Orientation

So far we have seen...

- The building blocks
  - MDPs, DP, Bellman, Policy and Value Iteration, Reinforcement Learning, Actor-Critic
  - LQR, Dynamic Optimization, Nonlinear Programming, MPC, Sensitivities
- MPC & RL
  - Synthesis - Different flavors of ML / RL and MPC
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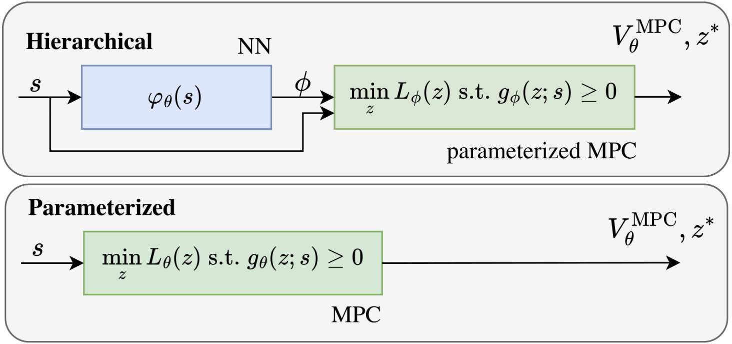
- Beyond MPC – Model-based Decisions and AI for decisions *(Tomorrow)*

Focus of today's lectures (Leap-c structure)



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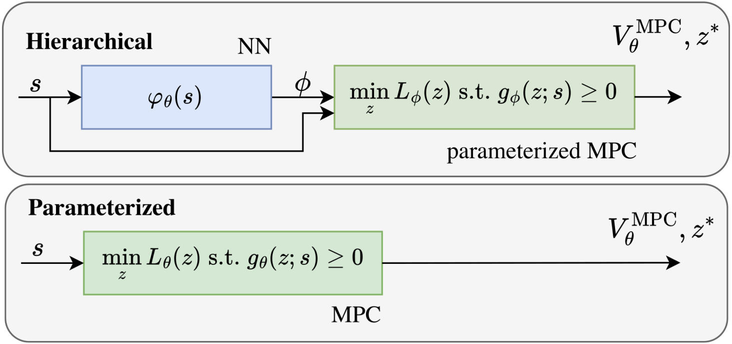

Normal thinking when using MPC

- Fit MPC model to reality as good as possible (SYSID)
- MPC cost is what we want to minimize (energy, time, money, reference)
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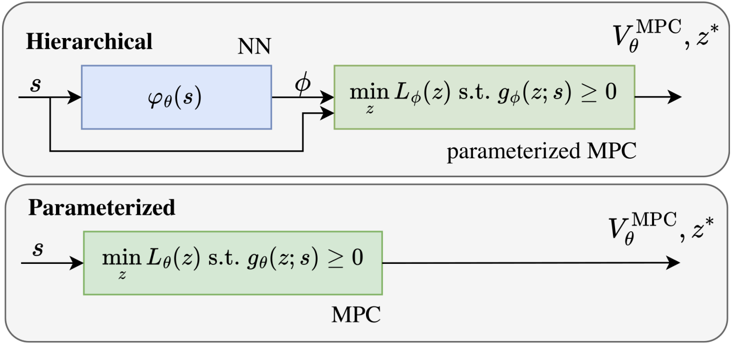

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We are talking about changing everything!!

Reinforcement Learning Over MPC
Why Does It Work?

Sebastien Gros

Dept. of Cybernetic, NTNU
Faculty of Information Tech.

Freiburg PhD School

Outline

Model Predictive Control (MPC)

Optimize a plan over finite horizon, apply first move, repeat

MPC: at current state $\mathbf{s}$

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\begin{aligned}
\min_{\mathbf{x},\mathbf{u}} \quad & T(\mathbf{x}_N) + \sum_{k=0}^{N-1} L(\mathbf{x}_k, \mathbf{u}_k) \\
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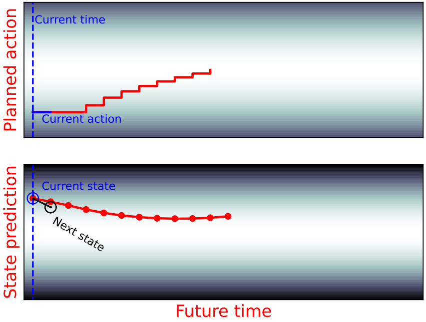

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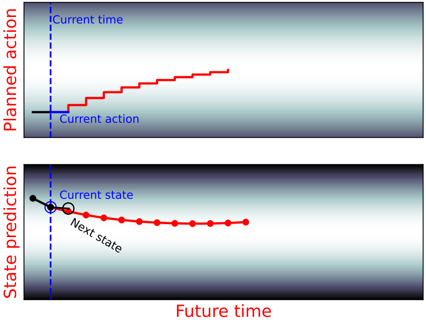

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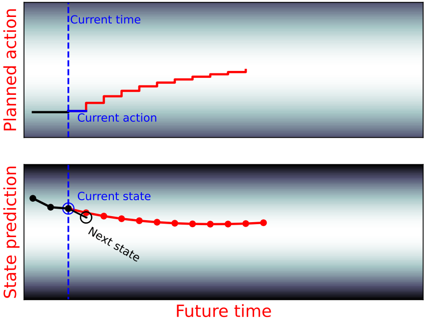

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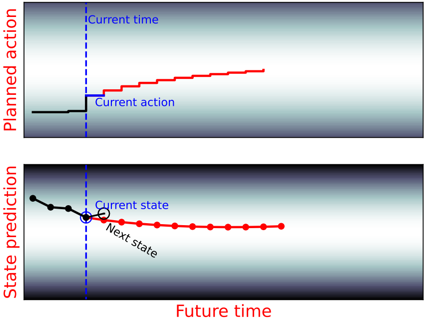

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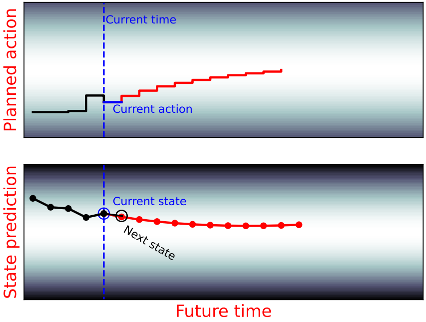

