Numerical Trajectory Optimization of Airborne Wind Energy Systems with Stroboscopic Averaging Methods

Jakob Harzer, Jochem De Schutter, Moritz Diehl

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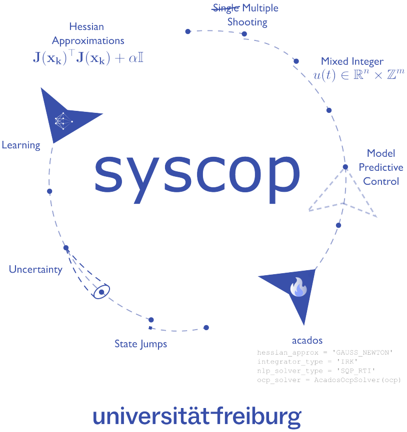
universität freiburg

Systems Control and Optimization Laboratory



Systems Control and Optimization Laboratory

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Airborne Wind Energy

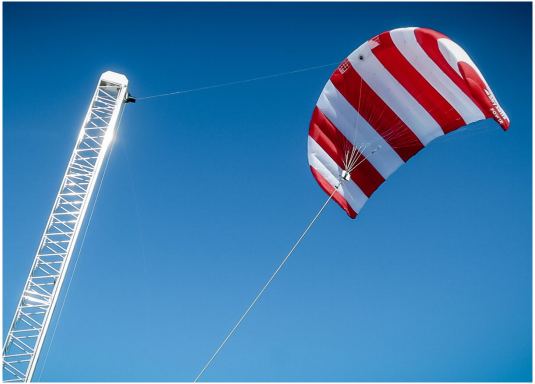
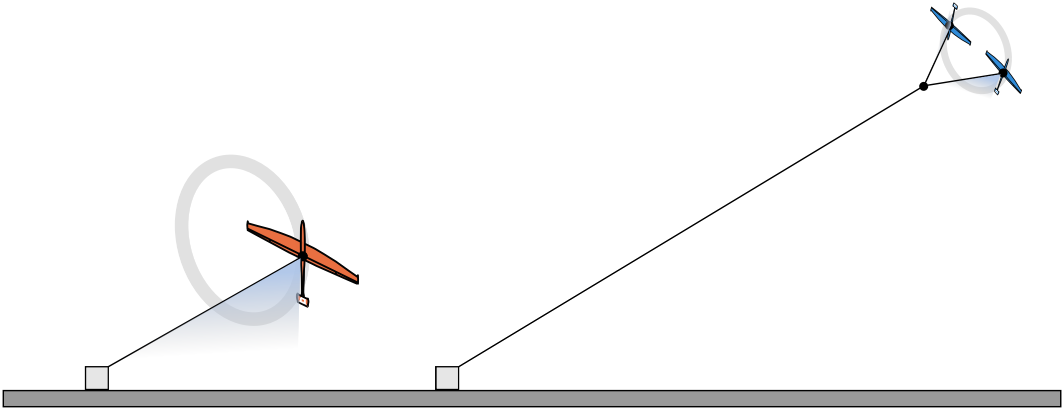


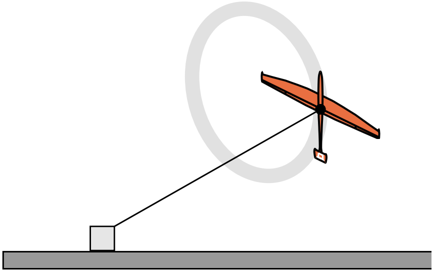
Image from Skysails Power

Image from Makani Power

Single vs Dual Kite Systems

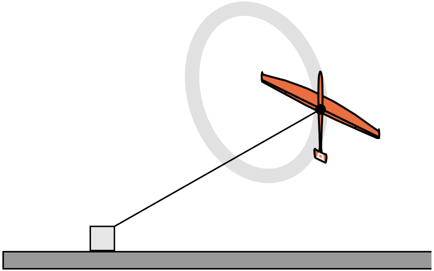


Model



- High-Fidelity 6-DOF model of the plane [5]

Model

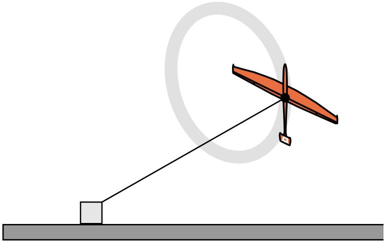


- High-Fidelity 6-DOF model of the plane [5]
- System Dynamics (Index-1 DAE)

$$f(x, u, z) = 0$$

with $x \in \mathbb{R}^{23}$, $u \in \mathbb{R}^4$, $z \in \mathbb{R}^1$, based on index-reduced Lagrangian dynamics

Trajectory Optimization



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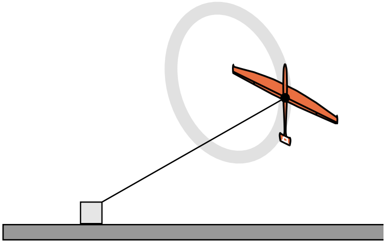
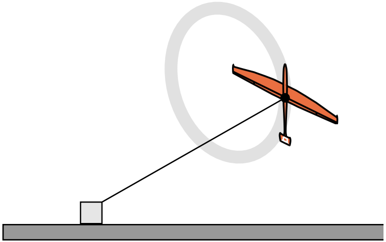


- Optimize a single pumping cycle of a single-kite AWE system

Trajectory Optimization



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- Formulate OCP

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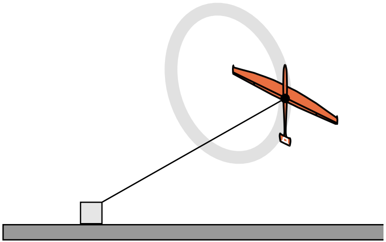
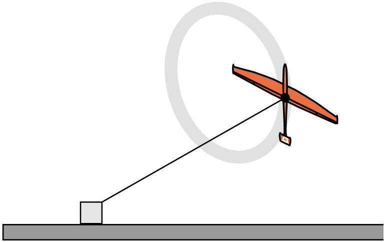


