

Optidef

A Latex library for optimization problems

Version - 1.0

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1 Introduction and features

This small library provides a standard set of environments for writing minimization problems. The most important features are:

1. It automatically aligns the problems in the most convenient way allowing even two different output formats:
 - (a) Alignment at the beginning of the words *minimize/argmin* and *subject to*.
 - (b) Double format for the location of the constraints: either to the right of *subject to* aligned with the objective function or below *subject to*.
 - (c) Alignment at the =, >, < signs of the constraints.
 - (d) Optionally, the user can add manually a double align character && to align some common constraints feature. A clear example could be the constraints names, e.g. (boundary constraint) alignment with (dynamic constraint), or the index of the constraints, e.g. in the case of having something like $h(x_k, u_k) \leq 0$, $k = 0, \dots, N$, align the indexes $k = 0, \dots, N$ across constraint lines.
2. It provides an easy interface to define optimization problem for three different reference situations:
 - (a) Where no equation is referenced/numbered.
 - (b) Where the problem is referenced with a single number.
 - (c) Where each equation has an individual reference.
3. It also allows a definition of any optimization problem without a limitless number of constraints.

2 Environment types

There are three basic environments depending on the type of referencing that should be used.

1. The **mini** environment for defining problems with a single reference label:

$$\begin{array}{ll} \underset{w}{\text{minimize}} & f(w) + R(w + 6x) \\ \text{subject to} & g(w) = 0 \end{array} \tag{1}$$

2. The **mini*** environment if the problem does not have to be referenced:

$$\begin{array}{ll} \underset{w}{\text{minimize}} & f(w) + R(w + 6x) \\ \text{subject to} & g(w) = 0 \end{array}$$

3. The **mini!** environment if each equation should be referenced:

$$\underset{w}{\text{minimize}} \quad f(w) + R(w + 6x) \quad (2a)$$

$$\text{subject to} \quad g(w) = 0 \quad (2b)$$

3 Environment types

There are two basic output formats. By default the constraints are aligned with the objective function and to the right of *subject to*. However the user can choose to locate them below *subject to*.

1. Standard format:

$$\underset{w}{\text{minimize}} \quad f(w) + R(w + 6x) \quad (3)$$

$$\text{subject to} \quad g(w) = 0$$

2. Alternative format:

$$\underset{w}{\text{minimize}} \quad f(w) + R(w + 6x)$$

$$\text{subject to} \quad g(w) = 0 \quad (4)$$

4 Environment Syntaxes

The three environments use the same syntax with the same number of parameters. In particular, considering that LHS stands for Left-hand-side and RHS for Right-hand-side, the basic structure to define a general optimization problem is:

```
\begin{mini#}[Format]
  {Optimization variable}
  {Objective function \label{Objective function referece}}
  {\label{Global referece of Optimization Problem}}
  {Result of the optimization problem or any expression on
  the left of the minimize word}

  \addConstraint{LHS Constraint 1}{RHS Constraint 1 \label{Reference Constraint 1}}
  \addConstraint{LHS Constraint 2}{RHS Constraint 2 \label{Reference Constraint 2}}
  :
  :
  \addConstraint{LHS N}{RHSConstraint N \label{Reference Constraint N}}
\end{mini#}
```

where # ∈ {*,!, }.

4.1 Parameters definition

The number of parameters that the environment uses is 6:

1. **Optimization variable**, e.g. $w \in \mathbb{R}^N$.
2. **Objective function**, e.g. $\|w\|_2$.
3. `\label{Global referece of Optimization Problem}`: it defines the main and general reference for the optimization problem. It is used for the `mini` and `mini!` enviroments. In the `mini*` environment leave it blank, i.e. `{}`.
4. **Result of the optimization problem**: optionally a term expressing the result of the optimization problem, e.g. $J(w^*) =$.
5. **Format**: optional parameter if the user want to start the constraints right below *subject to*, instead to the default location on the right of *subject to*.

