

Differentiable NMPC with acados

Katrin Baumgärtner

University of Freiburg

Predict, Control, Learn: Combining Model Predictive Control and Reinforcement Learning
Tutorial Session at ECC 2026, Reykjavík, July 2026

universität freiburg



NTNU

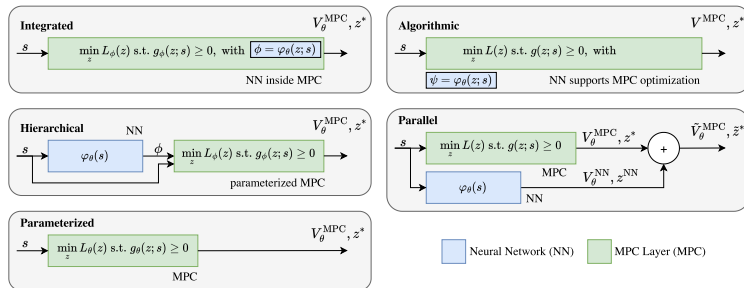
Norwegian University of
Science and Technology



**University of
Zurich** ^{UZH}



ROBOTICS &
PERCEPTION
GROUP



Building blocks for practical implementations.

- ▶ Efficient numerical solvers for optimal control problems.
- ▶ Availability of solution sensitivities with respect to problem parameters.

Tutorial Roadmap

<i>Talk</i>	<i>Presenter</i>	<i>Duration</i>
✓ Markov Decision Processes – a unifying perspective on MPC and RL	Jasper Hoffmann	20min
✓ Synthesis of Model Predictive Control and Reinforcement Learning	Rudolf Reiter	30min
✓ Reinforcement Learning over MPC: Why does it work?	Dirk Reinhardt	30min
➤ Differentiable NMPC with acados	Katrin Baumgärtner	20min
MPCRL with leap-c	Jasper Hoffmann	20min

At the end of each talk there will be time for questions!



Tutorial webpage for slides
and more!

Parameterized OCP with parameters ϕ .

OCP-structured NLP.

$$\begin{aligned} \min_{\substack{x_0, \dots, x_N, \\ u_0, \dots, u_{N-1}}} \quad & \bar{V}_\phi^{\text{MPC}}(x_N) + \sum_{k=0}^{N-1} l_\phi^{\text{MPC}}(x_k, u_k) \\ \text{s.t.} \quad & x_0 = s, \\ & x_{k+1} = f_\phi^{\text{MPC}}(x_k, u_k), \quad 0 \leq k < N, \\ & 0 \leq h_\phi^{\text{MPC}}(x_k, u_k), \quad 0 \leq k < N, \\ & 0 \leq \tilde{h}_\phi^{\text{MPC}}(x_N). \end{aligned}$$

Parameters ϕ can affect the objective L_ϕ , the dynamics f_ϕ^{MPC} , and the constraints h_ϕ^{MPC} .



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- ⚡ Written in C using high-performance linear algebra provided by BLASFEO
- ⚡ Fully exploits sparsity of optimal control structured NLPs



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- 🎯 NLP solvers: SQP, DDP, RTI, AS-RTI
- 🛡 Robust & stochastic MPC via zoRO
- 🧠 Exploit convex-over-nonlinear structures
- 🎨 Customize the solver by providing a tailored Hessian or Jacobian approximation



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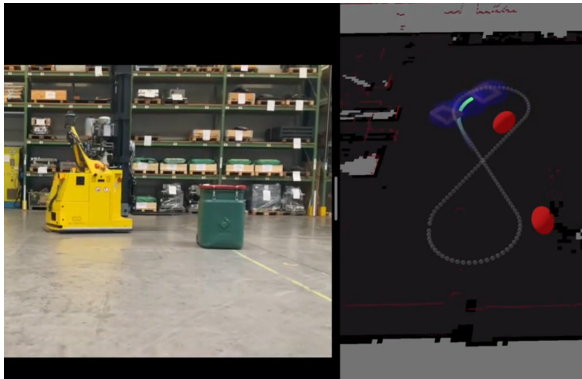
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★ github.com/acados/acados

📖 docs.acados.org

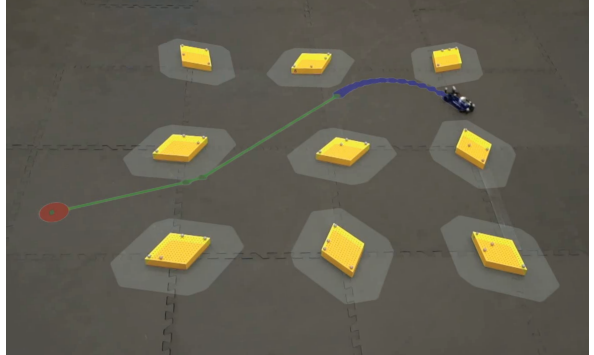
💬 discourse.acados.org

- Mobile robotics with obstacle avoidance



(Subash et al., 2024)