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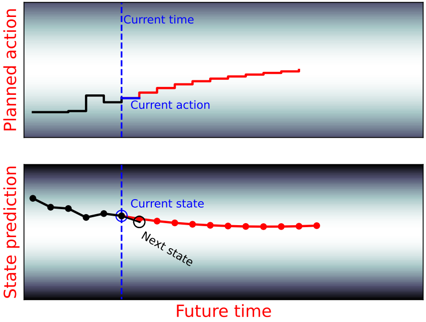

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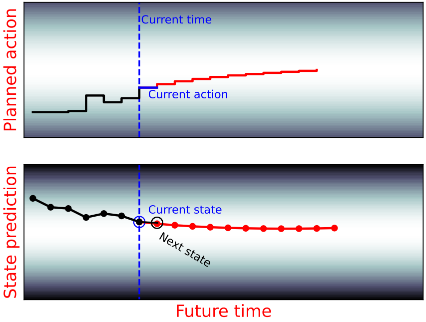

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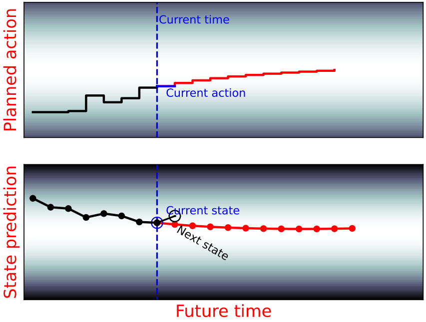

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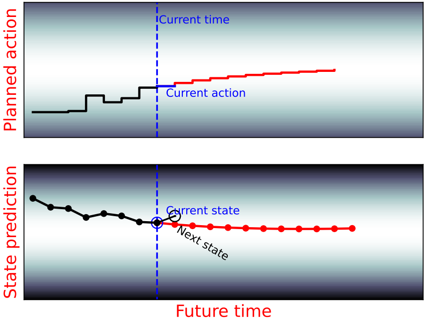

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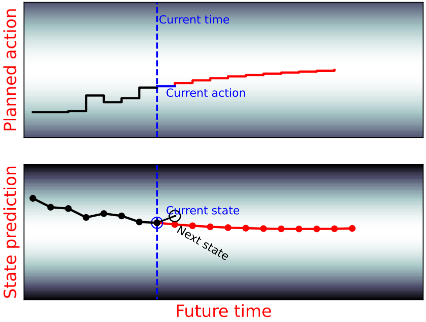

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What an odd thing to do: we build and throw away plans all the time knowing that they are wrong all along, but kinda use them anyway...

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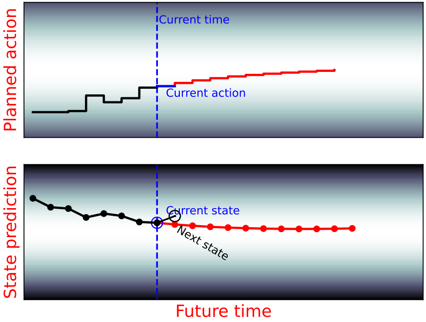

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MPC is a powerful tool to control constrained systems, increasingly used as a practical way of building optimal policies

Theoretical Framework to connect RL and MPC

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- Model driven
- Policies from planning
- Constraints oriented

Reinforcement Learning (RL)

- Data driven
- Optimal policies from learning
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But understanding what we are doing is about connecting MPC to MDPs!!

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$$\mathbf{s}, \mathbf{a} \rightarrow \mathbf{s}_+$$

(state-action $\rightarrow$ next state)

Cost function (instant performance)
$L(\mathbf{s}, \mathbf{a}) \in \mathbb{R}$

A (fairly) general way of describing optimal control

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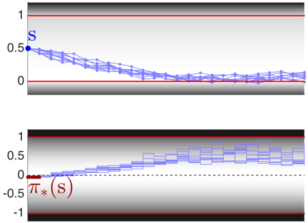

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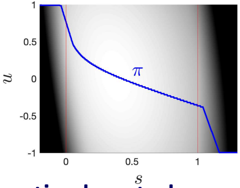

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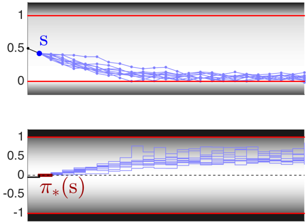

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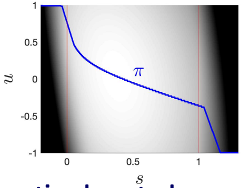

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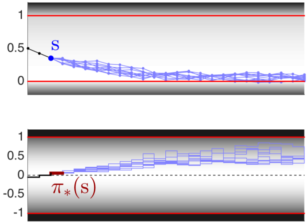

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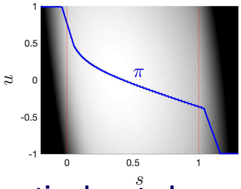

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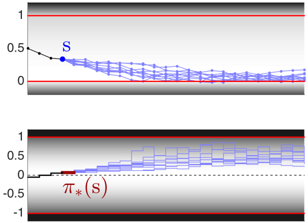

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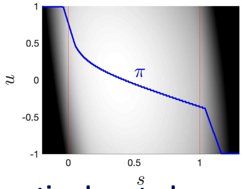

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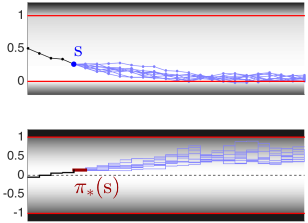
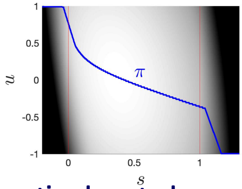




A (fairly) general way of describing optimal control

Markov Decision Processes (MDP)

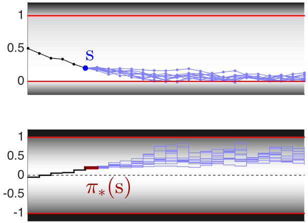
Stochastic state transitions (real world)

$$\mathbf{s}, \mathbf{a} \rightarrow \mathbf{s}_+$$

(state-action $\rightarrow$ next state)

Cost function (instant performance)

$$L(\mathbf{s}, \mathbf{a}) \in \mathbb{R}$$



- **Policy**

$$\mathbf{a} = \boldsymbol{\pi}(\mathbf{s})$$

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- **Closed-loop performance**