Notice that only the first two mandatory parameters (variable and objective) will be really necessary in every definition, nevertheless and for the sake of having homogeneous definitions, we opted for requiring the 4 parameters in every definition and expecting empty parameters definitions, i.e. `{}`, when they are not needed.

After the definition of this parameters, the environment accepts the definition of an infinite number of constraints. For this definitions the command:

```
\addConstraint{LHS Constraint k}
              {RHS Constraint k \label{Reference Constraint k}}
```

has to be used, where:

1. **LHS Constraint**: the left-hand side of the any constraint, e.g. $3w^\top w$.
2. **RHS Constraint**: the right-hand side of the first constraints including the equal, less and greater signs, e.g. $\leq \|w\|_\infty$, as well as the constraint label.

Constraints referencing

Notice that the label for the constraints is always included in the right hand side expression and it only makes sense for the case of using the `mini!` enviroment. The label of the objective function can also be included in a similar way.

Example 1 - mini environment

The code:

```
\begin{mini}
  {w}{f(w)+ R(w+6x)}
  {\label{eq:Example1}}{}
```

```

\addConstraint{g(w)}{=0}
\addConstraint{n(w)}{= 6}
\addConstraint{L(w)+r(x)}{=Kw+p}
\addConstraint{h(x)}{=0.}
\end{mini}

```

outputs:

$$\begin{aligned}
& \underset{w}{\text{minimize}} && f(w) + R(w + 6x) \\
& \text{subject to} && g(w) = 0, \\
& && n(w) = 6, \\
& && L(w) + r(x) = Kw + p, \\
& && h(x) = 0.
\end{aligned} \tag{5}$$

Example 2 - mini* environment

On the other hand:

```

\begin{mini*}
{w}{f(w)+ R(w+6x)}
{}{}

\addConstraint{g(w)}{=0}
\addConstraint{n(w)}{= 6,}
\addConstraint{L(w)+r(x)}{=Kw+p}
\addConstraint{h(x)}{=0.}
\end{mini*}

```

it is almost the same but removing the reference:

$$\begin{aligned}
& \underset{w}{\text{minimize}} && f(w) + R(w + 6x) \\
& \text{subject to} && g(w) = 0, \\
& && n(w) = 6, \\
& && L(w) + r(x) = Kw + p, \\
& && h(x) = 0.
\end{aligned}$$

Example 3 - mini! environment

Finally, the multireferencing environment outputs:

```

\begin{mini!}
{w}{f(w)+ R(w+6x) \label{eq:ObjectiveExample1}}

```

```

{\label{eq:Example1}}{

\addConstraint{g(w)}{=0 \label{eq:C1Example3}}
\addConstraint{n(w)}{= 6 \label{eq:C2Example1}}
\addConstraint{L(w)+r(x)}{=Kw+p \label{eq:C3Example1}}
\addConstraint{h(x)}{=0. \label{eq:C4Example1}}
\end{mini!}

```

$$\underset{w}{\text{minimize}} \quad f(w) + R(w + 6x) \quad (6a)$$

$$\text{subject to} \quad g(w) = 0, \quad (6b)$$

$$n(w) = 6, \quad (6c)$$

$$L(w) + r(x) = Kw + p, \quad (6d)$$

$$h(x) = 0. \quad (6e)$$

Example 4 - mini + problem result

Adding the problem result:

```

\begin{mini}
{w}{f(w)+ R(w+6x)}
{\label{eq:Example1}}
{J(w^*)=}

\addConstraint{g(w)}{=0}
\addConstraint{n(w)}{= 6}
\addConstraint{L(w)+r(x)}{=Kw+p}
\addConstraint{h(x)}{=0.}
\end{mini}

```

outputs:

$$\begin{aligned}
J(w^*) = \quad & \underset{w}{\text{minimize}} \quad f(w) + R(w + 6x) \\
& \text{subject to} \quad g(w) = 0, \\
& \quad \quad \quad n(w) = 6, \\
& \quad \quad \quad L(w) + r(x) = Kw + p, \\
& \quad \quad \quad h(x) = 0.
\end{aligned} \quad (7)$$