- Optimize a single pumping cycle of a single-kite AWE system
- Formulate OCP

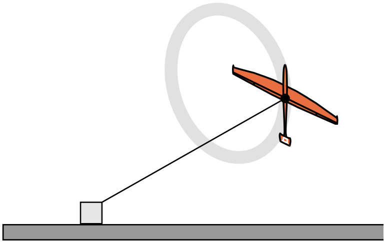
$$
\begin{aligned}
\max_{\substack{x(\cdot),u(\cdot),\\ z(\cdot),t_\mathrm{f}}} \quad & P_\mathrm{gen}(x,u,z,t_\mathrm{f}) \\
\text{s.t.} \quad & 0 = f(\dot{x}(t),x(t),u(t),z(t)), \quad \forall t \in [0,t_\mathrm{f}], \\
& 0 \leq h(\dot{x}(t),x(t),u(t),z(t)), \quad \forall t \in [0,t_\mathrm{f}], \\
& 0 = p(x(0),x(t_\mathrm{f}))
\end{aligned}
$$

Trajectory Optimization



- Optimize a single pumping cycle of a single-kite AWE system
- Formulate OCP $\rightarrow$ discretize to NLP

$$\begin{aligned} \min_{w} \quad & F(w) \\ \text{s.t.} \quad & 0 = G(w), \\ & 0 \leq H(w) \end{aligned}$$



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- Very large, complicated nonlinear problem, need good strategy and initialization to solve

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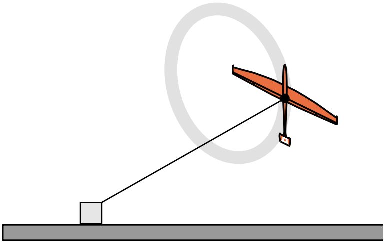

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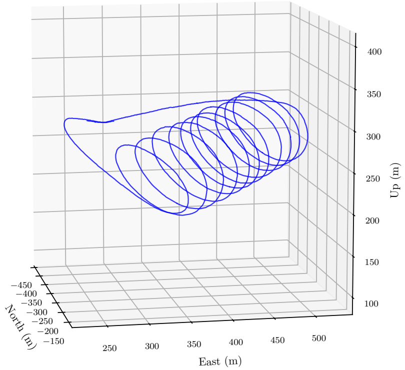
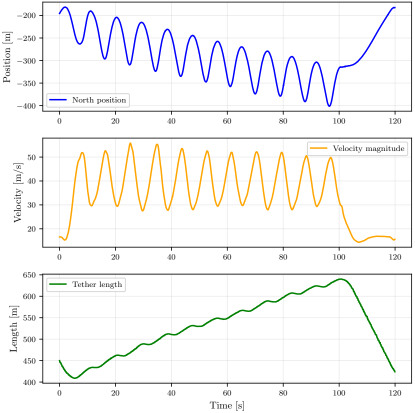
- Very large, complicated nonlinear problem, need good strategy and initialization to solve
- Software Packages such as the AWEBox[2]

[IPOPT Video]

Some Observations from Real Data



Produced from `https://github.com/kitemill/Flight-log`



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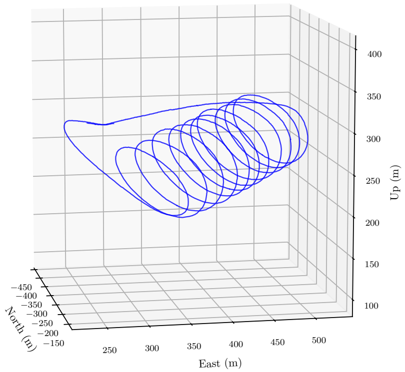

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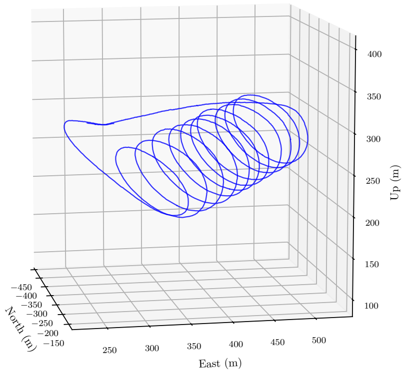

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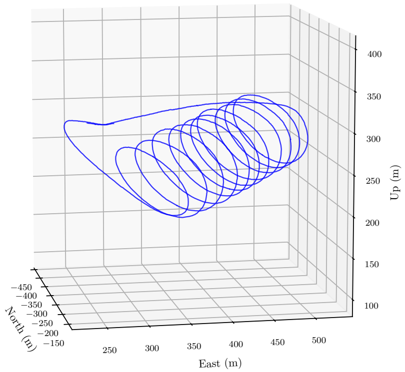

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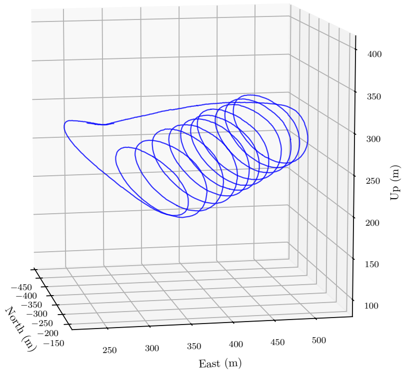

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Strong Assumption

In the reel-out phase, the power optimal trajectory $x^*(t)$ and the corresponding control $u^*(t)$ consist of many similar, slowly changing cycles.