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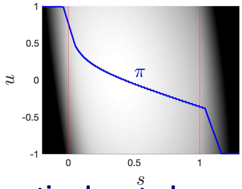

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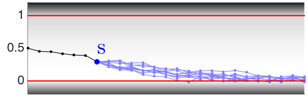

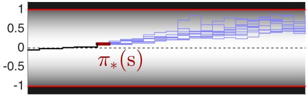

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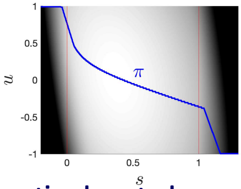

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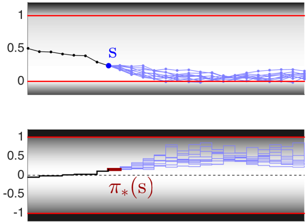

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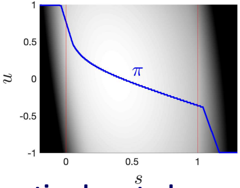

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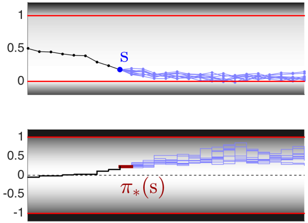

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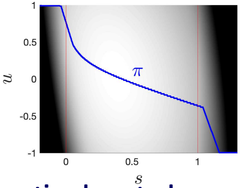

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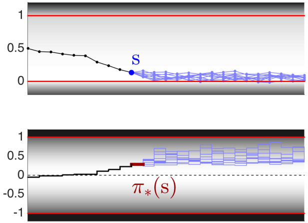

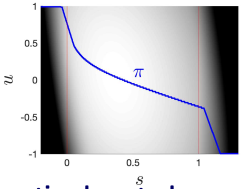

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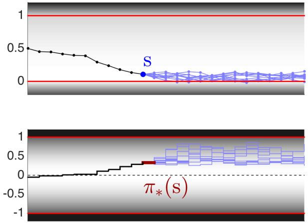

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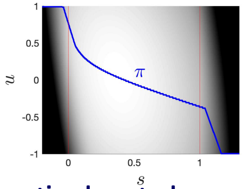

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$$L(\mathbf{s}, \mathbf{a}) = \begin{cases} \ell(\mathbf{s}, \mathbf{a}) & \text{if} \quad \mathbf{h}(\mathbf{s}, \mathbf{a}) \leq 0 \\ \infty & \text{if} \quad \mathbf{h}(\mathbf{s}, \mathbf{a}) > 0 \end{cases}$$

I will use the same view in MPC for a bit

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Solution of an MDP is described by "simple" equations, but solving them is very challenging

A (fairly) general way of describing optimal control

From Policy to Repeated Planning (MPC)

Infinite horizon & discounted

$$\boldsymbol{\pi}^\star_\infty = \arg\min_{\boldsymbol{\pi}} \mathbb{E}\left[\sum_{k=0}^{\infty} \gamma^k L(\mathbf{x}_k, \mathbf{a}_k)\right]$$

Policy $\boldsymbol{\pi}$: state $\rightarrow$ action
belongs to a function space

Let's transform an MDP into an MPC and understand the approximations we make

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Finite-horizon equivalent:

$$\boldsymbol{\pi}^\star_{0,\dots,N-1} = \arg\min_{\boldsymbol{\pi}_{0,\dots,N-1}} \mathbb{E}\left[T(\mathbf{s}_N) + \sum_{k=0}^{N-1} \gamma^k L(\mathbf{s}_k, \mathbf{a}_k)\right]$$

If $T = V^\star$, then $\boldsymbol{\pi}^\star_{0,\dots,N-1} = \boldsymbol{\pi}^\star_\infty$

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Planning instead of a policy:

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i.e. restrict policies to fixed $\mathbf{a}_{0,\ldots,N-1}$

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$$\begin{aligned} \min_{\boldsymbol{\pi}_{0,\ldots,N-1}} \quad & T(\mathbf{s}_N) + \sum_{k=0}^{N-1} \gamma^k L(\mathbf{s}_k, \mathbf{a}_k) \\ \text{s.t} \quad & \mathbf{s}_{k+1} = \mathbf{f}(\mathbf{s}_k, \mathbf{a}_k) \\ & \mathbf{s}_0 = \mathbf{s} \end{aligned}$$

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Why attacking the problem in these ways?

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Defines policy:

$$\boldsymbol{\pi}^{\mathrm{MPC}}(\mathbf{s}) = \mathbf{u}_0^\star$$

How does $\boldsymbol{\pi}^{\mathrm{MPC}}$ relate to $\boldsymbol{\pi}^\star$?

No reason to match:

- Planning rather than policing
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Infinite horizon & discounted

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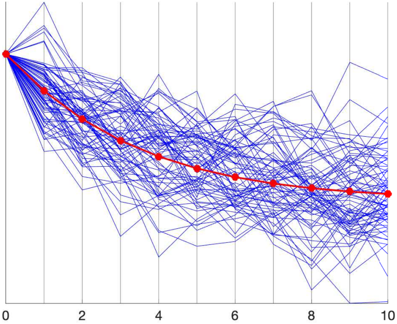

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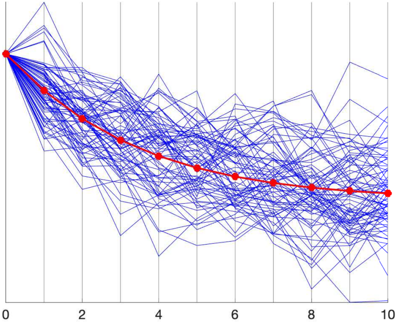

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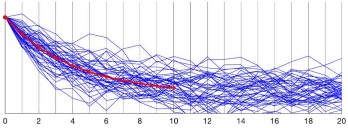

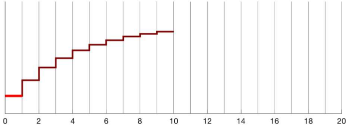

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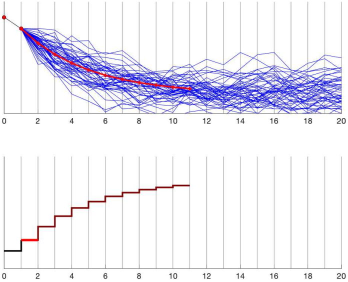

