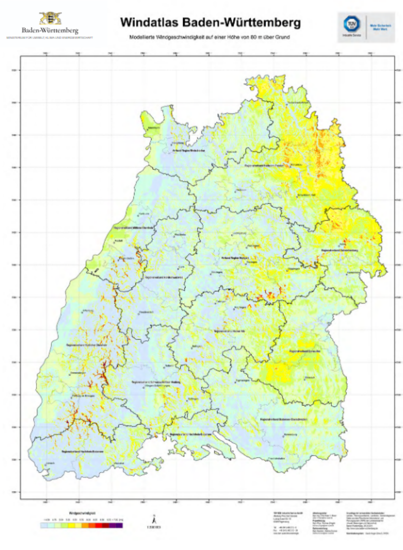
WES Concept Questions!

Rachel Leuthold

August 5, 2025

Energy content of the wind



WINDATLAS BADEN-WÜRTTEMBERG 2014 | 27

wind atlas images from: `https://um.baden-wuerttemberg.de/fileadmin/redaktion/m-um/intern/Dateien/Dokumente/2_Presse_und_Service/Publikationen/Energie/Windatlas.pdf`

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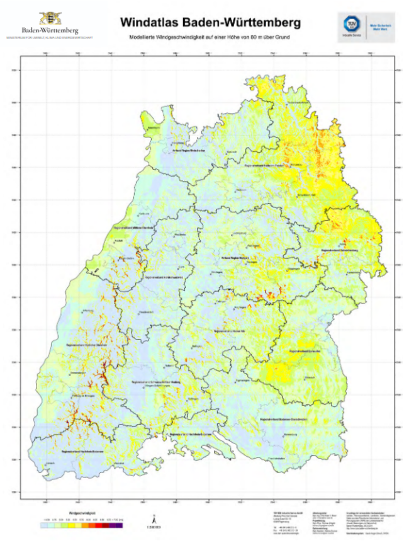
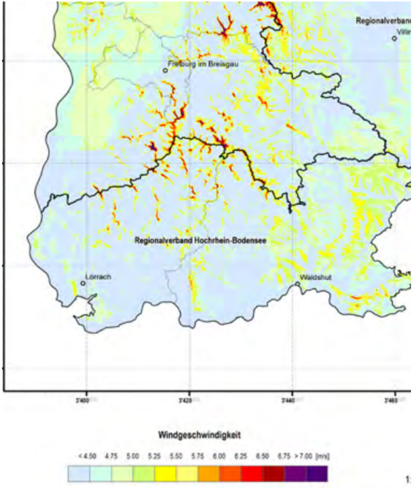




WINDATLAS BADEN-WÜRTTEMBERG 2014 | 27

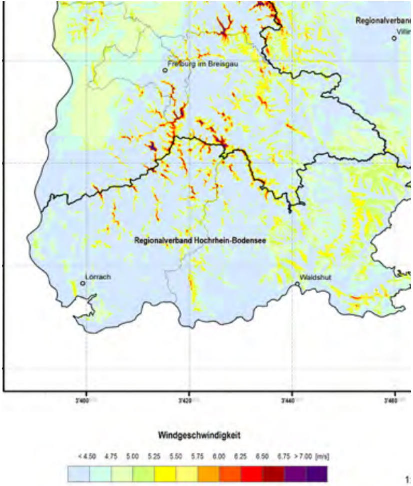
Energy content of the wind

Pick a nice "dark red" site w. $u_\infty = 6.5\mathrm{m/s}$. This suggests that there's a wind power available of

$$\mathrm{P}_\infty = \frac{1}{2}\rho_{\mathrm{air}} u_\infty^3 A$$

By roughly how much do you have to reduce this power to take into account the momentum loss in the flow? (For $P = C_\mathrm{P} P_\infty$, What is C_P?)

- (A) 1/60
- (B) 1/10
- (C) 1/2
- (D) 9/10



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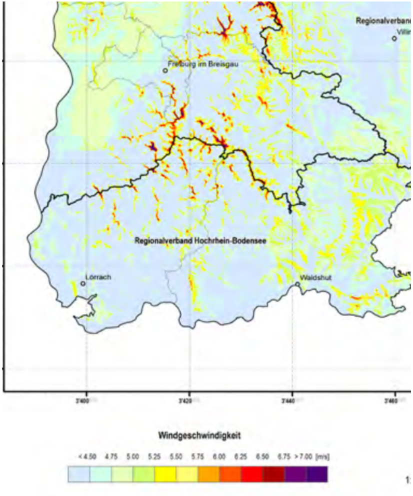
- (A) $1/60$
- (B) $1/10$
- (C) $1/2 \leftarrow$ Betz says $C_\mathrm{P} < 0.6$
- (D) $9/10$

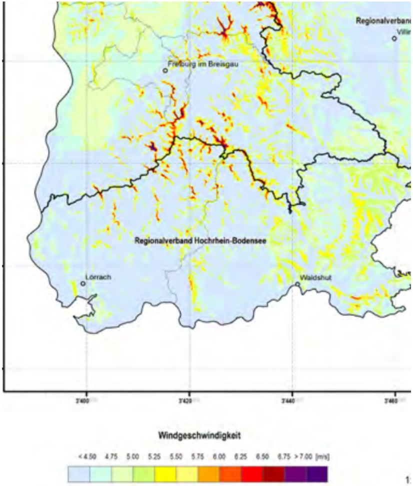


Energy content of the wind

$$P_\infty = \frac{1}{2}\rho_{\mathrm{air}} u_\infty^3 A$$

Suppose we have a near-optimal turbine, and we're thinking of putting up another one behind it. If the sites are both aligned with the flow, how much do you have to reduce the velocity at the site of the 2nd turbine before you build the 2nd turbine? (For $u_{\mathrm{site\ 1}} = k u_{\mathrm{turbine\ 1}}$, what is k?)

