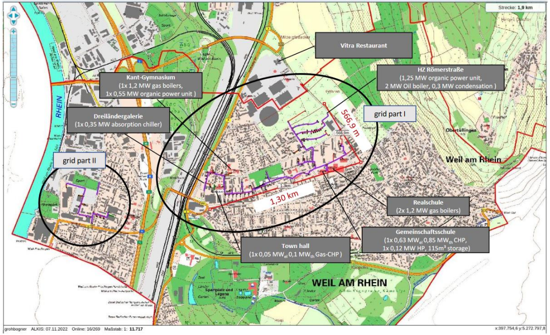


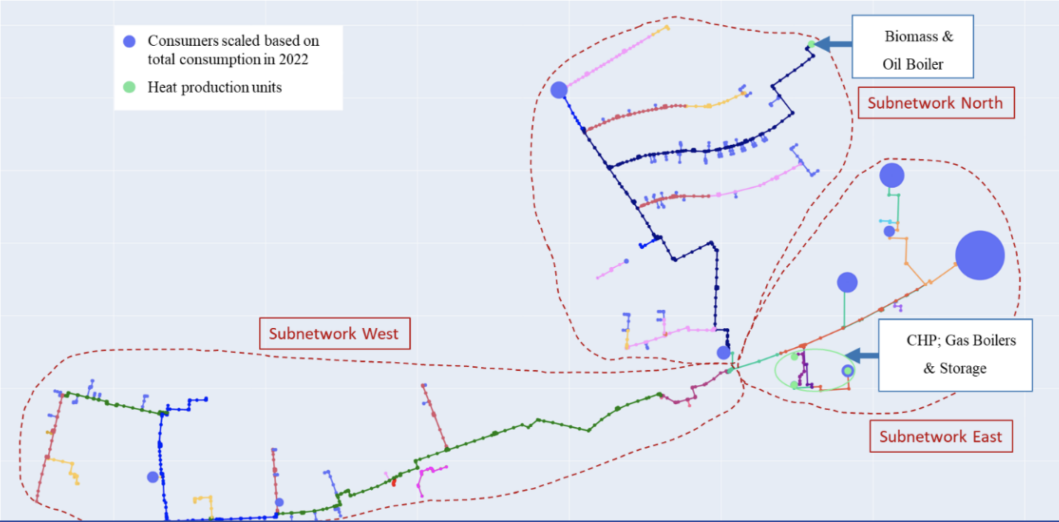
- MPC for the control of heating networks with decentralized energy sources/storage facilities
- Power and heat optimized operation

→ MI-LMPC, MI-NMPC

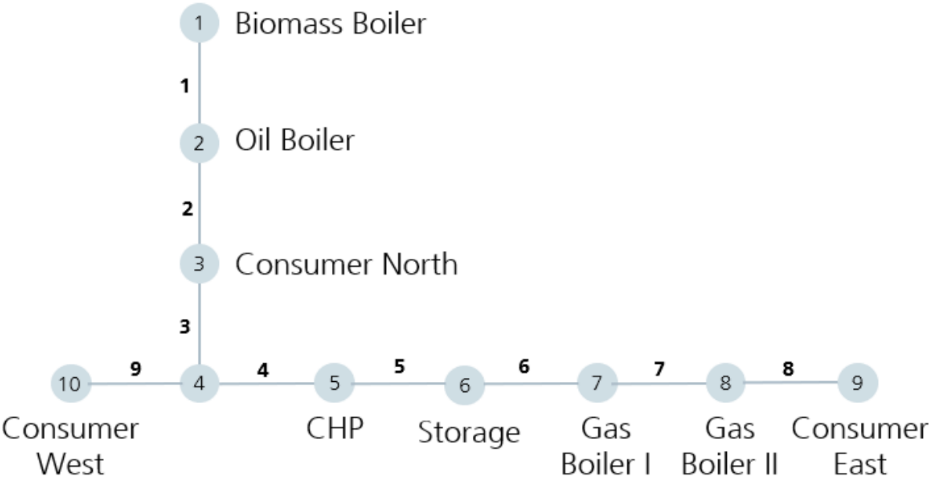
- **Demonstration sites Weil am Rhein & Rheinfelden**
  - 3-10 producers:
    - CHP, biomass, heat pump
    - gas and oil boiler for peak loads
    - waste heat source
  - storage tanks
  - Current control: Rule based prioritization between production units

Verbundvorhaben: EnEff:Wärme: WOpS - Wärmefluss-Optimierung zur Sektorenkopplung in Fernwärmenetzen mitttels MPC unter Berücksichtigung eines strommarktorientierten Betriebes

Network: Weil am Rhein



Network model



Parameters

Production unit	Unit	**Biomass boiler**	**Oil boiler**	**CHP**	**Gas boiler I/II**
Subnetwork		North	North	East	East
Maximum capacity	MW	2.0	1.6	0.87	1.16
Minimum capacity	MW	0.7	1.2	0.43	0.29
Ramp-up/-down	MW/h	0.2	26.1	1.32	26.1
Start-up time	min	15	15	15	15
Minimum runtime	h	350	0.25	1	0.25

MPC of an exemplary day in December