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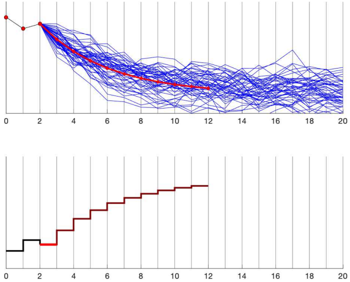

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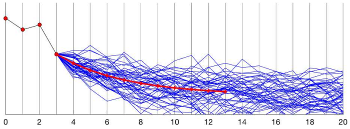

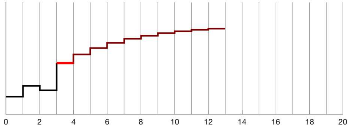

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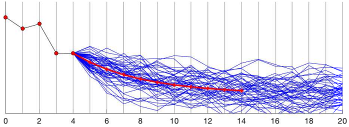

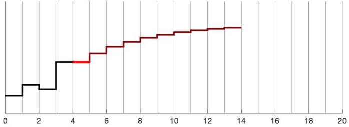

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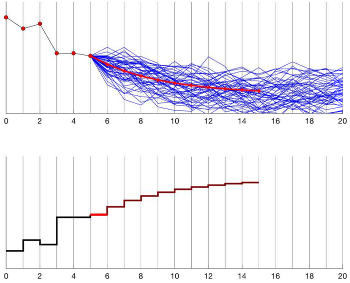

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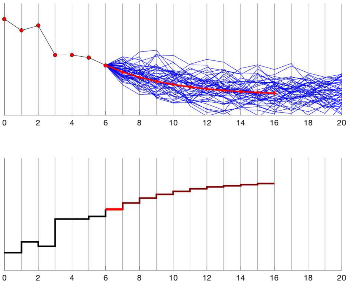

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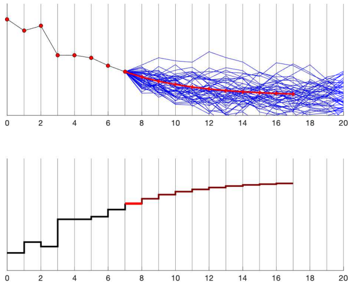

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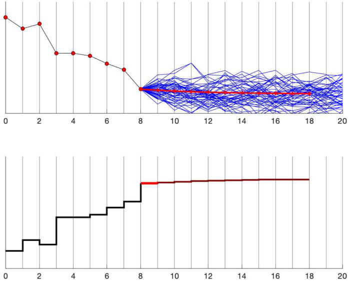

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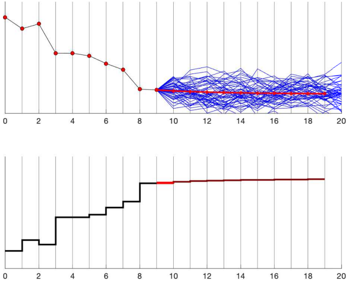

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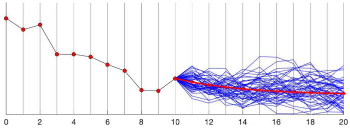

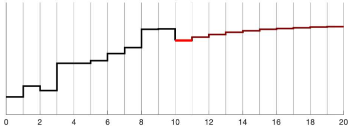

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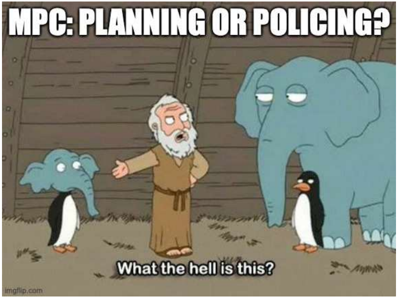

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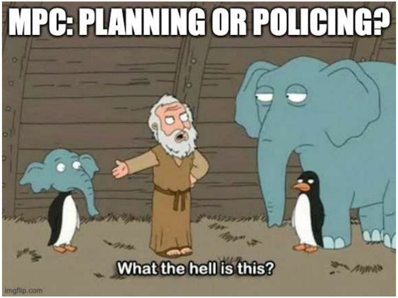

Can we clarify the relationship?

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Soft claim: MPC can be used as a practical toolbox to model the solution of MDPs. This view is the "best way" for understanding what we are doing with economic MPC

Outline

1. Let's rebuild some background – MPC and MDPs
2. MPC as a solution to MDPs
3. When is RL (most) beneficial for MPC?
4. A Deeper Look at the Theory

Learning for MPC - Machine Learning in-the-loop

MPC policy

$\pi_{\theta}^{\text{MPC}}(\mathbf{s})$

from model*, e.g.

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$\mathbf{a}$

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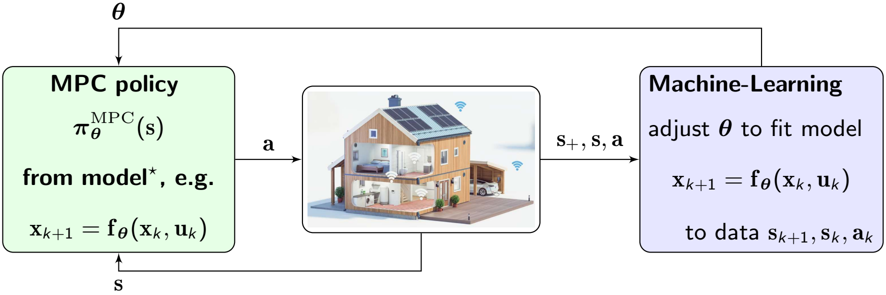

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- **Physics-based**: first principles + SYSID
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We focus on "breaking" this paradigm
Learning / RL plays a key role

How can MPC model an MDP?