- Mobile robotics with obstacle avoidance



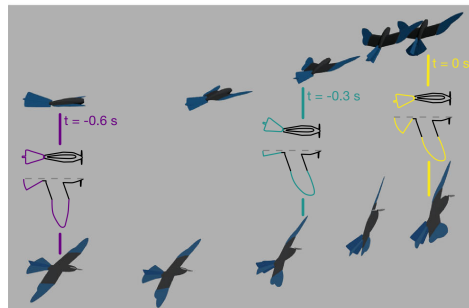
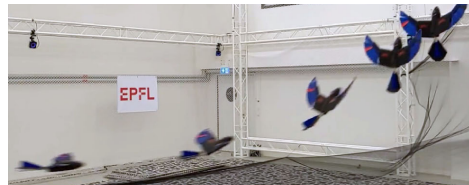
(Köhler et al., 2025)

- ▶ Mobile robotics with obstacle avoidance
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- ▶ Autonomous e-scooters



(Meister et al., 2025)

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Short notation:

$$\begin{aligned} z^{\text{sol}}(\phi) &:= \arg \min_{z \in \mathbb{R}^{n_z}} f(z; \phi) \\ \text{s.t.} \quad & g(z; \phi) = 0, \\ & h(z; \phi) \leq 0. \end{aligned}$$

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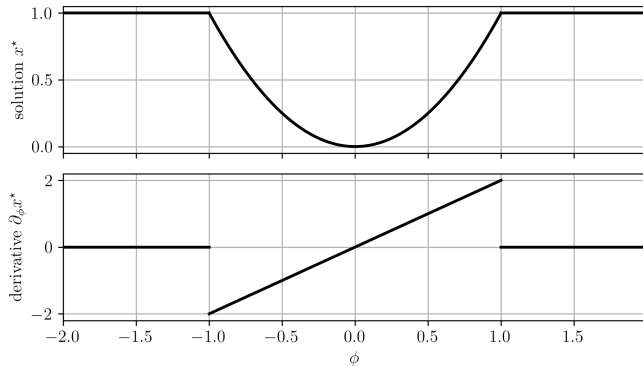
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Wanted: $\frac{\partial z^{\text{sol}}}{\partial \phi}(\phi)$ 🧐💰

Illustrative example

Scalar example

$$\begin{aligned} \min_x \quad & (x - \phi^2)^2 \\ \text{s.t.} \quad & -1 \leq x \leq 1. \end{aligned}$$



KKT conditions & smoothing

Parametric optimization problem.

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KKT conditions

$$\begin{aligned} \nabla_z f(z; \phi) + \nabla_z g(z; \phi) \lambda + \nabla_z h(z; \phi) \mu &= 0, \\ g(z; \phi) &= 0, \\ h(z; \phi) &\leq 0, \\ \mu &\geq 0, \\ \mu_i h_i(z; \phi) &= 0, \quad i = 1, \dots, n_h. \end{aligned}$$

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Interior-point smoothed KKT conditions.

$$\begin{aligned} \nabla_z f(z; \phi) + \nabla_z g(z; \phi) \lambda + \nabla_z h(z; \phi) \mu &= 0, \\ g(z; \phi) &= 0, \\ h(z; \phi) &\leq 0, \\ \mu &\geq 0, \\ \mu_i h_i(z; \phi) &= \tau, \quad i = 1, \dots, n_h. \end{aligned}$$

Interior-point methods (IPM)
solve this for $\tau \rightarrow 0$,
e.g. IPOPT, HPIPM, FORCES,
Clarabel, fmincon, ...

The implicit function theorem

Regularity assumptions.

- ▶ f, g, h are twice differentiable in z and once differentiable in ϕ .
- ▶ $z^* = z^{\text{sol}}(\bar{\phi})$ is a KKT point of the parametric NLP with parameter $\bar{\phi}$ satisfying LICQ, SOSC, and strict complementarity.