- (A) 1
- (B) 2/3
- (C) 1/3
- (D) 1/10



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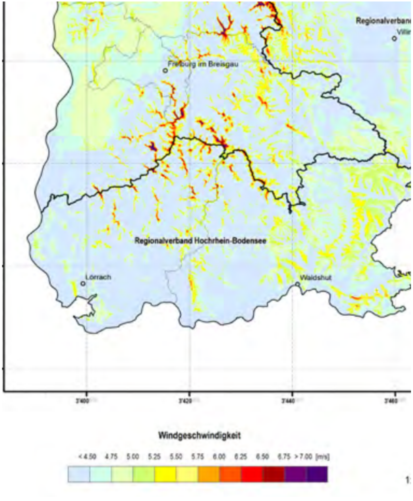
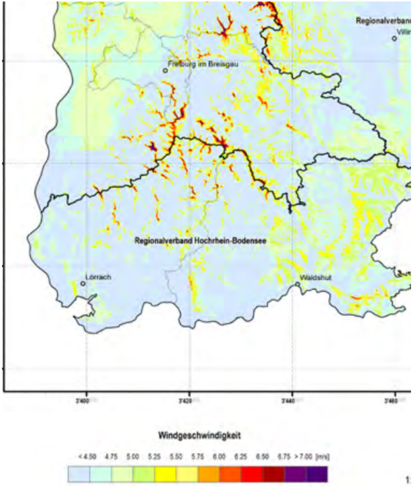
- (A) 1
- (B) 2/3
- (C) $1/3 \leftarrow (1-2a)u_0\big|_{a\to\frac{1}{3}}$
- (D) 1/10



$$P_\infty = \frac{1}{2}\rho_{\text{air}} u_\infty^3 A$$

Suppose you build this second Betz-optimal turbine behind the first. What power coefficient should you expect with respect to this free-stream power? (For $P_{\text{turbine 2}} = kP_\infty$, what is k?)

- (A) 1/60
- (B) 1/10
- (C) 1/2
- (D) 9/10



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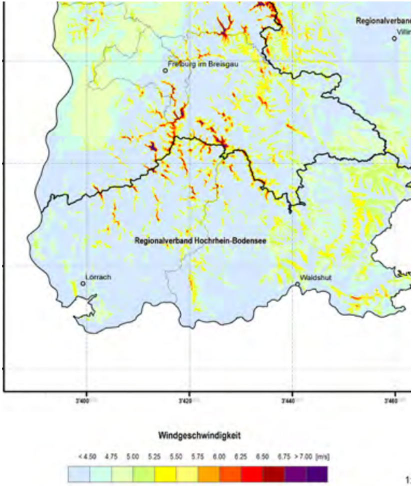
- (A) 1/60 ←
- (B) 1/10
- (C) 1/2
- (D) 9/10

$u_{\infty,\text{turbine 2}} = \frac{2}{3}u_\infty$

$power_{\text{turbine 2}} =$

$C_{\mathrm{P}}{}^\star(\frac{1}{2}\rho u^3_{3,\text{turbine}-1}A =$

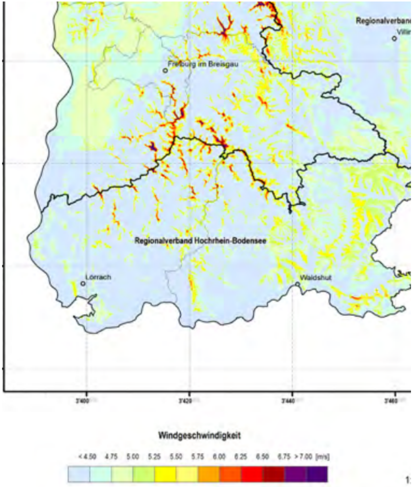
$C_{\mathrm{P}}{}^\star(1-2a)^3(\frac{1}{2}\rho u_\infty^3 A$



$$P_\infty = \frac{1}{2}\rho_{\text{air}} u_\infty^3 A$$

By (very roughly) how much do you still have to reduce this to describe the portion of the energy that will actually get delivered? What is k for: $\overline{E} = k \int_{t_1}^{t_2 \gg t_1} C_{\text{P}} P_\infty(t) \text{d}t$?

- (A) 1/10
- (B) 1/4
- (C) 3/4
- (D) 9/10



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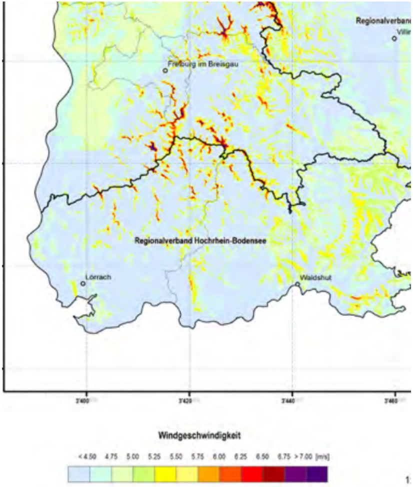
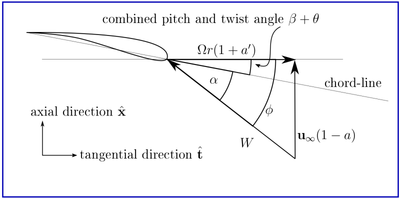
- (A) 1/10
- (B) 1/4 ← E-138!
- (C) 3/4
- (D) 9/10

wind turbines usually have capacity factors beween 20-50 percent.

`https://windeurope.org/about-wind/daily-wind/capacity-factors`



Tip Speed Ratio



We've defined this nondim. value called the tip speed ratio

$$\lambda = \frac{\Omega R}{u_\infty}$$

If $\lambda = 0$, what do we know about the situation?

- (A) turbine is not rotating
- (B) there's no freestream wind
- (C) it's hunting time for bats

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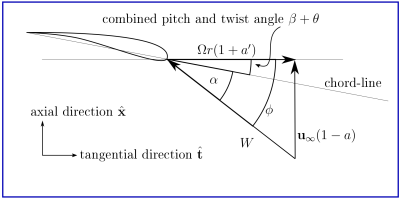

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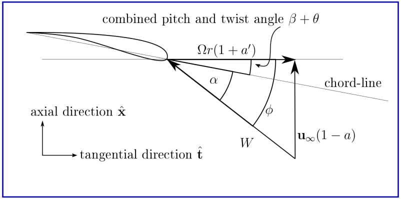

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If $\lambda = \infty$, what do we know about the situation?

