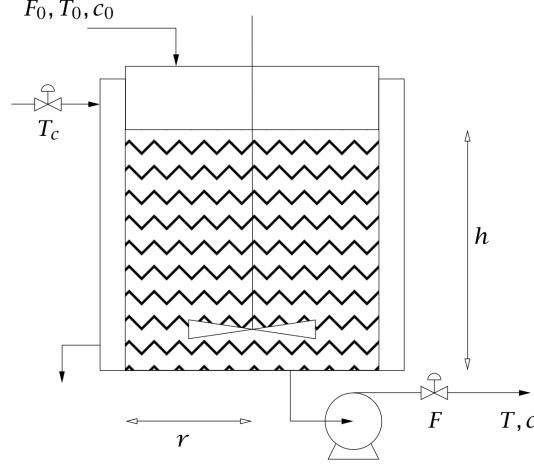


## Exercise 2: Numerical Optimal Control and NMPC with CasADi + IPOPT and acados

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In this exercise, we again consider the nonlinear continuous stirred-tank reactor (CSTR).<sup>1</sup>



An irreversible, first-order reaction  $A \rightarrow B$  occurs in the liquid phase and the reactor temperature is regulated with external cooling. Mass and energy balances lead to the following nonlinear model:

$$\begin{aligned}\dot{c} &= \frac{F_0(c_0 - c)}{\pi r^2 h} - k_0 \exp\left(-\frac{E}{RT}\right) c \\ \dot{T} &= \frac{F_0(T_0 - T)}{\pi r^2 h} - \frac{\Delta H}{\rho C_p} k_0 \exp\left(-\frac{E}{RT}\right) c + \frac{2U}{r \rho C_p} (T_c - T) \\ \dot{h} &= \frac{F_0 - F}{\pi r^2}\end{aligned}$$

with states  $x = (c, T, h)$  where  $c$  is the concentration of substance  $A$ ,  $T$  is the reactor temperature and  $h$  is the height. The controls  $u = (T_c, F)$  are the coolant liquid temperature  $T_c$  and the outlet flowrate  $F$ .

### Description of closed-loop simulation environment

- The `main.py` file is prepared to simulate the closed-loop system with `acados` and plot the trajectories.
- The file `cstr_model.py` defines the model equations stated above with values for all the parameter values.
- The file `setup_acados_integrator.py` uses this model to generate an `AcadosSimSolver` instance which we use as our plant model.

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<sup>1</sup>The example as well as the figure have been adopted from Example 1.11 in J. B. Rawlings, D. Q. Mayne, and M. M. Diehl. *Model Predictive Control: Theory, Computation, and Design*. Nob Hill, 2nd edition, 2017.

## Exercises

### Uncontrolled simulation:

- Simulate the system with a constant reference control input.

### Exact NMPC with `acados`

- Set `with_nmpc = True` in `main.py` to create an `AcadosOcpSolver` and run it in a closed loop simulation.

**Note:** check how the OCP is formulated using the `acados` interface using the `AcadosOcp` object. In comparison to the plain `CasADi` formulation from exercise 1, the multiple shooting discretization is done by the framework and does not need to be done by the user.

### Exact NMPC with `CasADi` and IPOPT using the `AcadosCasadiOcpSolver`

IPOPT is a very reliable solver for nonlinear programming (NLP) formulations. It is the most widely used solver in `CasADi`, and known to be very robust. IPOPT tackles general sparse formulations. In contrast `acados` exploits the structure of optimal control problems.

The `AcadosCasadiOcpSolver` class allows one to formulate OCPs with the `acados` interface and use solvers interfaced with `CasADi`. Through the `AcadosCasadiOcpSolver`, one can interact with those solver, similar to the `AcadosOcpSolver`. This is very useful for prototyping, comparisons etc. See also Bonus exercise 2.

- Set `with_nmpc_ipopt = True` in `main.py` to also create an `AcadosCasadiOcpSolver`.
- Set `with_timevar_ref_nmpc = True` and `with_timevar_ref_nmpc_ipopt = True` to enable time varying references in a closed loop simulation. How is this done?
- Compare the trajectories without time-varying references to the one from the previous exercise, and also compare both trajectories (with and without time-varying references) along with the solvers' computation times.

### Approximate and fast MPC

- Set `with_linear_mpc = True` in `main.py` to create an `AcadosOcpSolver` with a model linearized at the steady state.
- Set `with_nmpc_rti = True` in `main.py` to create an `AcadosOcpSolver` that uses the real-time iteration (RTI) algorithm. This algorithm performs one SQP iteration at each sampling time.
- Compare the resulting closed loop trajectories and the runtime of their controllers.

### Bonus exercise 1: NMPC with model plant mismatch

- Note that the model parameter  $F_0$  is implemented as a parameter in the `AcadosModel`.
- Task: Introduce a mismatch between the OCP model and the plant, by increasing  $F_0$  in the OCP model by 5%. How well are the references tracked?

### Bonus exercise 2: Differences between NLP solvers and warm starting

For this exercise the template `bonus_warm_start.py` is provided.

- Regard the first OCP of the closed-loop simulation. Solve it with the `AcadosCasadiOcpSolver` and `AcadosOcpSolver`.
- Compare the solutions. Can they be different if the problem is the same?

- Initialize the solver with the solution of the other. Is the IPOPT solution a solution for the **acados** solver?
- And vice versa?