An Update on the AWEbox

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What is the AWEbox anyways?

Ultimate goal:

open-source design tool for Airborne Wind Energy systems,
using low-order physics-based models,
in an optimal-control framework

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Ultimate goal:

open-source design tool for Airborne Wind Energy systems,
using low-order physics-based models,
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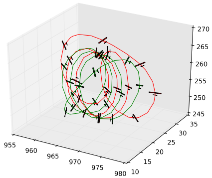
Current goal:

trajectory-optimization tool based on given kite geometry and wind profile...

For example...



dual kites with general 'kiteswarms' dimensions, dcm-rotations with stability-derivative aerodynamics coordinates from Schweizer SGS1-36,

direct surface deflection control, no induction, in isa standard atmosphere

The example OCP, decision variables

$\boldsymbol{x}$: [position $\boldsymbol{q}$, velocity $\dot{\boldsymbol{q}}$ (for tether nodes),
position $\boldsymbol{q}$, velocity $\dot{\boldsymbol{q}}$, angular velocity $\boldsymbol{\omega}$, dcm $\mathbf{R}$ (for kite nodes),
and main tether length l_{T}, main tether speed $\dot{l}_{\mathrm{T}}$, total energy E $]^{\top}$.

$\boldsymbol{u}$: [control surface deflections $\boldsymbol{\delta}$ (for kite nodes),
and tether acceleration $\ddot{l}_{\mathrm{T}}$ $]^{\top}$

$\boldsymbol{\lambda}$: [tether stress λ (for tether and kite nodes) $]^{\top}$,

$\boldsymbol{\theta}$: [period t_{final}, secondary tether length l_{S} $]^{\top}$

The example OCP

$\boldsymbol{x}$: [$\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ (for tether nodes), $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, $\boldsymbol{\omega}$, $\mathbf{R}$ (for kite nodes), and l_{T}, $\dot{l}_{\mathrm{T}}$, E]$^\top$.
$\boldsymbol{u}$: [$\boldsymbol{\delta}$ (for kite nodes), and $\ddot{l}_{\mathrm{T}}$]$^\top$
$\boldsymbol{\lambda}$: [λ (for tether and kite nodes)]$^\top$, $\boldsymbol{\theta}$: [t_{final}, l_{S}]$^\top$

$$
\begin{array}{lll}
\underset{\boldsymbol{x},\boldsymbol{u},\boldsymbol{\lambda},\boldsymbol{\theta}}{\min.} & \left(-\frac{E(t_{\text{final}})}{t_{\text{final}}}\right) & \\
\text{st.} & \text{collocation} & \dot{\boldsymbol{x}} - f(\boldsymbol{x},\boldsymbol{u},\boldsymbol{\lambda},\boldsymbol{\theta}) = 0 \\
 & \text{consistency} & \boldsymbol{x}(t)_{d+1} - \boldsymbol{x}(t+\Delta t)_1 = \mathbf{0} \\
 & \text{initial energy} & E(t_{\text{initial}}) = 0 \\
 & \text{periodicity} & [\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\omega},l_{\mathrm{T}},\dot{l}_{\mathrm{T}}]^\top(t_{\text{final}}) - [\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\omega},l_{\mathrm{T}},\dot{l}_{\mathrm{T}}]^\top(t_{\text{initial}}) = \mathbf{0} \\
 & & \mathscr{P}_{\text{lower-triangular}}\left(\mathbf{R}(t_{\text{final}}) - \mathbf{R}(t_{\text{initial}})\right) = \mathbf{0} \\
 & \text{orthonormality} & \mathscr{P}_{\text{upper-triangular}}\left(\mathbf{R}^\top\mathbf{R} - \mathbf{I}\right) = \mathbf{0} \\
 & \text{orientation} & \alpha_{\min} \leq \alpha(\boldsymbol{x}) \leq \alpha_{\max},\ \beta_{\min} \leq \beta(\boldsymbol{x}) \leq \beta_{\max} \\
 & \text{simple bounds} & z_{\mathrm{Ref}} \leq \boldsymbol{q}_z,\ \boldsymbol{\delta}_{\min} \leq \boldsymbol{\delta} \leq \boldsymbol{\delta}_{\max},\ 0 \leq l_{\mathrm{T}} \leq l_{\mathrm{T}\max}, \\
 & & \dot{l}_{\mathrm{T}\min} \leq \dot{l}_{\mathrm{T}} \leq \dot{l}_{\mathrm{T}\max},\ 0 \leq t_{\text{final}} \leq t_{\text{final}\max}, \\
 & & l_{\mathrm{S}\min} \leq l_{\mathrm{S}} \leq l_{\mathrm{S}\max}
\end{array}
$$