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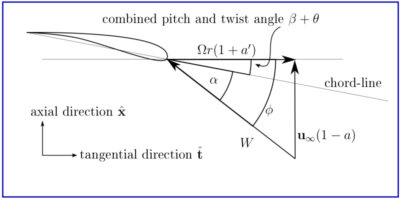

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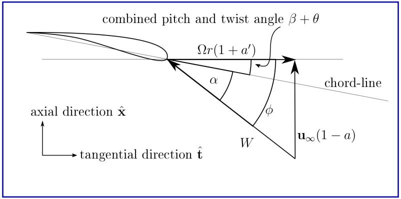

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Remembering visit to the E-138 turbine, what is λ if it's hunting time for bats?

- (A) (a positive value)/(another positive value)
- (B) (0)/(a positive value)
- (C) (a positive value)/(0)
- (D) (0)/(0), or undefined

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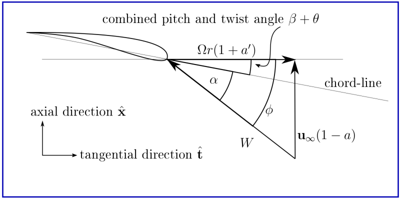

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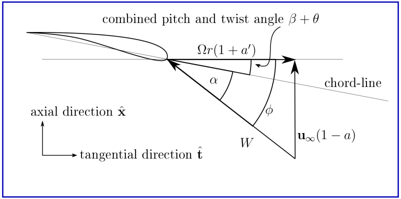
Trick question: if the wind speed is anyways low ($u_\infty \approx 0$), then they stop the turbine ($\Omega = 0$), otherwise normal operation. So, either (A) or (D)

Tip Speed Ratio



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If $\lambda = 0$, what direction is the effective speed W pointing?

- (A) along $+\hat{\boldsymbol{x}}$;
- (B) along $-\hat{\boldsymbol{x}}$;
- (C) along $+\hat{\boldsymbol{t}}$;
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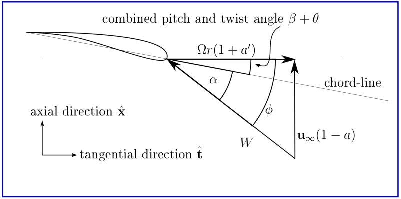

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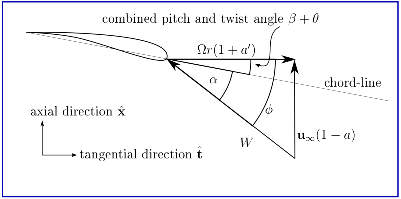

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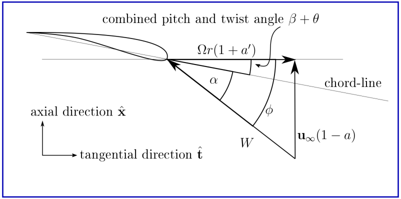

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because turbine is not rotating

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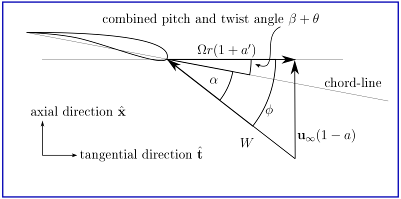

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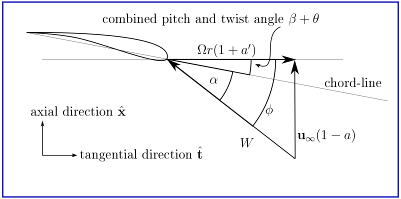

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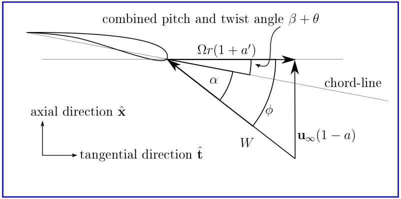

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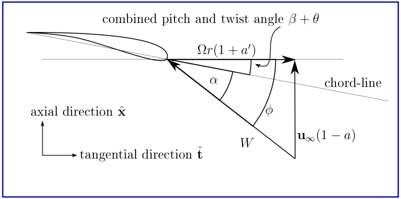

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because there's much more rotational speed than wind

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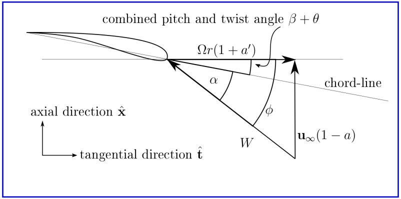

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If λ is very large but not ∞, what direction are a blade's lift $\boldsymbol{L}$ and drag $\boldsymbol{D}$ force?

(A) drag is axially downstream; lift is driving rotation;

(B) drag opposes rotation; lift points axially downstream;

(C) something intermediate to A and B, but the lift-force still drives forwards;

(D) no idea: drag does whatever is "bad" and lift does whatever is "good"

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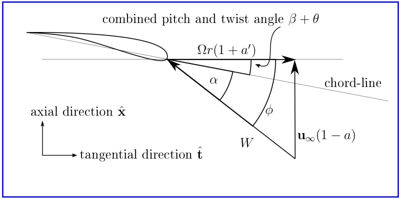

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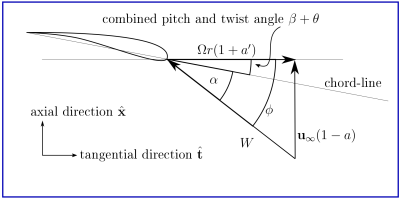

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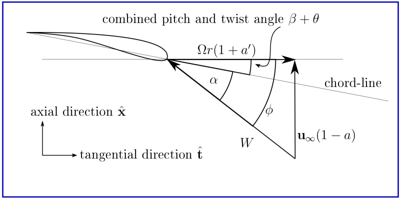
Very roughly (order of magnitude), how big do you expect the (2D) gliding ratio c_ℓ / c_d to be?