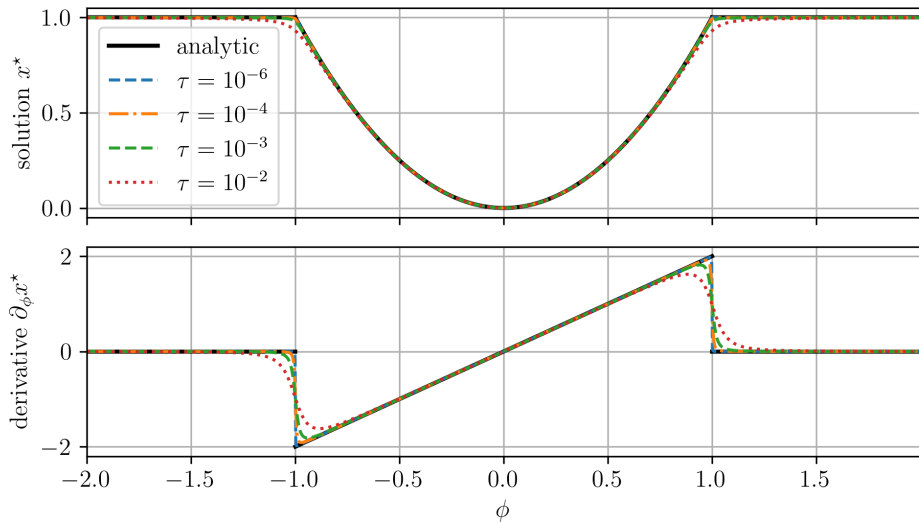
Differentiable solution map.

If SOSC holds at z^* with respect to the barrier problem with barrier parameter $\tau > 0$, the **implicit function theorem** implies that

1. There exists a solution of the smoothed interior-point KKT system $z_{\text{IPM}}^{\text{sol}}(\tau; \bar{\phi})$ with $\lim_{\tau \rightarrow 0^+} z_{\text{IPM}}^{\text{sol}}(\tau, \bar{\phi}) = z^*$ and $\|z_{\text{IPM}}^{\text{sol}}(\tau; \bar{\phi}) - z^*\| \in \mathcal{O}(\tau)$.
2. In a neighborhood of $\bar{\phi}$, there exists a **differentiable** function $v(\tau; \phi) = (z(\tau; \phi), \lambda(\tau; \phi), \mu(\tau; \phi))$ that corresponds to a locally unique solution of the smoothed interior-point KKT system.

Proofs are given in Pirnay, López-Negrete, and Biegler (2012); Wright (1992).

Illustrative example



Nominal solution. Smoothed interior-point KKT system is solved by iteratively solving **quadratic barrier problems** (barrier QPs):

1. Approximate the NLP by a QP.
2. Solve the smoothed interior-point KKT system associated with the QP with an interior-point solver for $\tau \rightarrow \bar{\tau}$ with $\bar{\tau} > 0$.

Equivalent to SQP
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Solution sensitivities. A linear system with the same matrix as for the nominal solution needs to be solved.

- ▶▶ Reuse efficient implementation via Riccati recursion provided by HPIPM (Frison & Diehl, 2020) exploiting the **OCP structure**.

Hessian approximations in SQP.

- ▶ (generalized and extended) Gauss-Newton Hessian
(Messerer et al., 2021; Baumgärtner & Diehl, 2022)
- ▶ regularization techniques such as project, mirror, Levenberg-Marquardt, convexify
(Verschuere et al., 2017)

Benefits are **reduced computational complexity** and **convex** QP subproblems.

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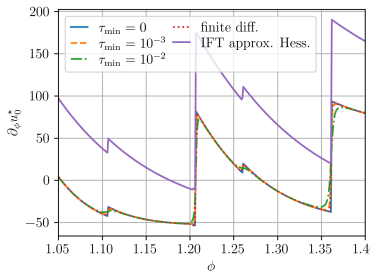
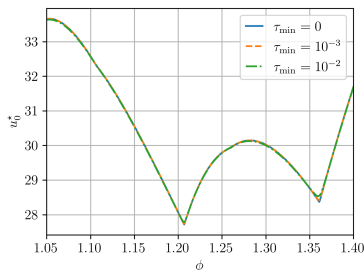
Hessian approximations & two-solver approach

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Nominal solver

Solve nominal problem using **any** algorithm.

- ▶ Hessian approximation
- ▶ regularization
- ▶ ...



Sensitivity solver

- ▶ Evaluate exact Hessian.
- ▶ Compute solution sensitivity via efficient linear solve based on the Riccati recursion provided by HPIPM.