Example 5 - second format

If including a 1 as optional parameter, the first constraint will appear aligned to the left right below *subject to*.

```

\begin{mini}[1]
{w}{f(w)+ R(w+6x)}
{\label{eq:Example1}}
{}

\addConstraint{g(w)}{=0}
\addConstraint{n(w)}{= 6}
\addConstraint{L(w)+r(x)}{=Kw+p}
\addConstraint{h(x)}{=0.}
\end{mini}

```

outputs:

$$\begin{aligned}
& \underset{w}{\text{minimize}} && f(w) + R(w + 6x) \\
& \text{subject to} && \\
& && g(w) = 0, && (8) \\
& && n(w) = 6, \\
& && L(w) + r(x) = Kw + p, \\
& && h(x) = 0.
\end{aligned}$$

Example 6 - mini* + extra constraint alignment

Adding the fourth optional alignment to add constraint names:

```

\begin{mini*}
{w}{f(w)+ R(w+6x)}
{}{}

\addConstraint{g(w)}{=0, \quad \&\& \text{(Dynamic constraint)}}
\addConstraint{n(w)}{= 6, \quad \&\& \text{(Boundary constraint)}}
\addConstraint{L(w)+r(x)}{=Kw+p, \quad \&\& \text{(Random constraint)}}
\addConstraint{h(x)}{=0.\quad \&\& \text{(Path constraint)}}
\end{mini*}

```

$$\begin{aligned}
& \underset{w}{\text{minimize}} && f(w) + R(w + 6x) \\
& \text{subject to} && g(w) = 0, && \text{(Dynamic constraint),} \\
& && n(w) = 6, && \text{(Boundary constraint),} \\
& && L(w) + r(x) = Kw + p, && \text{(Random constraint),} \\
& && h(x) = 0. && \text{(Path constraint).}
\end{aligned}$$

Example 7 - mini environment on the unconstrained case

```
\begin{mini}
  {w}{f(w)+ R(w+6x)}
  {\label{eq:Ex4}}
  {}
\end{mini}
```

outputs:

$$\underset{w}{\text{minimize}} \quad f(w) + R(w + 6x) \quad (9)$$

5 The *argmini* environment

Similar to the `mini`, `mini*` and `mini!` environments, the environments `argmini`, `argmini*` and `argmini!` are very similar environments that use the same syntax but the output is slightly different. The following code serves for illustration:

```
\begin{argmini}
  {w}{f(w)+ R(w+6x)}

  {\label{eq:Example1}}{w^*=}

  \addConstraint{g(w)}{=0}
  \addConstraint{n(w)}{= 6}
  \addConstraint{L(w)+r(x)}{=Kw+p}
  \addConstraint{h(x)}{=0.}
\end{argmini}
```

outputs:

$$\begin{aligned} w^* &= \underset{w}{\text{arg min}} \quad f(w) + R(w + 6x) \\ &\text{subject to} \quad \begin{aligned} g(w) &= 0, \\ n(w) &= 6, \\ L(w) + r(x) &= Kw + p, \\ h(x) &= 0. \end{aligned} \end{aligned} \quad (10)$$

6 The *maxi* and *argmaxi* environments

Exactly the same syntax and definition as the previous environments, but now for defining maximization environments. The following code serves for illustration:


```

\begin{maxi}
{w}{f(w)+ R(w+6x)}
{g(w)}{=0}

{\label{eq:Example1}}{}

\addConstraint{g(w)}{=0}
\addConstraint{n(w)}{= 6}
\addConstraint{L(w)+r(x)}{=Kw+p}
\addConstraint{h(x)}{=0.}
\end{maxi}

```

outputs:

$$\begin{aligned}
& \underset{w}{\text{maximize}} && f(w) + R(w + 6x) \\
& \text{subject to} && g(w) = 0, \\
& && n(w) = 6, \\
& && L(w) + r(x) = Kw + p, \\
& && h(x) = 0.
\end{aligned} \tag{11}$$