- (A) $1 \cdot 10^{-2}$
- (B) 1
- (C) $1 \cdot 10^{2}$
- (D) $1 \cdot 10^{4}$

Tip Speed Ratio



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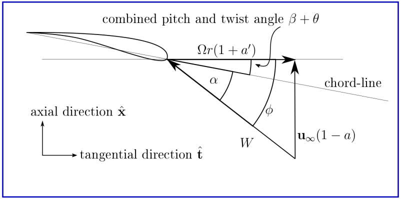
point is: it's big.

see airfoil profiles at the end of ch.3 of Burton

Tip Speed Ratio



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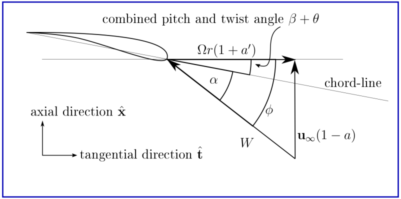
If a horizontal-axis wind turbine (HAWT) is not currently rotating, and the wind picks up: do you have to do anything specific to start the rotation?

- (A) no, HAWTs self-start!
- (B) yes, you have to disenguage the breaks!
- (C) yes, you have to pitch the blades!
- (D) all of the above is true!

Tip Speed Ratio



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even at $\lambda = 0$, the aerodynamic forces will drive rotation as long as you don't actively impede the rotation

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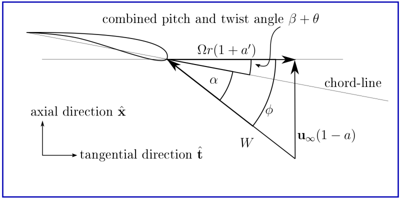

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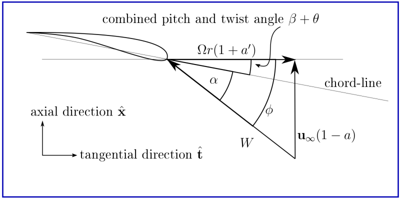
So, if the breaks fail (including that the generator stops providing resistance by harvesting electricity) on a horizontal-axis wind turbine (HAWT), what happens?

- (A) everything will be fine, the acceleration will stop on its own!
- (B) the blades will keep accelerating until the turbine explodes!

Tip Speed Ratio



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www.youtube.com/watch?app=desktop&v=WlPNyS8ApZI

en.wikipedia.org/wiki/Hornslet_wind-turbine_collapse

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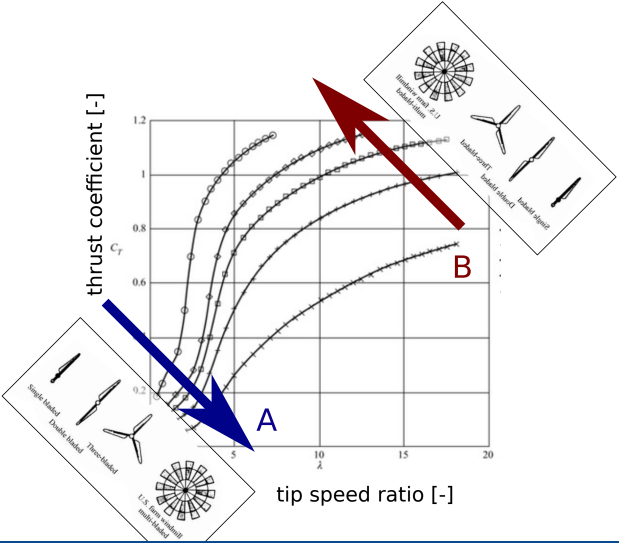

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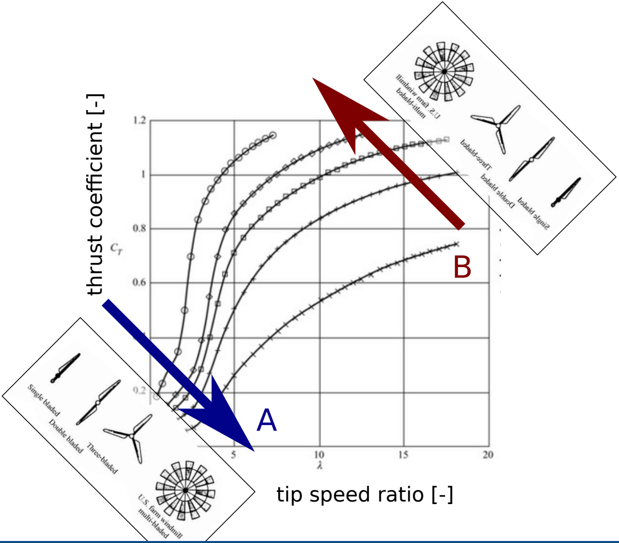
What's the relationship between solidity and tip speed ratio?

Burton et al, Fig. 3.54, and Rogers et al, Fig. 1.6



(B)! The fewer the blades, the...

- lower the solidity,
- faster the rotational speed,
- closer to Betz optimality!

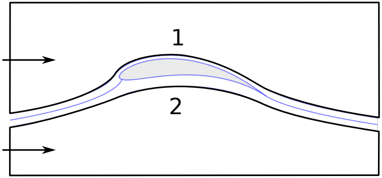
$$C_\mathrm{T} = 4a(1-a)$$

$$C_\mathrm{T} \to 4\frac{1}{3}\frac{2}{3} = \frac{8}{9}$$

Blade and airfoil nomenclature

Which side has the fastest flow speed? Which side has the highest pressure?

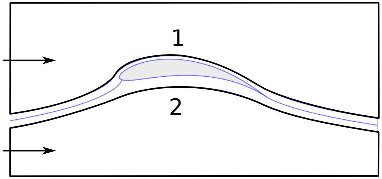
- (A) $u_1 > u_2$ & $p_1 > p_2$
- (B) $u_1 > u_2$ & $p_1 < p_2$
- (C) $u_1 < u_2$ & $p_1 < p_2$
- (D) $u_1 < u_2$ & $p_1 > p_2$



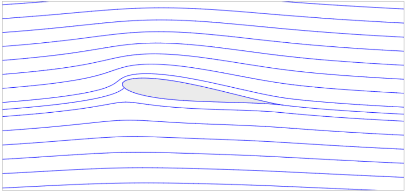
Blade and airfoil nomenclature

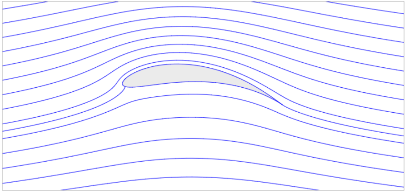
Which side has the fastest flow speed? Which side has the highest pressure?