The example OCP, rewritten as homotopy

$\boldsymbol{x}$: [$\boldsymbol{q}$, $\dot{\boldsymbol{q}}$ (for tether nodes), $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, $\boldsymbol{\omega}$, $\mathbf{R}$ (for kite nodes), and l_T, $\dot{l}_\mathrm{T}$, E $]^\top$.
$\boldsymbol{u}$: [$\boldsymbol{f}$, $\boldsymbol{m}$, $\boldsymbol{\delta}$ (for kite nodes), and $\ddot{l}_\mathrm{T}$ $]^\top$
$\boldsymbol{\lambda}$: [λ (for tether and kite nodes) $]^\top$, $\boldsymbol{\theta}$: [t_final, l_S, γ, ι $]^\top$

$$\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{\lambda},\boldsymbol{\theta}} \quad \iota\left(w_\text{gamma}\ \text{gamma} + w_\text{hmtp.}\ \text{homotopy} + w_\text{tracking}\ \text{tracking} + w_\text{time}\ \text{time}\right)$$
$$+(1-\iota)w_\text{energy}\left(-\frac{E(t_\text{final})}{t_\text{final}}\right) + w_\text{reg.}\ \text{regularization} + w_\text{iota}\ \text{iota}$$

st. collocation, consistency, initial energy, periodicity,...
orthonormality, orientation, simple bounds, $0 \leq \iota, \gamma \leq 1$

gamma : γ

homotopy : $[\boldsymbol{f}^\top, \boldsymbol{m}^\top]\ \mathbf{W}_\text{homotopy}\ [\boldsymbol{f}, \boldsymbol{m}]^\top$

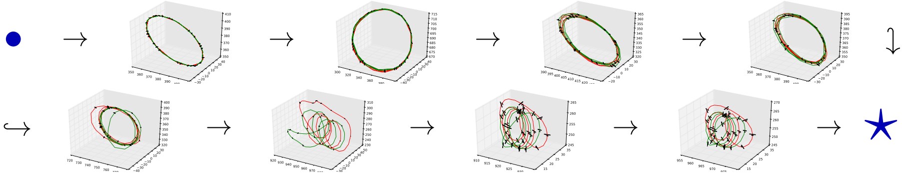
tracking : $\left([\boldsymbol{q}^\top, \dot{\boldsymbol{q}}^\top, \boldsymbol{\omega}^\top, \boldsymbol{R}^\top, l_\mathrm{T}, \dot{l}_\mathrm{T}] - [\boldsymbol{q}^\top, \dot{\boldsymbol{q}}^\top, \boldsymbol{\omega}^\top, \boldsymbol{R}^\top, l_\mathrm{T}, \dot{l}_\mathrm{T}]_\mathrm{Ref}\right) \mathbf{W}_\text{tracking}$
$\left([\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, \boldsymbol{R}, l_\mathrm{T}, \dot{l}_\mathrm{T}]^\top - [\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\omega}, \boldsymbol{R}, l_\mathrm{T}, \dot{l}_\mathrm{T}]^\top_\mathrm{Ref}\right)$

time : $(t_\text{final} - t_\text{finalRef})^2$

regulariz. : $[\boldsymbol{\delta}^\top, \ddot{l}_\mathrm{T}]\ \mathbf{W}_\text{regulariz.}\ [\boldsymbol{\delta}, \ddot{l}_\mathrm{T}]^\top$