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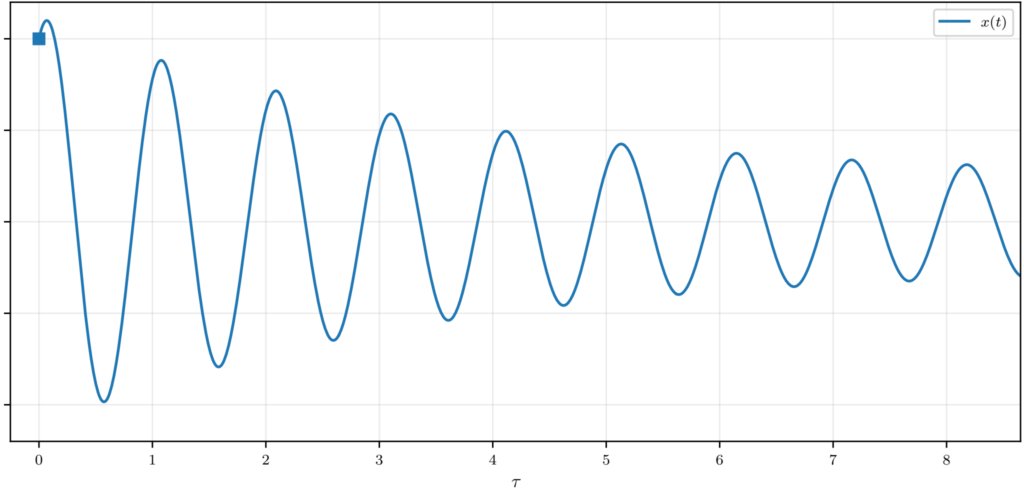
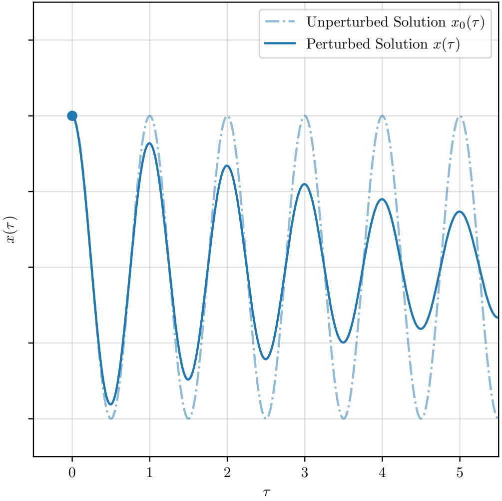
Stroboscopic Averaging Methods

Highly Oscillatory Systems

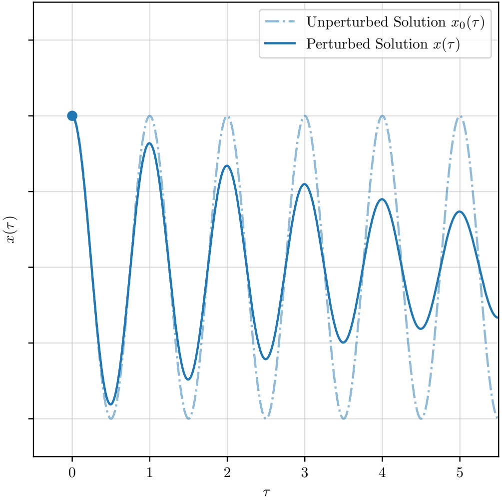


Averaging Methods for Highly Oscillatory Systems



$$\dot{x} = f(x)$$

Averaging Methods for Highly Oscillatory Systems



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High Order Stroboscopic Averaging

$$\dot{X} = F(x)$$

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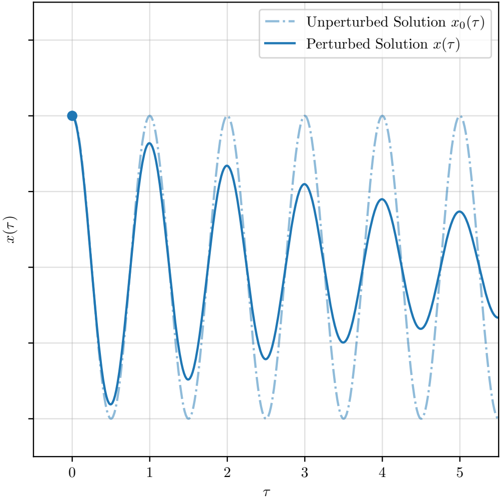

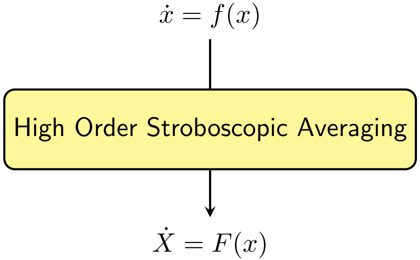

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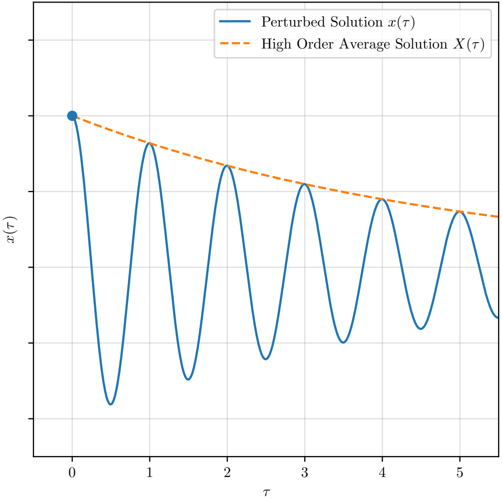