MDP optimal policy

$$\boldsymbol{\pi}^\star = \arg\min_{\boldsymbol{\pi}} \mathbb{E}\left[\sum_{k=0}^{\infty} \gamma^k L(\mathbf{x}_k, \mathbf{u}_k)\right]$$

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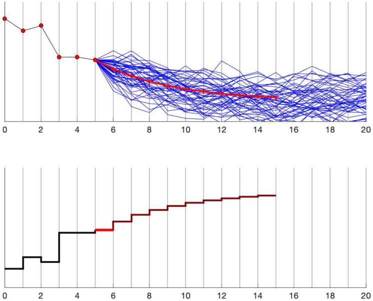

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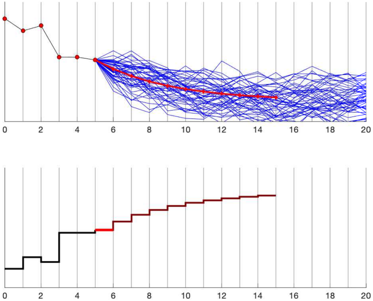

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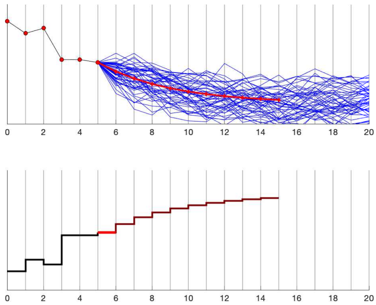
- Solving MDP $\leftarrow$ building model of $Q^\star$
- E.g. Q-learning does that from data
- MPC can model a value function...
- Can it model an action-value function?



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$$\begin{aligned}V^{\mathrm{MPC}}(\mathbf{s}) := \min_{\mathbf{x},\mathbf{u}} \; & T(\mathbf{x}_N) + \sum_{k=0}^{N-1} \gamma^k L(\mathbf{x}_k, \mathbf{u}_k)\\ \mathrm{s.t} \quad & \mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k), \quad \mathbf{x}_0 = \mathbf{s}\\ & \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k) \leq 0\end{aligned}$$

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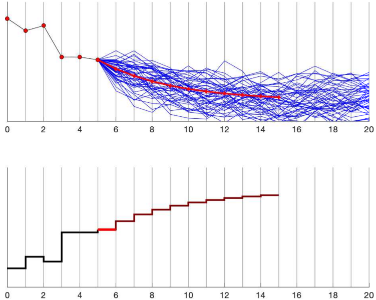

How can MPC model an MDP?

MDP optimal policy

$$\pi^\star = \arg\min_{\pi} \mathbb{E}\left[\sum_{k=0}^{\infty} \gamma^k L(\mathbf{x}_k, \mathbf{u}_k)\right]$$

is entirely described by $Q^\star(\mathbf{s}, \mathbf{a})$

MPC policy $\pi^{\mathrm{MPC}}(\mathbf{s}) = \mathbf{u}_0^\star$ from

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MPC is consistent (for correct T):

$$V^{\mathrm{MPC}}(\mathbf{s}) = \min_{\mathbf{a}} Q^{\mathrm{MPC}}(\mathbf{s}, \mathbf{a})$$

$$\pi^{\mathrm{MPC}}(\mathbf{s}) = \arg\min_{\mathbf{a}} Q^{\mathrm{MPC}}(\mathbf{s}, \mathbf{a})$$

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MPC is a complete model of MDP if:

$$Q^{\mathrm{MPC}}(\mathbf{s}, \mathbf{a}) = Q^\star(\mathbf{s}, \mathbf{a})$$

for all $\mathbf{s}, \mathbf{a}$. Then **optimality** holds:

$$\boldsymbol{\pi}^{\mathrm{MPC}}(\mathbf{s}) = \boldsymbol{\pi}^\star(\mathbf{s})$$

Modelling the MDP solution with MPC? Paradigm shifts...

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Shift 2: "holistic" parametrization

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i.e. cost and constraints are part of the model

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"Holistic" parametrization - Is that formally justified? Yes...

Full MPC parametrization:

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Theorem: under some technical conditions and for a “rich” parametrization of the MPC, there is a $\boldsymbol{\theta}$ such that

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even if the model $\mathbf{f}_{\boldsymbol{\theta}}$ *cannot* describe the real system accurately

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Remarks

- Compensate MPC model deficiencies in the cost + constraints

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MPC is a model of the optimal policy
not a policy approximation using open-loop (model-based) predictions

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How to use this? Reinforcement Learning

Policy $\boldsymbol{\pi}_{\boldsymbol{\theta}}^{\mathrm{MPC}}(\mathbf{s}) = \mathbf{u}_0^\star$ from

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- $\min_{\boldsymbol{\theta}} J\left(\boldsymbol{\pi}_{\boldsymbol{\theta}}^{\mathrm{MPC}}\right)$ using data
- $\boldsymbol{\theta} \rightarrow J\left(\boldsymbol{\pi}_{\boldsymbol{\theta}}^{\mathrm{MPC}}\right)$ very implicit
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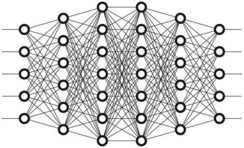
Reinforcement Learning

Tools to approximate $\pi^\star$ from data
This is not (necessarily) about DNNs



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Reinforcement Learning

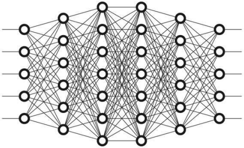
Tools to approximate $\pi^\star$ from data
This is not (necessarily) about DNNs

For MPC: tools to find best $\boldsymbol{\theta}$, e.g.

- **Policy Gradient**: estimations of

$$\nabla_{\boldsymbol{\theta}} J\left(\boldsymbol{\pi}_{\boldsymbol{\theta}}^{\mathrm{MPC}}\right), \quad \text{possibly} \quad \nabla_{\boldsymbol{\theta}}^2 J\left(\boldsymbol{\pi}_{\boldsymbol{\theta}}^{\mathrm{MPC}}\right)$$

- **Q-learning**: direct “shaping” of MPC

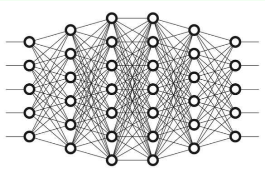


Reinforcement Learning & MPC

Classical Reinforcement Learning

$\boldsymbol{\theta}$

$\boldsymbol{\pi}_{\boldsymbol{\theta}}$ given by



$\mathbf{a}$

$\mathbf{s}_+, \mathbf{s}, \mathbf{a}$

RL adjusts $\boldsymbol{\theta}$ to achieve

$$\nabla_{\boldsymbol{\theta}} J\left(\boldsymbol{\pi}_{\boldsymbol{\theta}}^{\mathrm{MPC}}\right) = 0$$

from data

$\mathbf{s}$

Reinforcement Learning over MPC

$\boldsymbol{\theta}$

MPC policy

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Why is it useful?