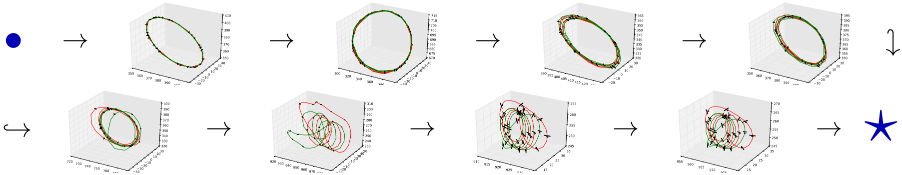
iota : ι

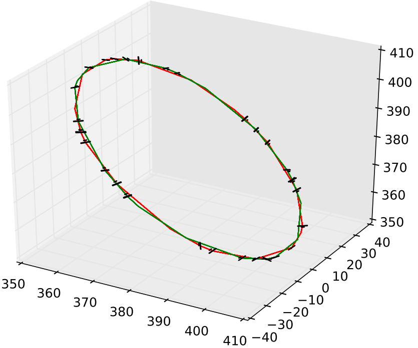
Homotopy path



... solve with the hippo strategy!

Homotopy path

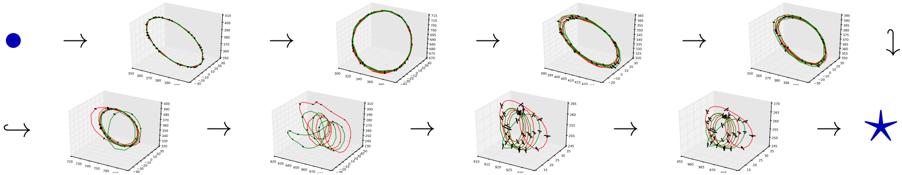


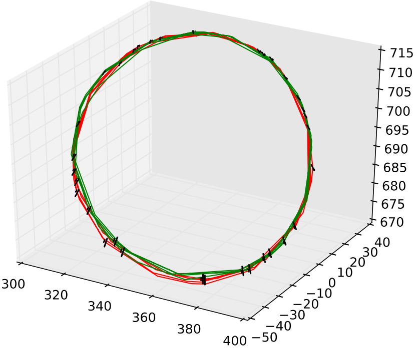


Find Feasible Start

pure tracking problem
$(\gamma = 1,\ \iota = 1)$

Homotopy path

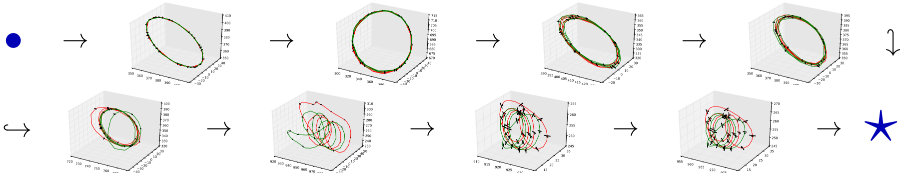


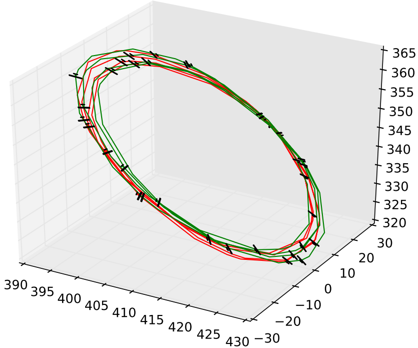


Aerodynamic Force Homotopy

switch from fictitious to modelled forces $(0 \leq \gamma \leq 1)$

Homotopy path

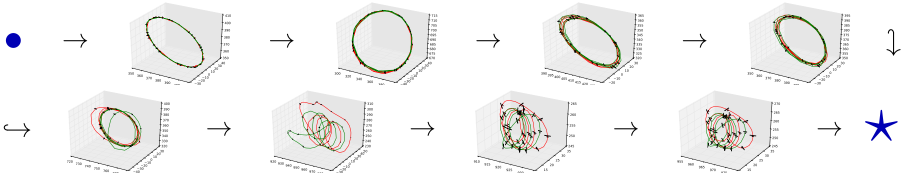


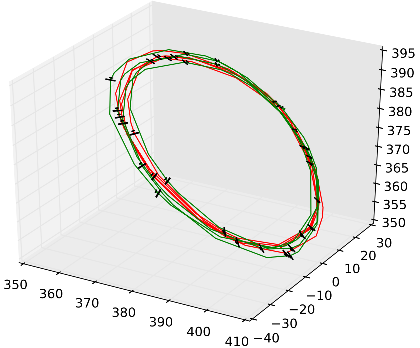


Enforce Aerodynamic Forces

force end of aerodynamic force homotopy ($\gamma = 0$)