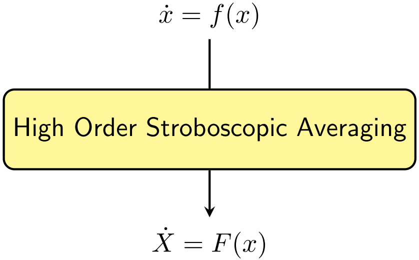

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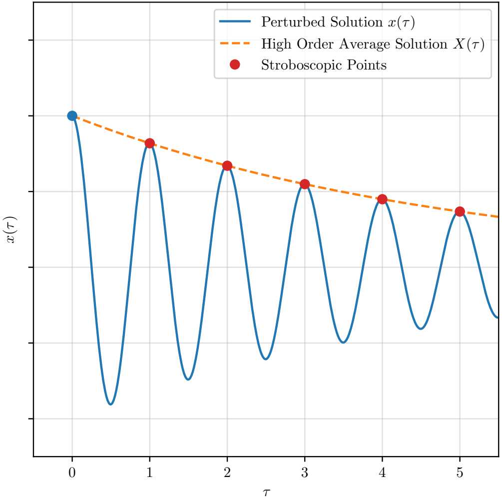

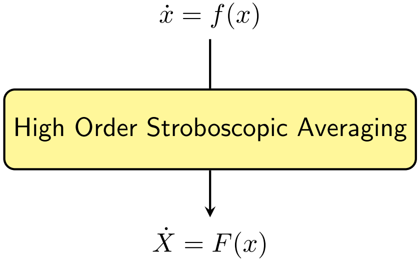

- If $x(0) = X(0)$ then the solution to averaged system satisfies

$$x(k) = X(k), \quad k \in \mathbb{Z}$$

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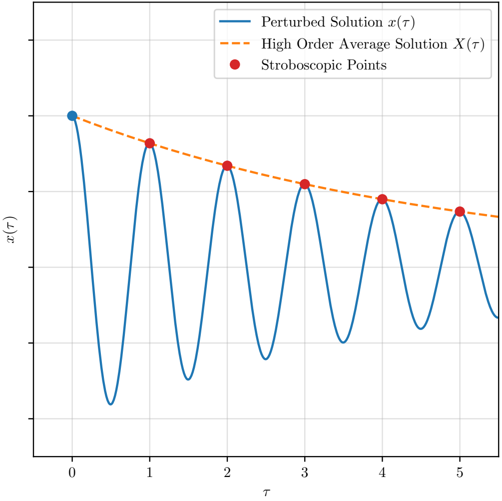

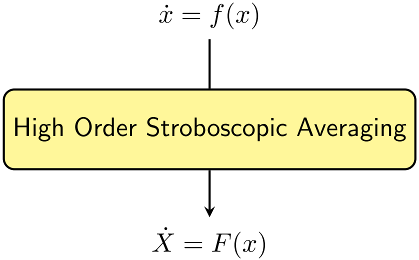

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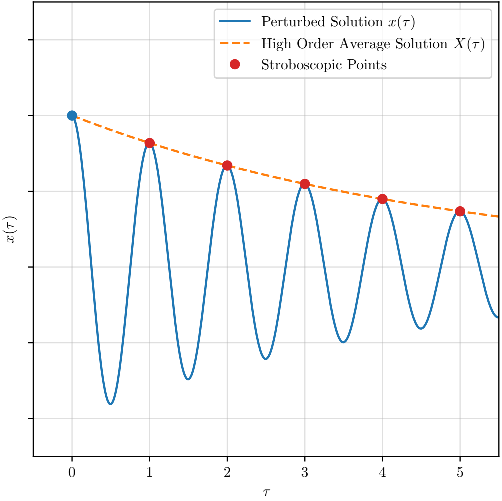
- Original system f on a fast timescale

Averaging Methods for Highly Oscillatory Systems



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High Order Stroboscopic Averaging

$$\dot{X} = F(x)$$

- If $x(0) = X(0)$ then the solution to averaged system satisfies

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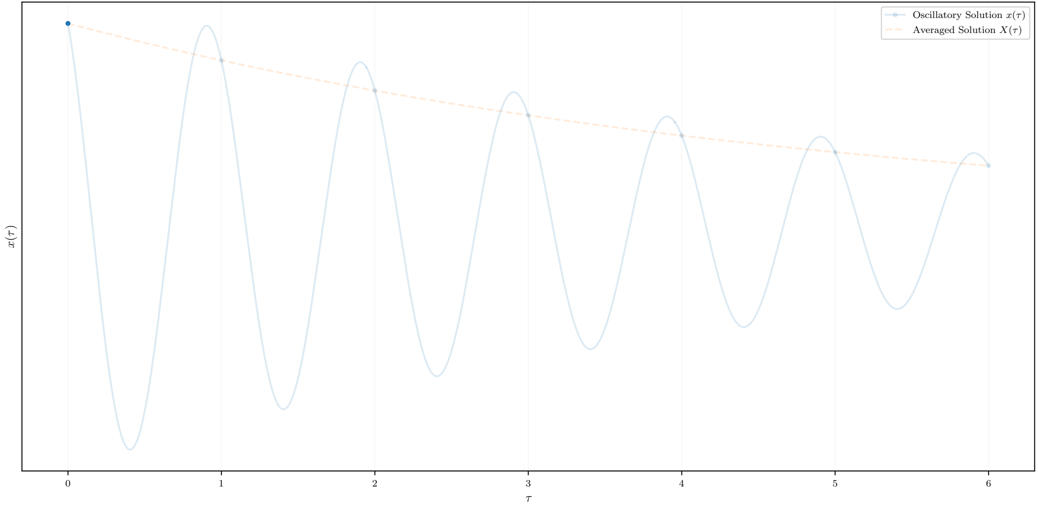
- Averaged system F on slow timescale

Stroboscopic Average Method (SAM)