- (A) $u_1 > u_2$ & $\mathrm{p}_1 > \mathrm{p}_2$
- (B) $u_1 > u_2$ & $\mathrm{p}_1 < \mathrm{p}_2 \leftarrow$
- (C) $u_1 < u_2$ & $\mathrm{p}_1 < \mathrm{p}_2$
- (D) $u_1 < u_2$ & $\mathrm{p}_1 > \mathrm{p}_2$



Blade and airfoil nomenclature





If we keep the angle of attack constant, but increase the camber of the blade (go from left image to right image), what happens? (hint: blue lines are streamlines!)

(A) more lift, more drag

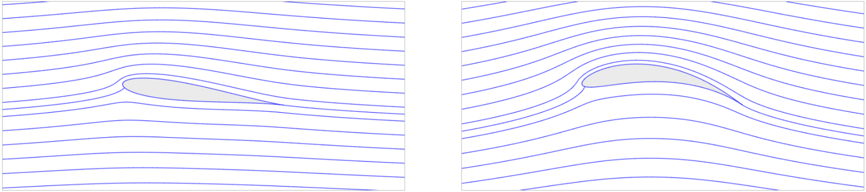
(B) more lift, less drag

(C) less lift, more drag

(D) less lift, less drag

all images of streamlines-over-2d-airfoils similar to these, are from
`http://dimanov.com/airfoil/af_prj.html`

Blade and airfoil nomenclature



If we keep the angle of attack constant, but increase the camber of the blade (go from left image to right image), what happens? (hint: blue lines are streamlines!)

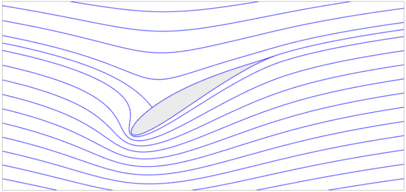
- (A) more lift, more drag ←
- (B) more lift, less drag
- (C) less lift, more drag
- (D) less lift, less drag

because flow is more redirected downwards!

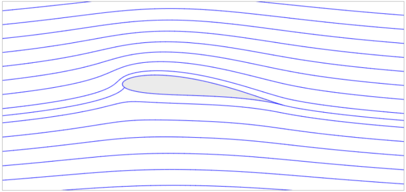
Blade and airfoil nomenclature

Which of the following situations has the greatest lift force upwards (in page)? (Hint: blue lines are streamlines!)

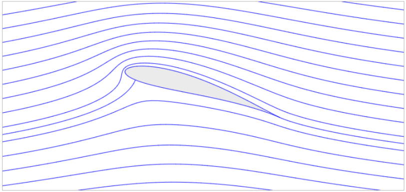
Ⓐ negative angle of attack



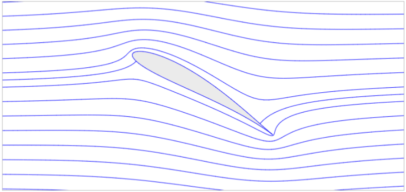
Ⓑ zero angle of attack



Ⓒ medium angle of attack



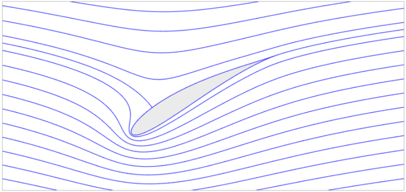
Ⓓ high angle of attack (stall)



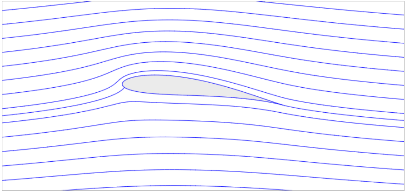
Blade and airfoil nomenclature

Which of the following situations has the greatest lift force upwards (in page)? (Hint: blue lines are streamlines!) Look at redirection!

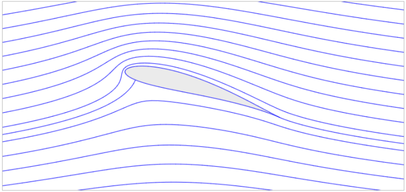
(A) negative angle of attack



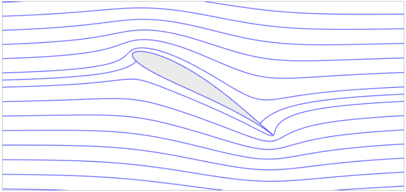
(B) zero angle of attack



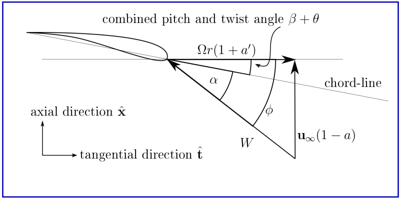
(C) medium angle of attack ←



(D) high angle of attack (stall)



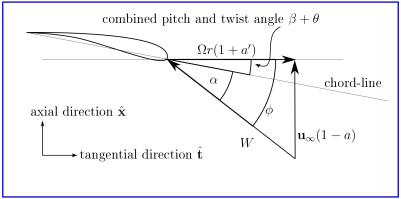
Blade and airfoil nomenclature



If you pitch the blade (increase the blade pitch angle), what will happen to the angle of attack α?

- (A) α increases
- (B) α stays the same
- (C) α decreases

Blade and airfoil nomenclature

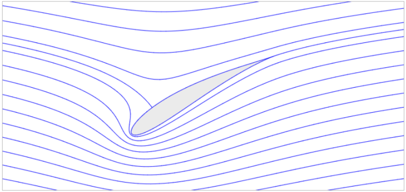


If you pitch the blade (increase the blade pitch angle), what will happen to the angle of attack α?