Homotopy path

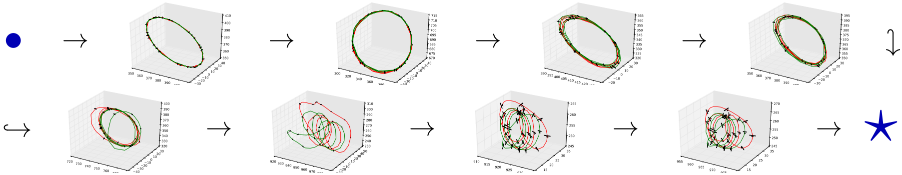


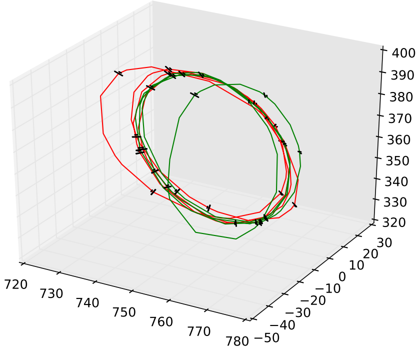


Free Period

release t_{final} equality constraints $(0 \leq t_{\text{final}} \leq t_{\text{final}_{\max}})$

Homotopy path

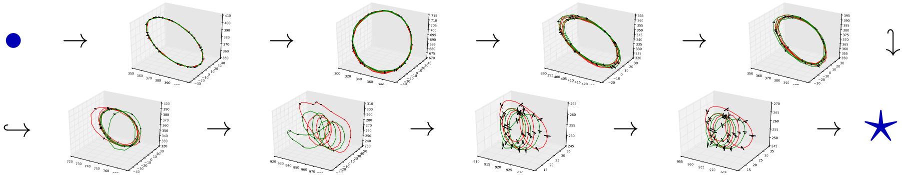


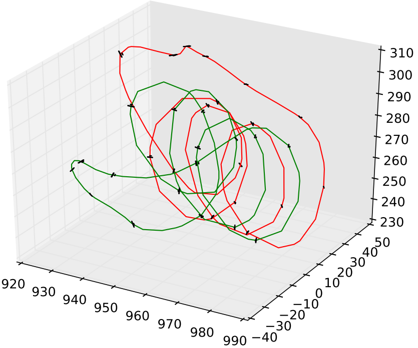


Energy Homotopy

switch from tracking to energy problem $(0 \leq \iota \leq 1)$

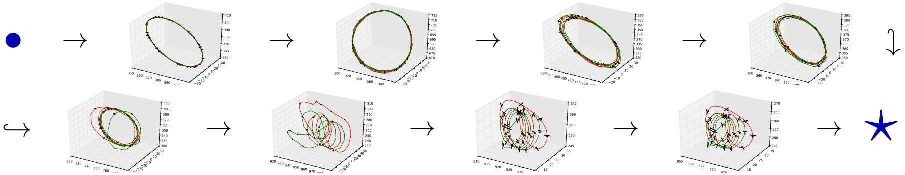
Homotopy path

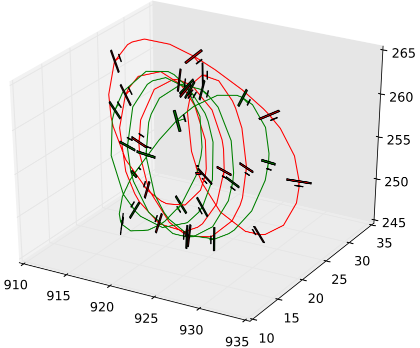




Enforce Energy Maximization

force end of energy homotopy $(\iota = 0)$

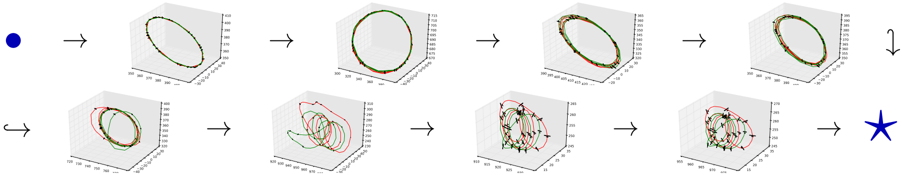


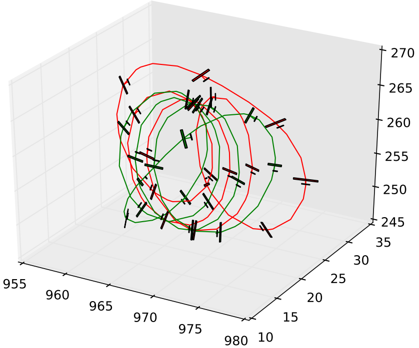


Free Geometry

release l_S equality constraints $(l_{S\min} \leq l_S \leq l_{S\max})$

Homotopy path





Final solution

solve to full tolerance and interior-point parameter

Concerning the aerodynamic forces

$$\frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{\boldsymbol{q}}} - \frac{\partial \mathscr{L}}{\partial \boldsymbol{q}} = \boldsymbol{F} \quad = \quad \gamma \boldsymbol{f} + (1-\gamma)\boldsymbol{f}_{\mathrm{aero}}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{J}\boldsymbol{\omega}) \quad = \quad \gamma \boldsymbol{m} + (1-\gamma)\boldsymbol{m}_{\mathrm{aero}}$$

How do we find the aerodynamic forces and moments?

Concerning the aerodynamic forces

$$\frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{\boldsymbol{q}}} - \frac{\partial \mathscr{L}}{\partial \boldsymbol{q}} = \boldsymbol{F} = \gamma \boldsymbol{f} + (1-\gamma)\boldsymbol{f}_{\text{aero}}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{J}\boldsymbol{\omega}) = \gamma \boldsymbol{m} + (1-\gamma)\boldsymbol{m}_{\text{aero}}$$