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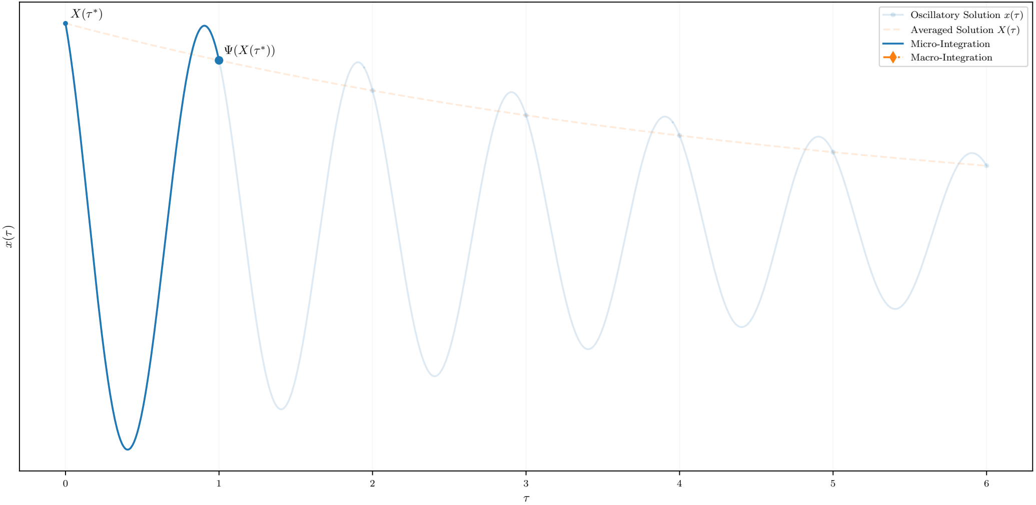

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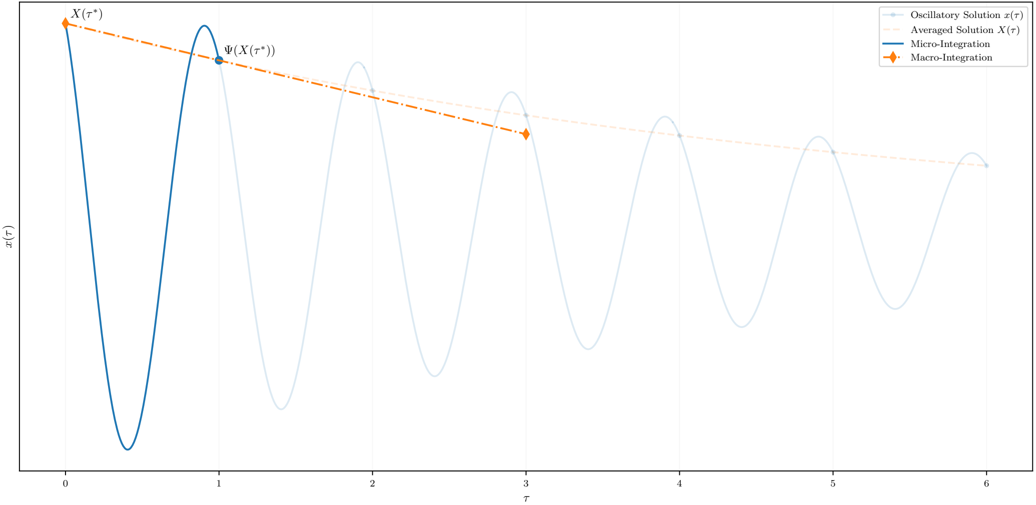

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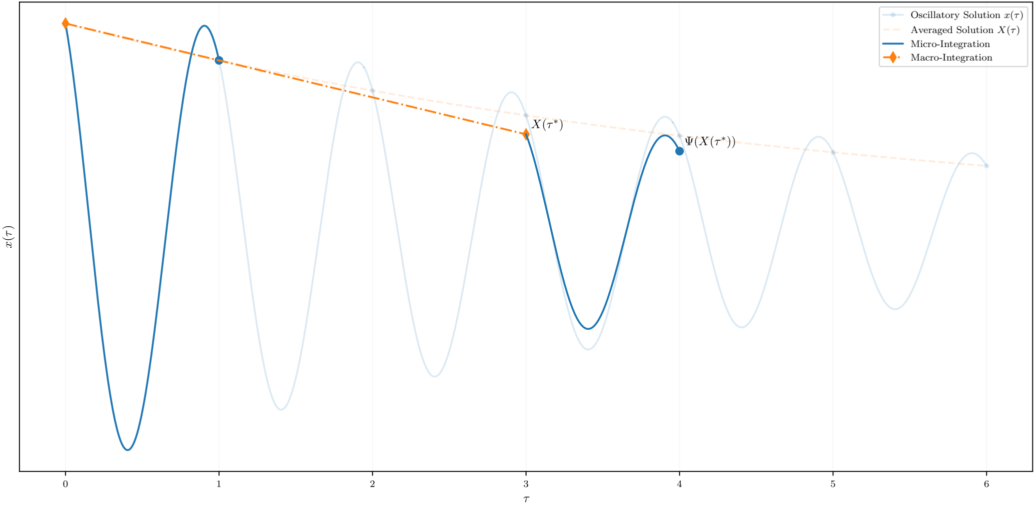

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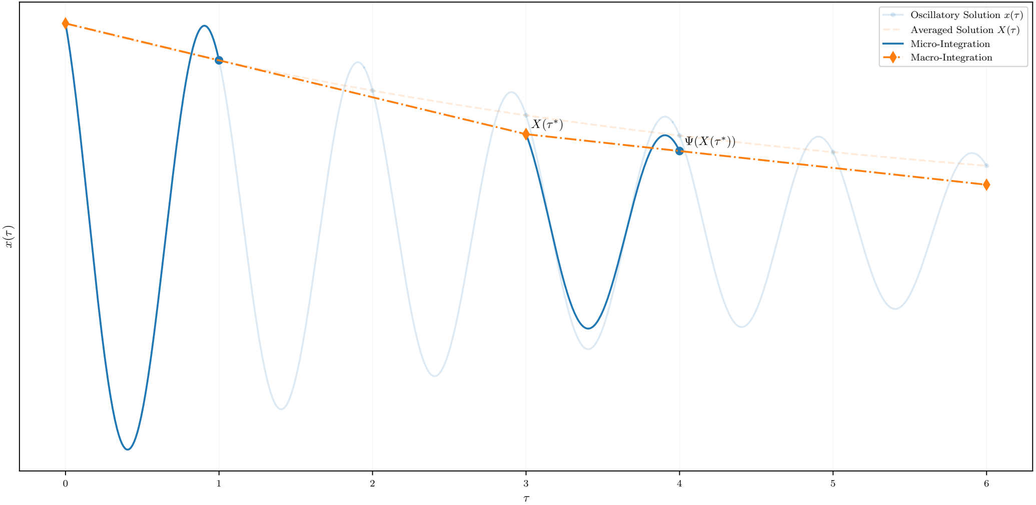