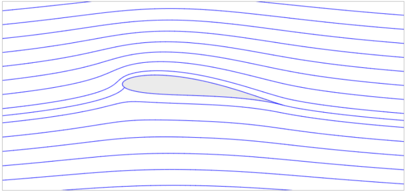
- (A) α increases
- (B) α stays the same
- (C) α decreases $\leftarrow$

Wait, that's weird: why are we deliberately decreasing the angle of attack? (Ie, which situation are we trying to avoid?)

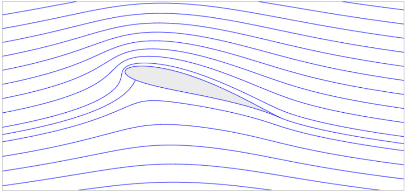
(A) negative angle of attack



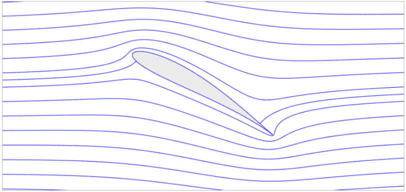
(B) zero angle of attack



(C) medium angle of attack



(D) high angle of attack (stall)



Blade and airfoil nomenclature

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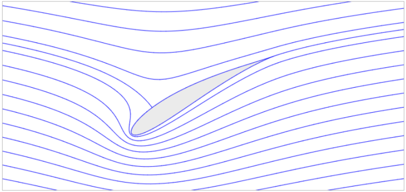

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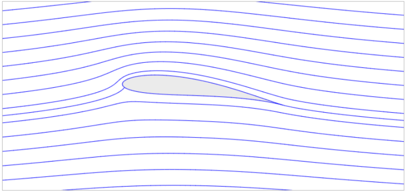
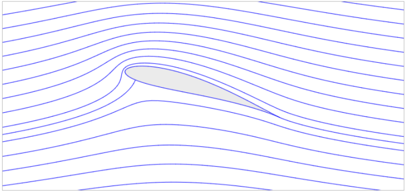
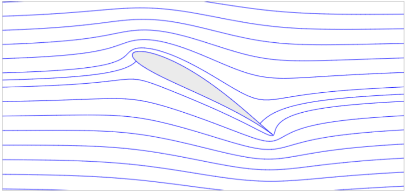


(C) medium angle of attack



(D) high angle of attack (stall) ←



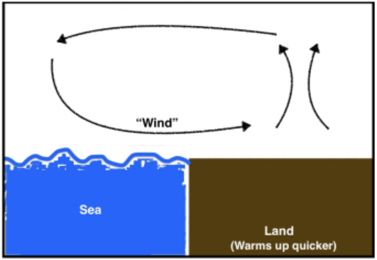


Figure 2.1 Sunny day at coast

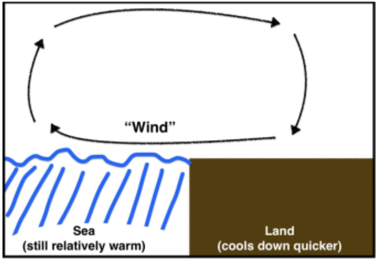


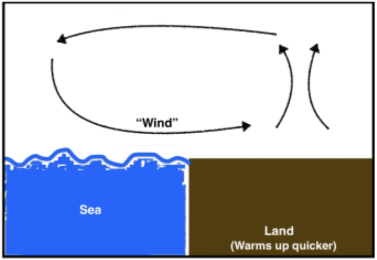
Figure 2.2 Clear night at coast

I know of people who were testing kites by towing them behind their cars during still-wind conditions, to mimic a wind tunnel. If you had to guess, what time of day would you (roughly) think they were doing this?

(A) 06:00 (or whatever part of the day had the most even surface temperatures)

(B) 15:00 (or whatever part of the day had the biggest surface-temperature differences)

Global patterns



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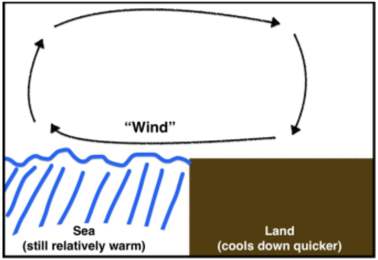


Figure 2.2 Clear night at coast

I know of people who were testing AWE systems by towing them behind their cars during still-wind conditions. If you had to guess, what time of day would you (roughly) think they were doing this?

(A) 06:00 (or whatever part of the day had the most even surface temperatures) ←

(B) 15:00 (or whatever part of the day had the biggest surface-temperature differences)

Global patterns

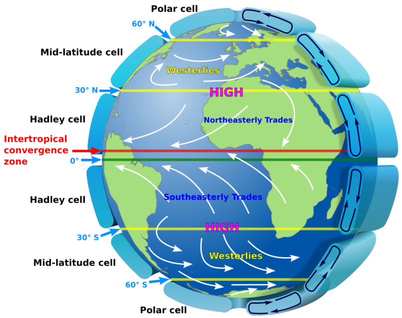
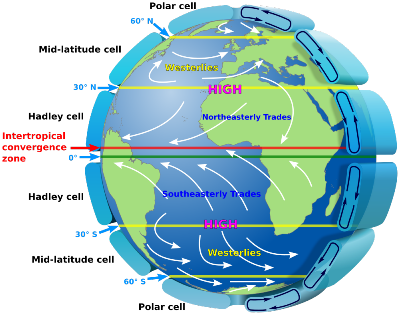


image from `https://en.wikipedia.org/wiki/Trade_winds#/media/File:Earth_Global_Circulation_-_en.svg`

Say you remember that three are three cells per hemisphere. Where can you be absolutely sure that the 'global trend' will be for air masses to rise?