7 Problems and limitations

One current limitation is that the size of the optimization variables must be smaller than the word minimize, otherwise, the output is not properly aligned. Example:

$$\underset{x_0, u_0, x_1, \dots, u_{N-1}, x_N}{\text{minimize}} \quad \sum_{k=0}^{N-1} L(x_k, u_k) + E(x_N) \tag{12a}$$

$$\text{subject to} \quad x_{k+1} - f(x_k, u_k) = 0, \quad k = 0, \dots, N-1, \tag{12b}$$

$$h(x_k, u_k) \leq 0, \quad k = 0, \dots, N-1, \tag{12c}$$

$$r(x_0, x_N) = 0. \tag{12d}$$

A possible way to avoid this is to stack them with the command:

```

\substack{x_0, u_0, x_1, \hdots, \u_{N-1}, x_N}

```

$$\underset{\substack{x_0, u_0, x_1, \dots, \\ u_{N-1}, x_N}}{\text{minimize}} \quad \sum_{k=0}^{N-1} L(x_k, u_k) + E(x_N) \tag{13a}$$

$$\text{subject to} \quad x_{k+1} - f(x_k, u_k) = 0, \quad k = 0, \dots, N-1, \tag{13b}$$

$$h(x_k, u_k) \leq 0, \quad k = 0, \dots, N-1, \tag{13c}$$

$$r(x_0, x_N) = 0. \tag{13d}$$

If you want to increase the size of the optimization variables: A possible way to avoid this is to stack them with the command:

`\substack{\displaystyle x_0,u_0,x_1,\hdots,\ \displaystyle u_{N-1},x_N}`

$$\begin{array}{l} \text{minimize} \\ x_0, u_0, x_1, \dots, \\ u_{N-1}, x_N \end{array} \sum_{k=0}^{N-1} L(x_k, u_k) + E(x_N) \quad (14a)$$

$$\text{subject to } x_{k+1} - f(x_k, u_k) = 0, \quad k = 0, \dots, N-1, \quad (14b)$$

$$h(x_k, u_k) \leq 0, \quad k = 0, \dots, N-1, \quad (14c)$$

$$r(x_0, x_N) = 0. \quad (14d)$$

8 Code definition

```
% optidef - Version 08/06/2016
%
%Copyright 2016 J. Lago Garcia
%
%This work may be distributed and/or modified under the conditions of the LaTeX Project
%The latest version of this license is in http://www.latex-project.org/lppl.txt and vers
%
%This work has the LPPL maintenance status 'maintained'. The Current Maintainer of this
%
%E-mail: jesus.lago.garcia@venus.uni-freiburg.de
%
%This work consists of the file optidef.sty.

\NeedsTeXFormat{LaTeX2e}
\ProvidesPackage{optidef}[2016/06/06 Package for defining optimization problems]

\RequirePackage{environ}
\RequirePackage{mathtools}
\RequirePackage{xifthen}
\RequirePackage{etoolbox}
\newtoggle{bodyCon}
\toggletrue{bodyCon}

% Input minimization environments

% Macros for objective definition, constraint definition and extra constraint definition
\newcommand{\bodyobj}[4]
```

```

{
\ifthenelse{\isempty{#4}}
{
&\underset{\displaystyle #1}{\mbox{#3}} \quad #2\span\span\span\span
}
{
#4 ~ ~ &\underset{\displaystyle #1}{\mbox{#3}} \quad #2\span\span\span\span
}
}

```

```

\newcommand{\bodyconst}[2]
{\ \ &\text{subject to} \quad \#1 & \#2}

```

```

\newcommand{\bodyconstRight}[2]
{\ \ &\text{subject to} \quad \#1 & \#2}

```

```

\newcommand{\bodyconstBelow}[2]
{\ \ &\text{subject to} \span\span \ \
&\quad\#1 & \#2}