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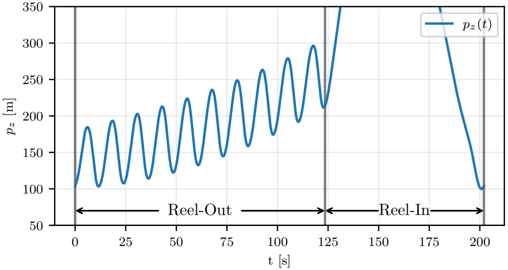
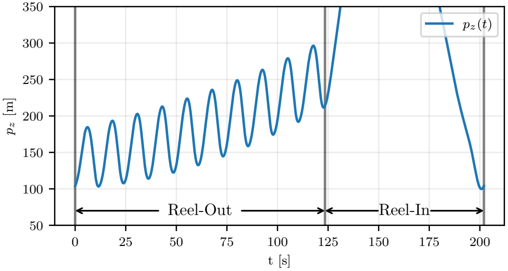
OCP Timescaling



OCP Timescaling

- Conceptually divide the trajectory into $N + 1$ section



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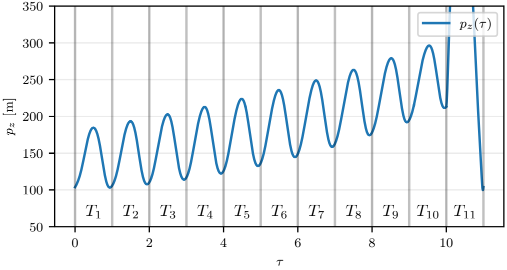

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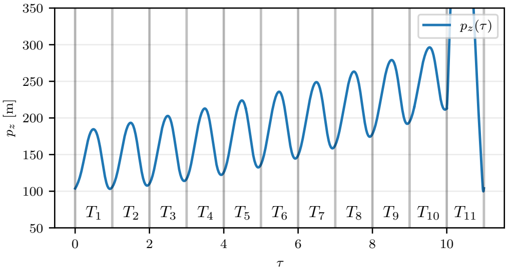
- Conceptually divide the trajectory into $N + 1$ section
- Of duration $T_1, \ldots, T_N, T_{N+1}$



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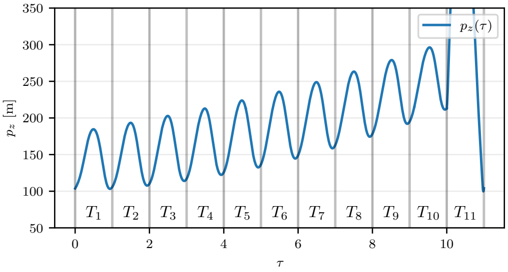

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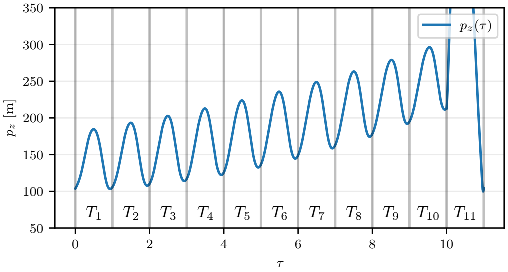
$$\min_{\substack{x(\cdot),u(\cdot),z(\cdot)\\ T_1,\ldots,T_{N+1}}} \int_0^{N+1} l(x(\tau), u(\tau), z(\tau))\, d\tau$$

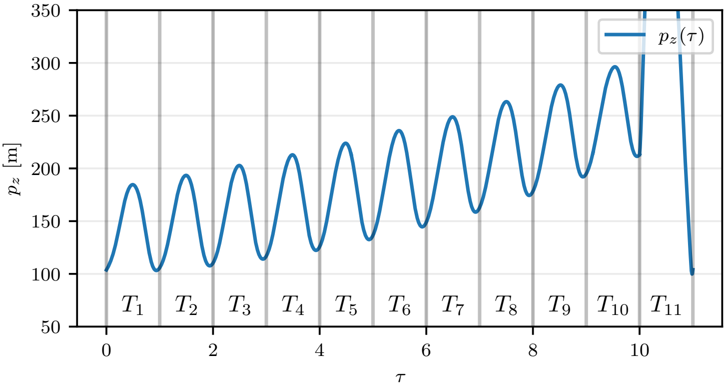
$$\text{s.t.} \quad 0 = f\left(\frac{1}{T(\tau)} \frac{dx(\tau)}{d\tau}, x(\tau), u(\tau), z(\tau)\right),$$

$$0 \leq h\left(x(\tau), u(\tau), z(\tau)\right),$$

$$0 = x(0) - x(N+1)$$



Stroboscopic Averaging for AWE System



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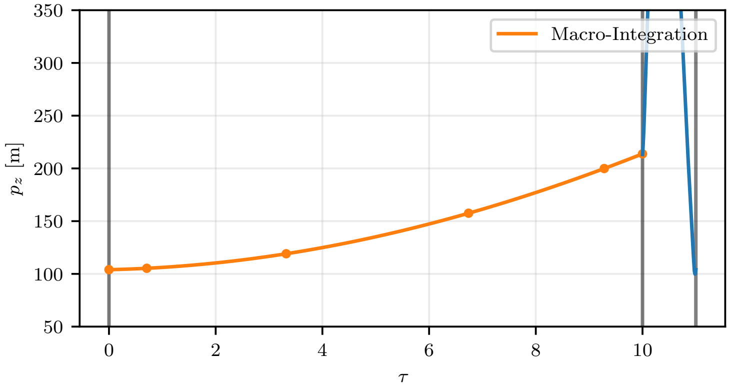