- (A) the North/South pole
- (B) 45 deg. North/South
- (C) the equator

Global patterns



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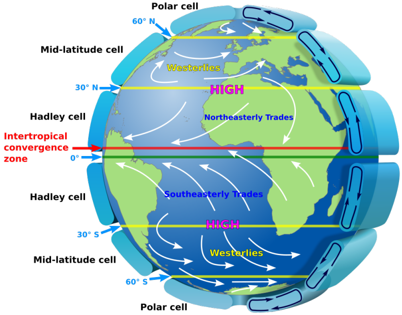
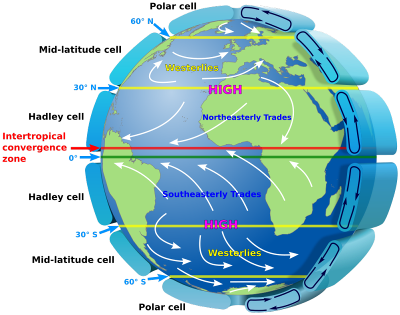
Global patterns



Say you remember that three are three cells per hemisphere. Where can you be absolutely sure that the 'global trend' will be for air masses to be sinking?

- (A) the North/South pole
- (B) 45 deg. North/South
- (C) the equator



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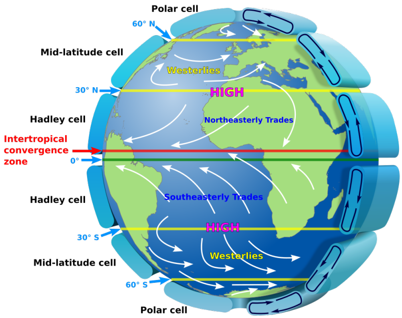
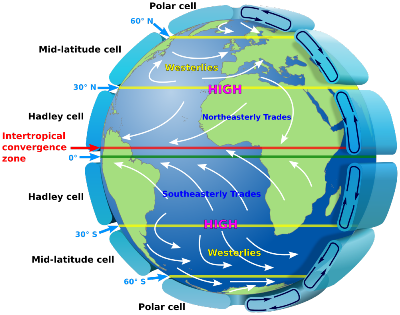
Global patterns



Freiburg is at 48 deg. North. If you see a storm 'approaching', what direction is it normally (in your personal experience) coming from?

- (A) the North, down the Rhine valley?
- (B) the East, over the Black Forest?
- (C) the South, over the Alps?
- (D) the West, over the Vosges?

Global patterns



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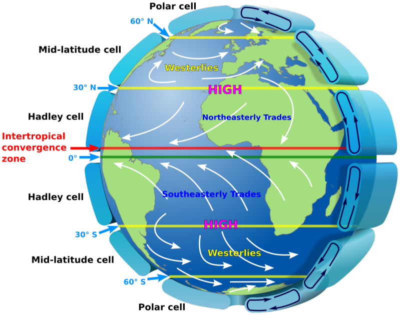
Global patterns



What significant historical event can you use to remember the global wind patterns?

- (A) the invasions of the Roman empire
- (B) the Protestant Reformation
- (C) the Atlantic slave trade
- (D) the separation of Germany

Global patterns

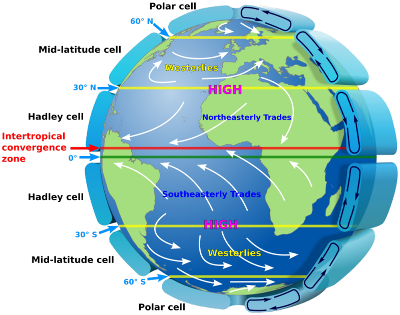
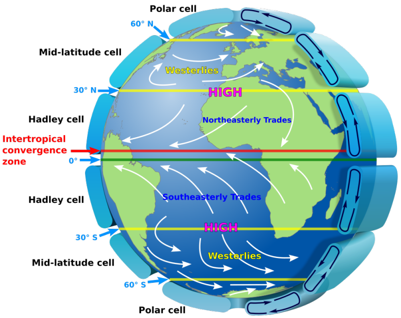


image from https://upload.wikimedia.org/wikipedia/commons/c/ca/Triangle_trade2.png

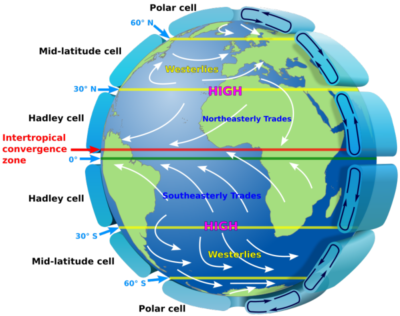
Global patterns



In the northern hemisphere, Coriolis force must always turn motions:

- (A) clockwise
- (B) anticlockwise
- (C) to the left
- (D) to the right ←

Global patterns



So, when flow is rushing from high pressure to fill a region of low pressure (in the Northern Hermisphere), which direction will the wind end up traveling around the point-of-lowest-pressure?

- (A) clockwise
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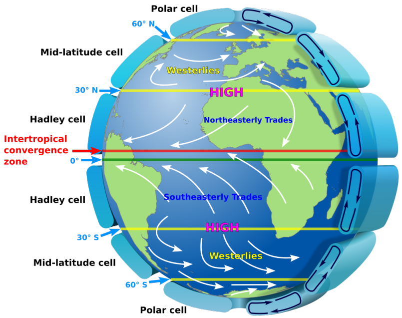
Global patterns



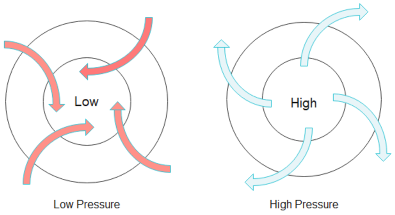


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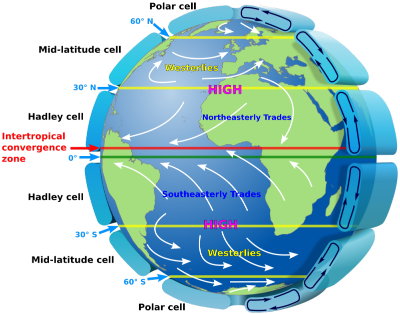
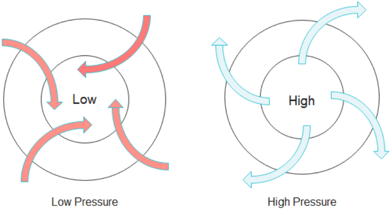
image from `https://www.vedantu.com/question-answer/in-the-northern-hemisphere-how-do-winds-`

Global patterns

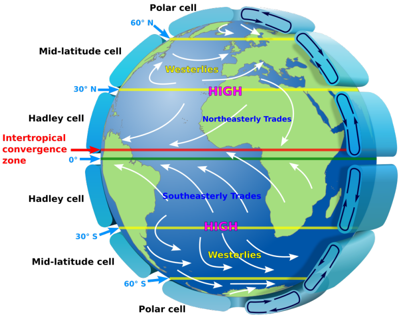


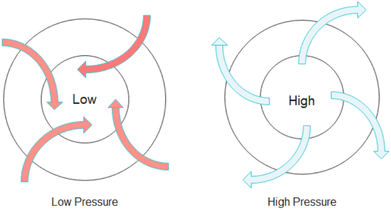


If you're out for a walk and there's a strong wind's at your back, which direction would you have to go to find the local high-pressure air-mass?