- ✓ Tunes your optimization model for real-world performance
- ✓ MPC: 25 years of results on formal guarantees & methods
- ✓ Easy to inject knowledge
- ✓ Learning does not start from scratch
- ✓ Planning provides explainability
- ✓ Software now available

Reinforcement Learning & MPC

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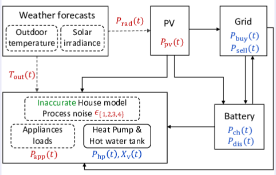
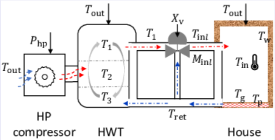
Example - Home Energy Management (simulated)

Setup





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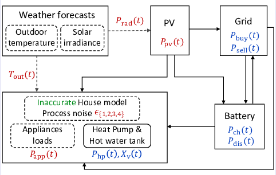

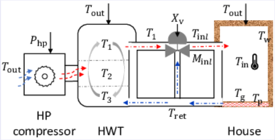

Learning process: aligned followed with closed-loop

Real world data

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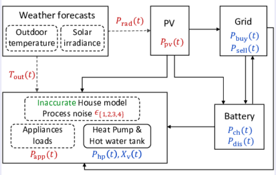

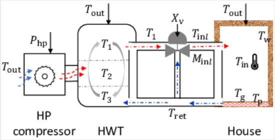

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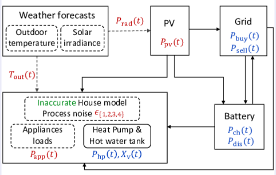

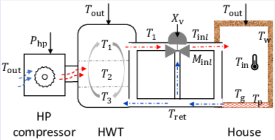

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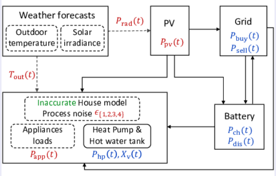

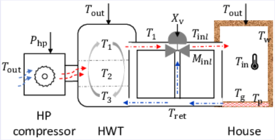

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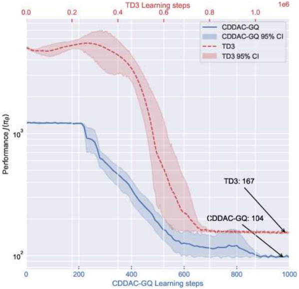

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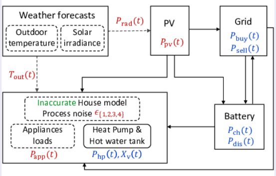

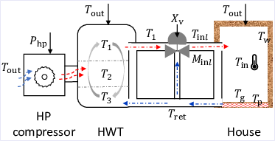

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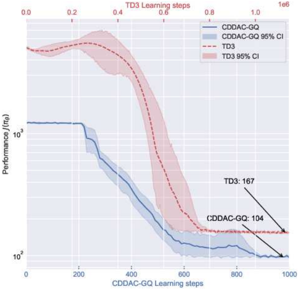

The RL step improves the MPC performance significantly from only model fitting

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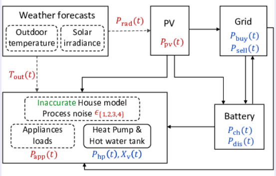

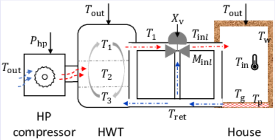

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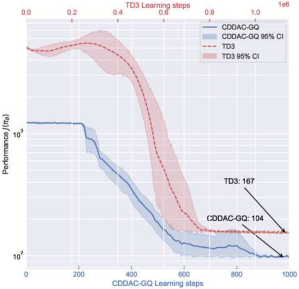

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RL over MPC can tune your MPC for performance beyond what classical methods can do!

Outline

Model fitting?

Proposed paradigm

Policy $\boldsymbol{\pi}_{\boldsymbol{\theta}}^{\mathrm{MPC}}(\mathbf{s}) = \mathbf{u}_0^\star$ from

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A step back: **model adjustment?**

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Adjust $\boldsymbol{\theta}$ according to (e.g.)

1. $\min_{\boldsymbol{\theta}} \mathbb{E}\left[\left\|\mathbf{f}_{\boldsymbol{\theta}}(\mathbf{s}, \mathbf{a}) - \mathbf{s}_+\right\|^2\right]$ vs.
2. $\min_{\boldsymbol{\theta}} J\left(\boldsymbol{\pi}_{\boldsymbol{\theta}}^{\mathrm{MPC}}\right)$

... **is that different?**

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... **is that different?**

Empirical answers:

- In general it is different...
- Good to do 1. first, then 2.
- Is step 2 necessary?
  - Improves closed-loop performance
  - But gain is *very* case dependent (also with full parametrization)

What causes that?

Sufficient Condition for MPC Optimality (for MDPs)

MPC policy $\boldsymbol{\pi}^{\mathrm{MPC}}(\mathbf{s}) = \mathbf{u}_0^\star$ from

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relates model $\mathbf{f}$ to optimal value function and conditional distribution $\mathbf{s}, \mathbf{a} \rightarrow \mathbf{s}_+$ of the MDP (+ condition on T)

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In general

- ridge regression

$$\mathbf{f}(\mathbf{s}, \mathbf{a}) = \mathbb{E}\left[\mathbf{s}_+ \,|\, \mathbf{s}, \mathbf{a}\right]$$

- max likelihood

$$\mathbf{f}(\mathbf{s}, \mathbf{a}) = \max \rho\left[\mathbf{s}_+ \,|\, \mathbf{s}, \mathbf{a}\right]$$

do not satisfy the optimality conditions

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Notable exceptions: model gives $\mathbb{E}\left[\,\mathbf{s}_+ \,|\, \mathbf{s}, \mathbf{a}\,\right]$ and

- LQR with process noise
- By extension, locally optimal policies for
  - dissipative MDP and
  - $V^\star \approx$ quadratic in positive invariant set

Smooth tracking MPC, fixed reference away from constraints, "reasonable" stochasticity

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When is RL (most) useful for MPC? *"practical" view*

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- Model is not rich enough to predict the (relevant) expected state transition
- Economic problem with "low dissipativity" (trajectories spread over the state space)
- Value function curvature changes a lot, problem is non-smooth
- Optimal steady-state near / at constraints
- Varying exogeneous inputs (e.g. varying prices in energy-related problems, changing reference)
- Task-based problems: racing, minimum time, termination conditions, etc.