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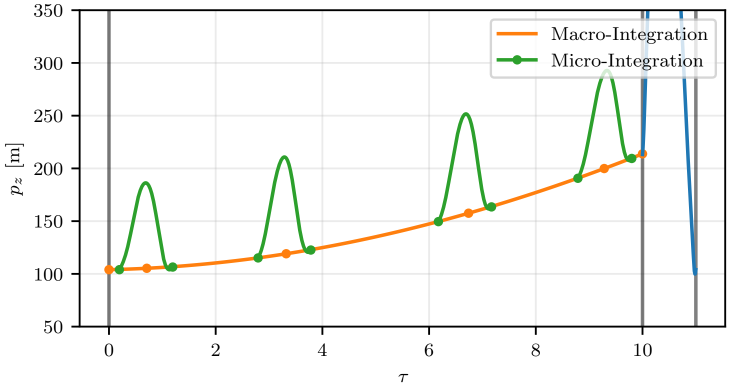

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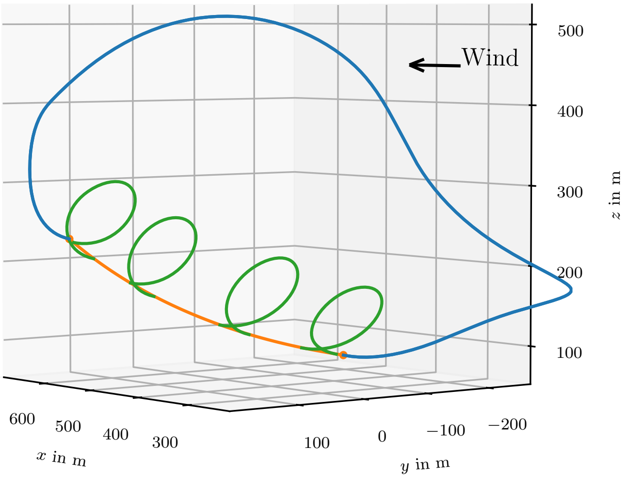

Regularization

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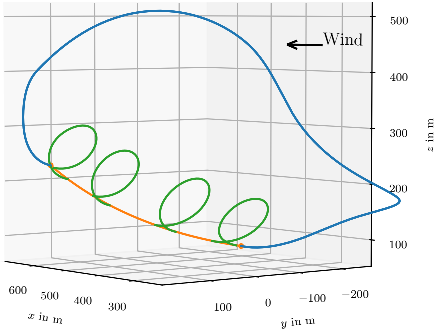
Strong Assumption

In the reel-out phase, the power optimal trajectory $x^*(t)$ and the corresponding control $u^*(t)$ consist of many similar, slowly changing cycles.



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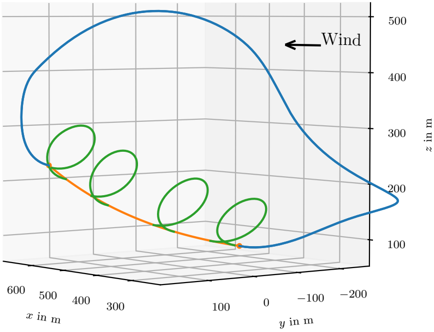

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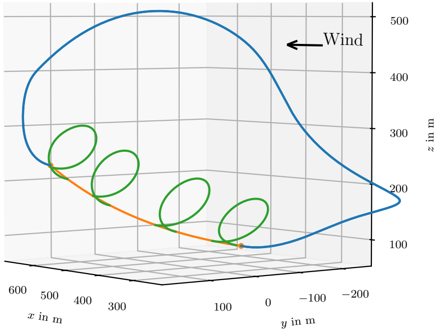

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Enforce via Regularization:

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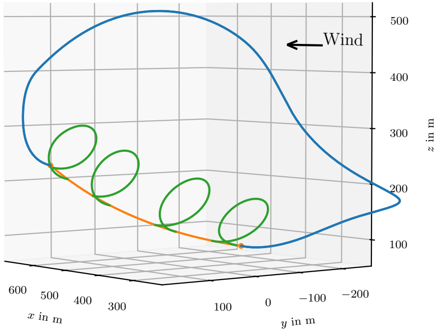
2) The change over a single cycle should be small



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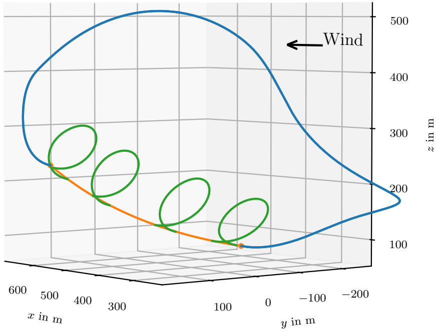

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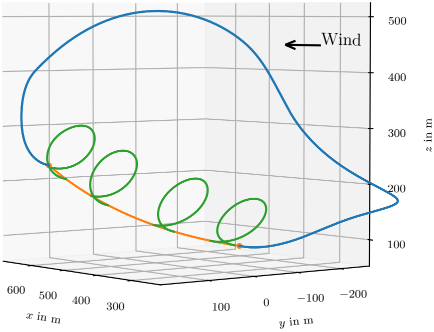