Forward vs. adjoint sensitivities

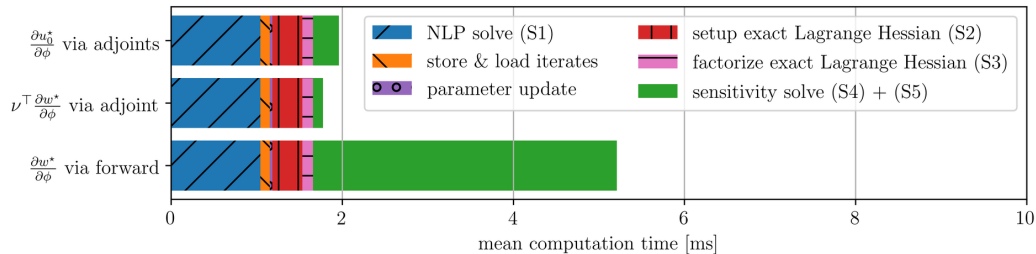


Figure: Computation times for a nominal solve plus forward or adjoint sensitivities. In this example, $n_x = 9$, $n_u = 3$, $n_\phi = 113$, $N = 40$

Forward sensitivity. Compute full Jacobian $\frac{\partial w^*}{\partial \phi} \in \mathbb{R}^{n_w \times n_\phi}$.

Adjoint sensitivity. Compute single adjoint direction $\nu^\top \frac{\partial w^*}{\partial \phi} \in \mathbb{R}^{1 \times n_\phi}$. ← backpropagation

Here, $w^* = (z^*, \lambda^*, \mu^*)$.

Benchmark Results

Table: Timings in [ms] for solving $n_{\text{batch}} = 128$ bounded LQR problems with $N = 20$, $n_x = 8$, $n_u = 4$, $n_\phi = 248$. In parentheses are multiples of the acados runtime.

u_{max}	acados	Nominal solution		Solution + adjoint sens.		
		mpc.pytorch	cvxpygen	acados	mpc.pytorch	cvxpygen
10^4	8.5	78 ($\times 9.2$)	262 ($\times 31$)	34.5	125 ($\times 3.6$)	658 ($\times 19$)
1.0	17.6	21024 ($\times 1200$)	6402 ($\times 360$)	42.0	21899 ($\times 520$)	6845 ($\times 160$)

Remark: mpc.pytorch is not faster on GPU than CPU (both in our experiments and in Adabag et al. (2025))



Smoothed interior-point KKT conditions to differentiate across active-set changes

Summary

- 🔮 Smoothed interior-point KKT conditions to differentiate across active-set changes
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Thanks for your attention! 🙏

Looking forward to questions, discussions and collaborations!

Preprint. Jonathan Frey, Katrin Baumgärtner, Gianluca Frison, Dirk Reinhardt, Jasper Hoffmann, Leonard Fichtner, Sebastien Gros, Moritz Diehl: *Differentiable Nonlinear Model Predictive Control* ([arxiv link](#))

Code for figures and benchmark. [GitHub repository](#)

★ Give us a star on GitHub: github.com/acados/acados ★

Related work: no other differentiable solver tailored to OCPs with inequality support

Software / publication	OCP-tailored	adjoints	ineq. support	Nonlinearity in cost & constraints	exact results	parallelization	permissive license
acados	✓	✓	✓	✓	✓	OpenMP	✓ BSD-2
mpc.pytorch (Amos et al., 2018)	✓	✓	✗ ^a	✗	✗	PyTorch	✓ MIT
cvxpygen + cvxpylayers (Schaller et al., 2022) (Schaller & Boyd, 2025)	✗	✓	✓	⚪ ^b	✓	Python multithreading	✓ Apache 2.0
diffmpc (Toyota) (Adabag et al., 2025)	✓	✓	✗	✓	✗	JAX	✗ CC BY- NC-ND 4.0
trajax (Google) no paper	✓	✓	✗	✓	⚪ ^c	JAX	✓ Apache 2.0
theseus (Facebook) (Pineda et al., 2022)	✗	✓	✗	✗	✗	PyTorch	✓ MIT
sIPOPT (Pirnay et al., 2012) ^d	✗	✗	✓	✓	✓	–	✓ EPL
CasADi native (Andersson & Rawlings, 2018)	✗	✓	✓	✓	✓	–	✓ LGPL

^a while simple control bounds can be added, experiments revealed that the solver fails to converge if active set is nontrivial.

^b only convex cost and constraints.

^c a set of options seems to exist that allow using the exact Hessian. The documentation does not indicate in which case the sensitivities are accurate.

^d functionality extremely limited in practice. Not usable in CasADi and abandoned rewrite effort in <https://github.com/athrpf/sipopt2>.

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