Outline

Fully Parametrized MPC Can Describe the MDP Solution

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The theory is about equivalences between MDPs

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World MDP

World MDP definition

- States $\mathbf{s}$ and actions $\mathbf{a}$
- Cost $L(\mathbf{s}, \mathbf{a})$
- Transition $\mathbf{s}_+ \sim \rho[\,\cdot\,|\mathbf{s}, \mathbf{a}]$

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$$V^\star(\mathbf{s}) = \min_{\mathbf{a}} Q^\star(\mathbf{s}, \mathbf{a})$$

$$Q^\star(\mathbf{s}, \mathbf{a}) = L(\mathbf{s}, \mathbf{a}) + \gamma \mathbb{E}_{\mathbf{s}_+ \sim \rho}\left[V^\star(\mathbf{s}_+)\,|\mathbf{s}, \mathbf{a}\right]$$

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Optimal value functions

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Fully Parametrized MPC Can Describe the MDP Solution

The theory is about equivalences between MDPs

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World MDP definition

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Proof: telescopic sum, some non-trivial assumptions to prevent $\infty - \infty$ cancellations

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MPC is a “weird” undiscounted Model MDP

- Deterministic model is a trivial $\hat{\rho}$, then optimal policy for Model MDP $\equiv$ planning
- Finite horizon: ok if correct terminal cost (strong assumption, but learning can tune it)
- Scenario tree: rough $\hat{\rho}$ and policy.
- Robust MPC: carries support $\hat{\rho}$, worst-case cost is a bit off...

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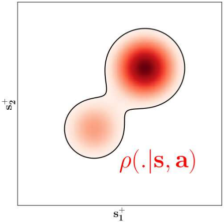

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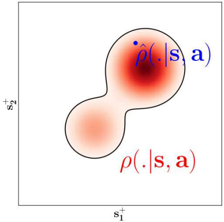

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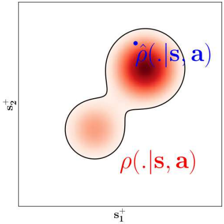

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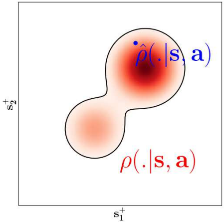
- Cost modification promotes "simplicity".



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Technical assumptions? They are very technical and not intuitive at all. But not very demanding, and there is a "self-fulfilling" effect in learning, i.e. RL "learns" to satisfy them.

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- I.e. we need to know the state of the World MDP to build a correct Model MDP
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The end of a nice theory? No, but it is a word of caution...

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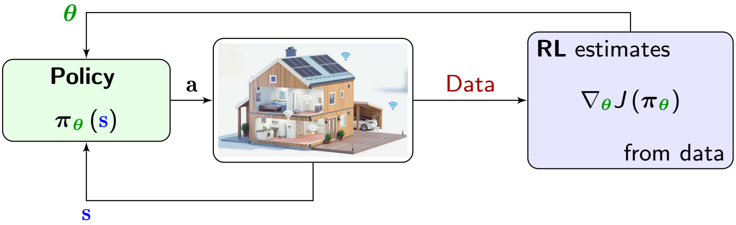
Options 2 and 3 are part of the decision-making process, back-propagate through policy + state "constructor".

State Estimation in the Learning Loop



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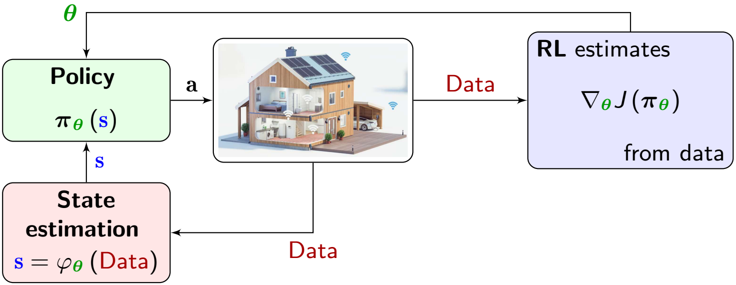

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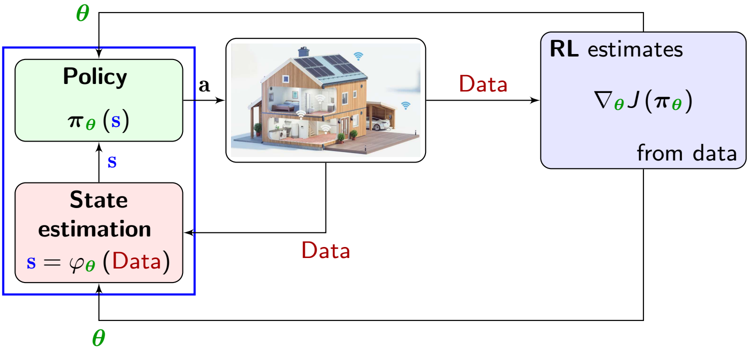

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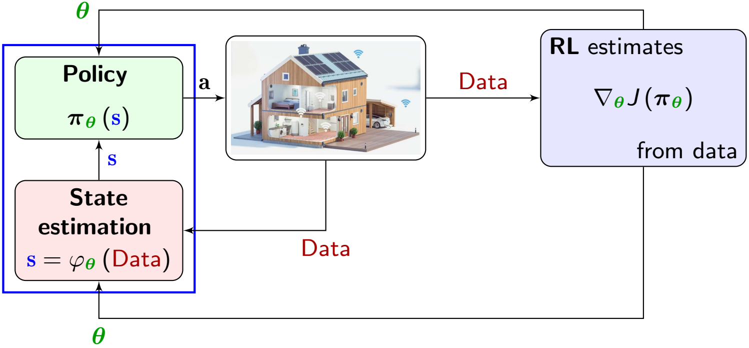

Remarks

- Policy is $\boldsymbol{\pi}^{\text{D}}_{\boldsymbol{\theta}}(\text{Data}) = \boldsymbol{\pi}_{\boldsymbol{\theta}}(\varphi_{\boldsymbol{\theta}}(\text{Data}))$

