Exercises for Course on Modeling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2025-2026

Exercise 1: Resistance Estimation Example

(to be returned before October 27th, 8:15)

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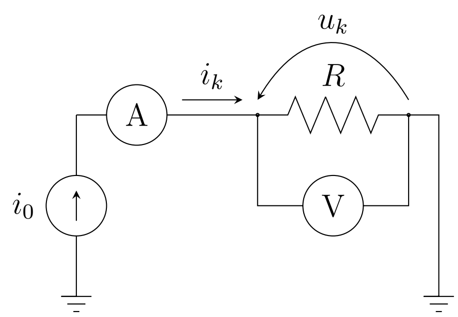
In this exercise you investigate some important facts from statistics in numerical experiments.

Exercise Tasks

1. We consider the following experimental setup:



Imagine you are sitting in a class of 200 electrical engineering students and you want to estimate the value of R using Ohm's law. Since the value of the current i_0 flowing through the resistor is not known exactly, an ammeter is used to measure the current i_k and a voltmeter to measure u_k. Every student is taking 1000 measurements. The measurement number is represented by k. We assume that the measurements are noisy:

$$i_k = i_0 + n_{i,k} \qquad \text{and} \qquad u_k = u_0 + n_{u,k}$$

where $u_0 = 10\,\mathrm{V}$ is the true values of the voltage across the resistor, $i_0 = 5\,\mathrm{A}$ is the true value of the current flowing through the resistor and $n_{i,k}$ and $n_{u,k}$ are the values of the noise.
Please consider the data-set with all measurements of all students provided in the repository.

Let us now investigate the behaviour of the three different estimators which were introduced in the lecture:

$$\hat{R}_{\mathrm{SA}}^{[N]} = \frac{1}{N}\sum_{k=1}^{N}\frac{u_k}{i_k} \qquad \hat{R}_{\mathrm{LS}}^{[N]} = \frac{\frac{1}{N}\sum_{k=1}^{N} u_k i_k}{\frac{1}{N}\sum_{k=1}^{n} i_k^2} \qquad \hat{R}_{\mathrm{EV}}^{[N]} = \frac{\frac{1}{N}\sum_{k=1}^{N} u_k}{\frac{1}{N}\sum_{k=1}^{N} i_k}$$

We will write code to simulate the behavior of these estimators. For each of the three estimators, carry out the following tasks.

(a) CODE: Compute the result of the function $\hat{R}_{*}^{[N]}$, for $N = 1, \dots, N_{\max}$ using your personal measurements (student 1 or experiment 1). Do this for each estimator ($*$ can be either SA, LS or EV). Plot the three curves in one plot.
ON PAPER: Do the estimators converge for $N \to \infty$? (5 points)

(b) CODE: It is good practice to analyze the results of several experiments to cancel noise. Luckily, you get the datasets of all other students. Plot the function $\hat{R}_*^{[N]}$, $N = 1, \dots, N_{\text{max}}$ for each estimator ($*$ can be either SA, LS or EV). To see the stochastic variations, plot all these functions in one graph per estimator.
ON PAPER: Do you see any difference to the plot from task (b)? (2 points)

(c) CODE: Compute the mean of $\hat{R}_*^{[N]}$ over all experiments (all 200 students) and plot it for N from 1 to N_{max}. (1 point)

(d) CODE: Plot a histogram containing all values of $\hat{R}_*^{[N_{\text{max}}]}$.
ON PAPER: Discuss what makes the difference. (2 points)

This sheet gives in total 10 points.