

**Exercise 1: Resistance Estimation Example**  
(to be returned before November 2nd, 8:15)

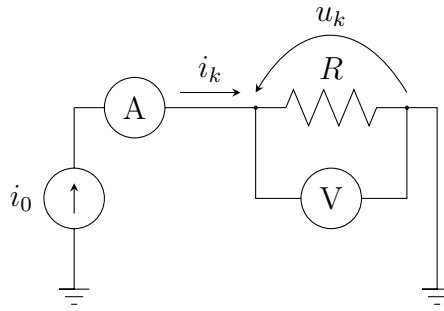
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In this exercise you investigate some important facts from statistics in numerical experiments.

**Exercise Tasks**

1. We consider the following experimental setup:



Imagine you are sitting in a class of 200 electrical engineering students and you want to estimate the value of  $R$  using Ohm's law. Since the value of the current  $i_0$  flowing through the resistor is not known exactly, an ammeter is used to measure the current  $i_k$  and a voltmeter to measure  $u_k$ . Every student is taking 1000 measurements. The measurement number is represented by  $k$ . We assume that the measurements are noisy:

$$i_k = i_0 + n_{i,k} \quad \text{and} \quad u_k = u_0 + n_{u,k}$$

where  $u_0 = 10 \text{ V}$  is the true values of the voltage across the resistor,  $i_0 = 5 \text{ A}$  is the true value of the current flowing through the resistor and  $n_{i,k}$  and  $n_{u,k}$  are the values of the noise.

Please consider the data-set with all measurements of all students provided in the repository.

Let us now investigate the behaviour of the three different estimators which were introduced in the lecture:

$$\hat{R}_{\text{SA}}^{[N]} = \frac{1}{N} \sum_{k=1}^N \frac{u_k}{i_k} \quad \hat{R}_{\text{LS}}^{[N]} = \frac{\frac{1}{N} \sum_{k=1}^N u_k i_k}{\frac{1}{N} \sum_{k=1}^N i_k^2} \quad \hat{R}_{\text{EV}}^{[N]} = \frac{\frac{1}{N} \sum_{k=1}^N u_k}{\frac{1}{N} \sum_{k=1}^N i_k}$$

We will write code to simulate the behavior of these estimators. For each of the three estimators, carry out the following tasks.

- (a) **CODE:** Compute the result of the function  $\hat{R}_*^{[N]}$ , for  $N = 1, \dots, N_{\text{max}}$  using your personal measurements (student 1 or experiment 1). Do this for each estimator ( $*$  can be either SA, LS or EV). Plot the three curves in one plot.  
**ON PAPER:** Do the estimators converge for  $N \rightarrow \infty$ ? (5 points)

- (b) CODE: It is good practice to analyze the results of several experiments to cancel noise. Luckily, you get the datasets of all other students. Plot the function  $\hat{R}_*^{[N]}$ ,  $N = 1, \dots, N_{\max}$  for each estimator (\* can be either SA, LS or EV). To see the stochastic variations, plot all these functions in one graph per estimator.  
ON PAPER: Do you see any difference to the plot from task (b)? (2 points)
- (c) CODE: Compute the mean of  $\hat{R}_*^{[N]}$  over all experiments (all 200 students) and plot it for  $N$  from 1 to  $N_{\max}$ . (1 point)
- (d) CODE: Plot a histogram containing all values of  $\hat{R}_*^{[N_{\max}]}$ .  
ON PAPER: Discuss what makes the difference. (2 points)