How do we find the aerodynamic forces and moments?

Direct Coefficient Control
with roll-control, as in [Zanon2013a]

Pre-Computed Aerodynamics
stability derivatives, computed remotely.

Geometry-Based Aerodynamics
in-built potential flow methods

(necessary for geometry optimization)

Concerning the aerodynamic forces

Direct Coefficient Control	Pre-Computed Aerodynamics	Geometry-Based Aerodynamics
with roll-control, as in [Zanon2013a]	stability derivatives, computed remotely.	in-built potential flow methods
NI	NI, SB (- NT, - CT, - KT), LLT	LLT, QPW, UFW (maybe)

NI = 'no-induction'

NT = 'no-tilt'

KT = 'MAWES-specific tilt model'

QPW = 'presecribed-wake VLM'

SB = 'single-element BEM'

CT = 'Glauert/Coleman tilt model'

LLT = 'lifting-line theory'

UFW = 'unsteady free-wake VLM'

Concerning the aerodynamic forces

Direct Coefficient Control
with roll-control, as in [Zanon2013a]

NI ✓

Pre-Computed Aerodynamics
stability derivatives, computed remotely.

NI ✓

SB
- NT *
- CT
- KT

LLT

∗ hopefully soon!

Geometry-Based Aerodynamics
in-built potential flow methods

LLT

QPW

UFW (maybe)

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Concerning the aerodynamic forces

Direct Coefficient Control	Pre-Computed Aerodynamics	Geometry-Based Aerodynamics
with roll-control, as in [Zanon2013a]	stability derivatives, computed remotely.	in-built potential flow methods

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□ and □ - two verification projects in progress with Thomas Haas.

Conclusion

AWEbox development slowly (but surely) progresses...

thanks for the attention! any questions?