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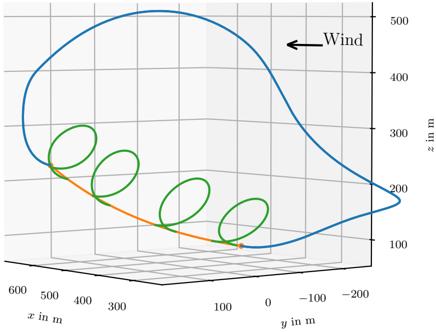
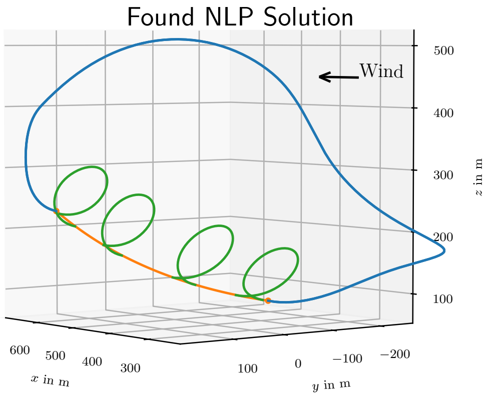
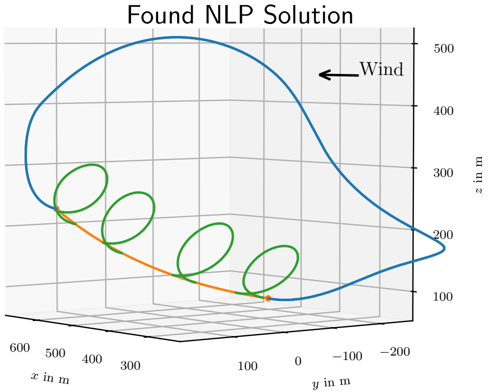


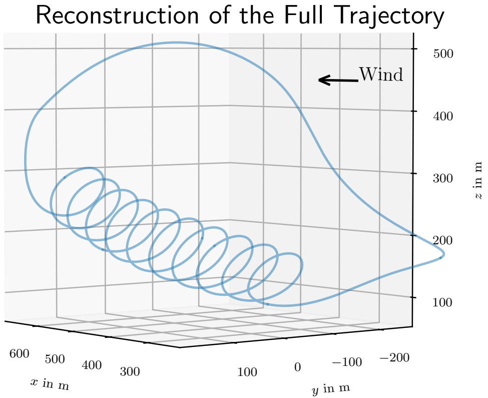
- Keeps the errors small
- Strong geometric assumption!

Stroboscopic Averaging for AWE System (cont.)

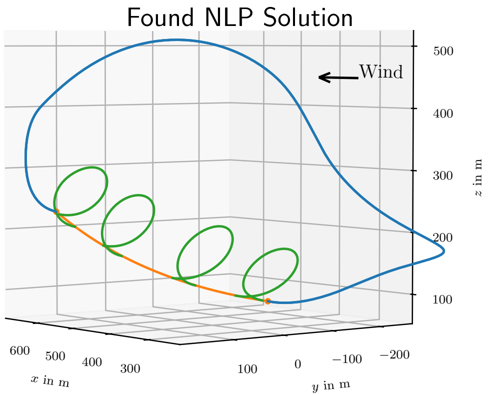


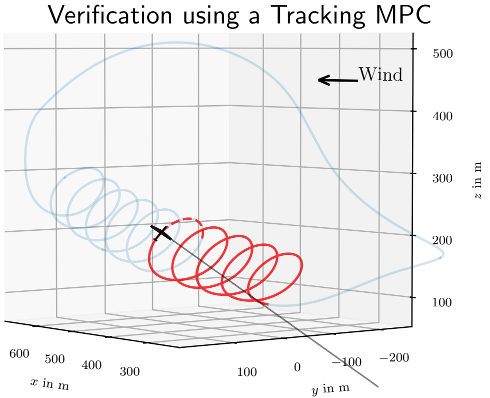
Stroboscopic Averaging for AWE System (cont.)



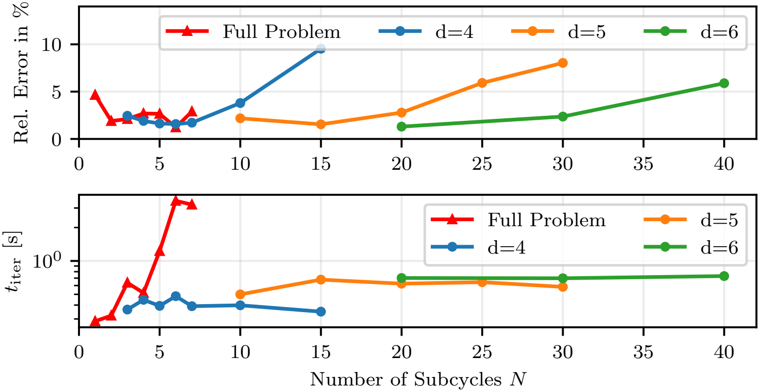


Stroboscopic Averaging for AWE System (cont.)

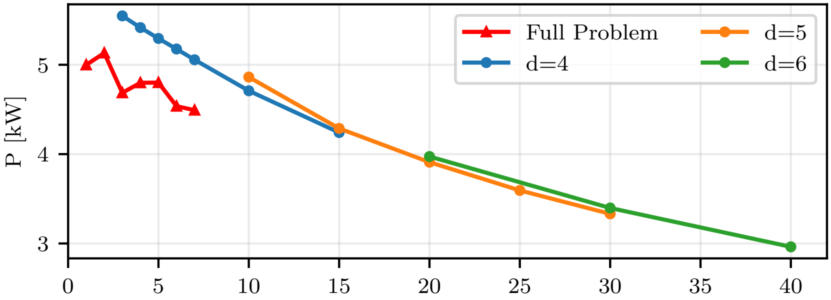




Results 1 - Computational Efficiency



Results 2 - Generated Power



Wrap Up

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Thank you for your attention!

Useful Sources

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