```

```

\newcommand{\bodyconstBelowMult}[2]
{\ \ &\text{subject to} \span\span\span\span \nonumber \ \
&\quad\#1 & \#2}

```

```

\newcommand{\addConstraint}[2]{
\iftoggle{bodyCon}{
\bodyconst{#1}{#2}
\togglefalse{bodyCon}
}{
,\ \&\quad \#1 & \#2
\togglefalse{bodyCon}
}
}

```

```

\newcommand{\breakObjectiveOneConstraint}[1]{&&\#1\ \}

```

%MINIMIZATION ENVIRONMENTS

```

\NewEnviron{mini}[5] [] {%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelow
}{
}
\begin{equation}

```

```

#4
\begin{alignedat}{4}
\bodyobj{#2}{#3}{minimize}{#5}
\BODY
\end{alignedat}
\end{equation}
\toggletrue{bodyCon}
\let\bodyconst\bodyconstRight
}

```

```

\NewEnviron{argmini}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelow
}{}

```

```

\begin{equation}
#4
\begin{alignedat}{4}
\bodyobj{#2}{#3}{arg min}{#5}
\BODY
\end{alignedat}
\end{equation}
}

```

```

\NewEnviron{mini*}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelow
}{}

```

```

\begin{equation*}
#4
\begin{alignedat}{4}
\bodyobj{#2}{#3}{minimize}{#5}
\BODY
\end{alignedat}
\end{equation*}
\toggletrue{bodyCon}
\let\bodyconst\bodyconstRight
}

```

```

\NewEnviron{argmini*}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelow
}{}

```

```

\begin{equation*}
#4
\begin{alignedat}{4}
\bodyobj{#2}{#3}{arg min}{#5}

```

```

\BODY
\end{alignedat}
\end{equation*}
}

\NewEnviron{mini!}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelowMult
}{
\begin{subequations}
#4
\begin{alignat}{4}
\bodyobj{#2}{#3}{minimize}{#5}
\BODY
\end{alignat}
\end{subequations}
\toggletrue{bodyCon}
\let\bodyconst\bodyconstRight
}

\NewEnviron{argmini!}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelowMult
}{
\begin{subequations}
#4
\begin{alignat}{4}
\bodyobj{#2}{#3}{arg min}{#5}
\BODY
\end{alignat}
\end{subequations}
}

%MAXIMIZATION ENVIRONMENTS

\NewEnviron{maxi}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelow
}{
\begin{equation}
#4
\begin{alignedat}{4}
\bodyobj{#2}{#3}{maximize}{#5}
\BODY
\end{alignedat}

```

```

\end{equation}
}

\NewEnviron{argmaxi}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelow
}{
\begin{equation}
#4
\begin{alignedat}{4}
\bodyobj{#2}{#3}{arg maxi}{#5}
\BODY
\end{alignedat}
\end{equation}
}

\NewEnviron{maxi*}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelow
}{
\begin{equation*}
#4
\begin{alignedat}{4}
\bodyobj{#2}{#3}{maximize}{#5}
\BODY
\end{alignedat}
\end{equation*}
}

\NewEnviron{argmaxi*}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelow
}{
\begin{equation*}
#4
\begin{alignedat}{4}
\bodyobj{#2}{#3}{arg maxi}{#5}
\BODY
\end{alignedat}
\end{equation*}
}

\NewEnviron{maxi!}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelowMult

```

```

}{}
\begin{subequations}
#4
\begin{alignat}{4}
\bodyobj{#2}{#3}{maximize}{#5}
\BODY
\end{alignat}
\end{subequations}
}

\NewEnviron{argmaxi!}[5][]{%
\ifthenelse{\equal{#1}{1}}{
\let\bodyconst\bodyconstBelowMult
}{}
\begin{subequations}
#4
\begin{alignat}{4}
\bodyobj{#2}{#3}{arg maxi}{#5}
\BODY
\end{alignat}
\end{subequations}
}

```