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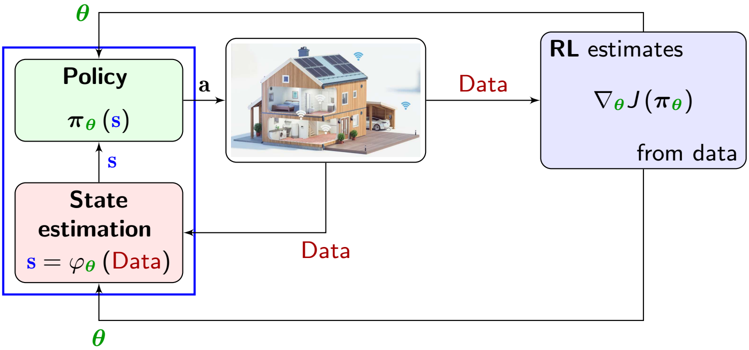

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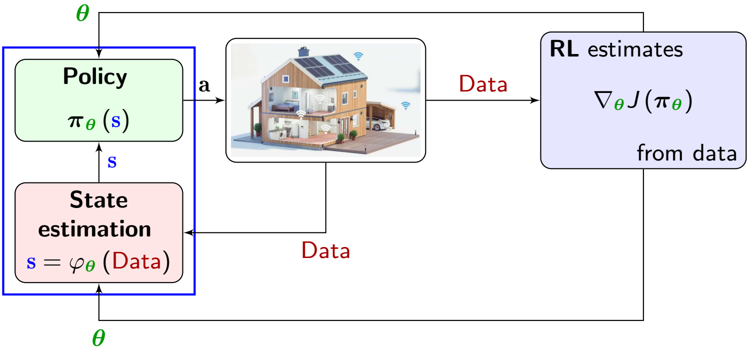

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- Critic is looking at Data $\xrightarrow{\boldsymbol{\theta}} \mathbf{a}$, i.e. $Q^{\boldsymbol{\pi}^{\mathsf{D}}_{\boldsymbol{\theta}}}(\text{Data}, \mathbf{a})$

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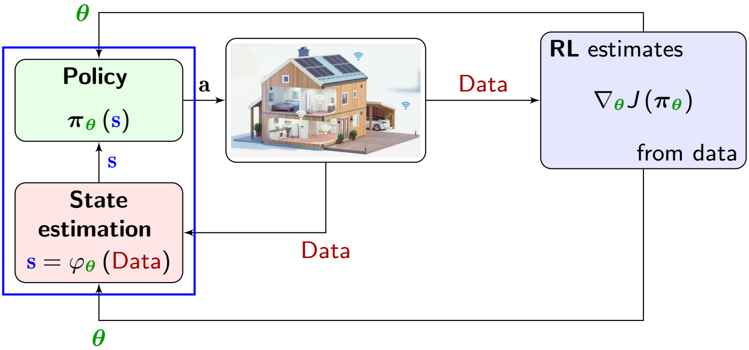

If φ_{θ} (Data) is

- MHE, then its gradient is computed using NLP sensitivities (same as MPC)
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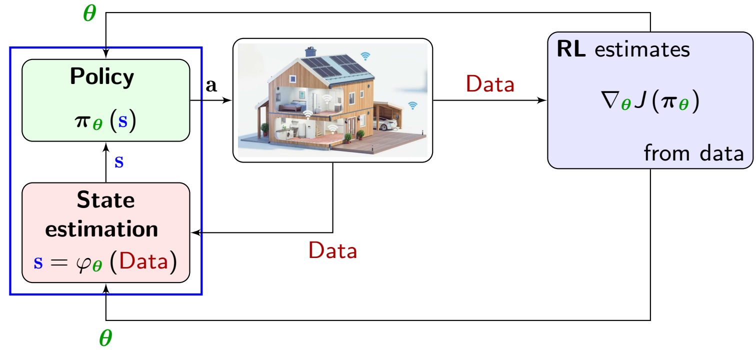
State Estimation in the Learning Loop



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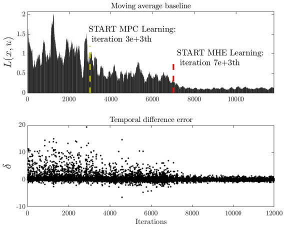

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Systems with

- $\sim$Linear dynamics
- Input-output data
- Significant stochasticity
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- Recent history of input-output sequence $\mathbf{u}, \mathbf{y}$
- Planned input sequence $\mathbf{u}$
- Predicted output sequence $\hat{\mathbf{y}}$

Multi-Step Predictive Control

SPC for $\mathbf{u}$, $\mathbf{y}$ given

$$\min_{\mathbf{u},\,\hat{\mathbf{y}}} \quad \sum_{k=0}^{N} L\left(\hat{\mathbf{y}}_k, \mathbf{u}_k\right)$$

$$\text{s.t.} \quad \hat{\mathbf{y}} = \Phi \begin{bmatrix} \mathbf{u} \\ \mathbf{y} \\ \mathbf{u} \end{bmatrix}$$

$$\mathbf{h}\left(\hat{\mathbf{y}}_k, \mathbf{u}_k\right) \leq 0$$

yields policy $\boldsymbol{\pi}\left(\mathbf{u}, \mathbf{y}\right) = \mathbf{u}_0^\star$

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matrix Φ can be **built from past data** $\mathcal{D}$, e.g.

$$\min_{\Phi} \quad \sum_{i\in\mathcal{D}} \frac{1}{2} \left\| \mathbf{y}_i - \Phi \begin{bmatrix} \mathbf{u}_i \\ \mathbf{y}_i \\ \mathbf{u}_i \end{bmatrix} \right\|^2 + R(\Phi)$$

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If R promotes a dynamic system structure, regression well-posed even with limited data (and Φ high-dimensional)

But best model fit $\not\Rightarrow$ best policy

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Can we close the gap via RL? Yes!

- RL-MPC theory applies with some twists
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MSPC for $\mathbf{u}$, $\mathbf{y}$ given

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What we will do next: RL over MPC

- Safe & Stable RL over MPC *(In the afternoon)*
- RL over MPC with belief states – a future prospect *(In the afternoon)*
- Beyond MPC – Model-based Decisions and AI for decisions *(Tomorrow)*

Thanks for your attention!

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