- (A) somewhere in front of you
- (B) somewhere to your right
- (C) somewhere behind you
- (D) somewhere to your left

Global patterns





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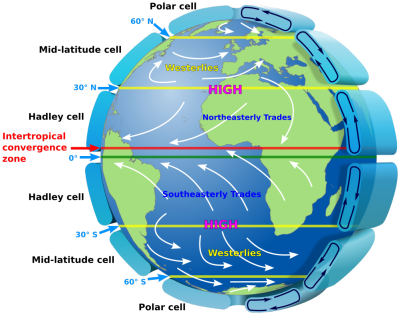
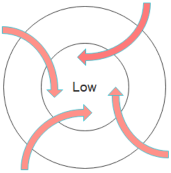
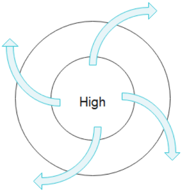
(if you're right handed, "Hi!")

Global patterns





Low Pressure



High Pressure

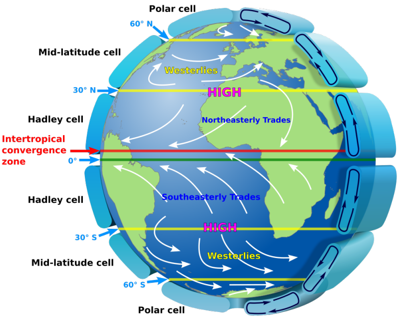
Remember that Coriolis force is

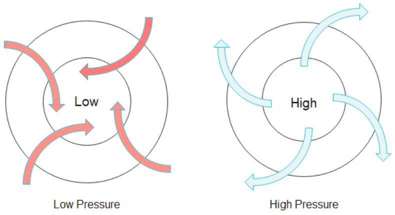
$$F = -2m(\boldsymbol{\Omega}_{\text{Earth}} \times \boldsymbol{v}')$$

Where do you have to go, for the high pressure to be exactly to your right?

- (A) lay down on the ground
- (B) stand beneath some trees/mountains
- (C) go above the trees/mountains
- (D) go far enough up that the atmosphere doesn't notice the trees/mountains anymore

Global patterns





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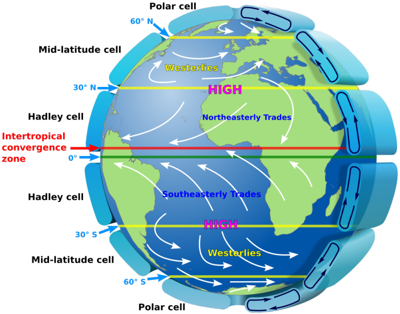
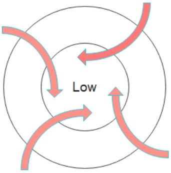
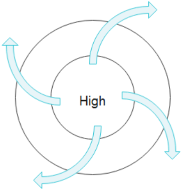
Geostrophic wind conditions only apply outside the ABL!

Global patterns





Low Pressure



High Pressure

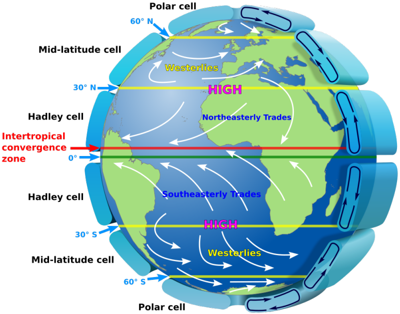
Remember that Coriolis force is

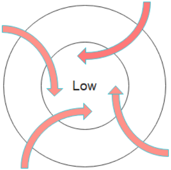
$$F = -2m(\boldsymbol{\Omega}_{\mathrm{Earth}} \times \boldsymbol{v}')$$

Suppose you're actually in a geostrophic wind condition, and you're moving with your back always to the wind. What line are you moving on? The line of constant...

- (A) temperature
- (B) density
- (C) pressure
- (D) CO_2 concentration

Global patterns





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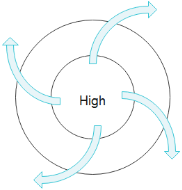

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- (D) CO_2 concentration

these are the black circles in this sketch.

Stable and unstable atmospheric stratification

image from
`https://mountwashington.org/life-cycle-of-a-cumulus-cloud-thunderstorm/`

If you're out for a walk and you see cumulus clouds, is the atmosphere stable or unstable? How much "mixing" is there air near the ground?

- (A) stable, lots of mixing
- (B) stable, very little mixing
- (C) unstable, lots of mixing ←
- (D) unstable, very little mixing

Stable and unstable atmospheric stratification

image from
https://mountwashington.org/life-cycle-of-a-cumulus-cloud-thunderstorm/

When should you expect the atmospheric boundary layer to be the thickest?

- (A) when there's lots of mixing (on a hot summer day)
- (B) when there's very little mixing (on a cold winter night)
- (C) eh, the whole thing is just kinda random

Stable and unstable atmospheric stratification

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Stable and unstable atmospheric stratification

image from
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If the atmosphere is unstable ('lots of mixing'), will you reach your "reference wind speed" at a lower altitude or a higher one, than when the atmosphere is stable ("very little" mixing)?

- (A) at higher altitudes
- (B) about the same
- (C) at lower altitudes

Stable and unstable atmospheric stratification

image from
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- (A) at higher altitudes ←
- (B) about the same
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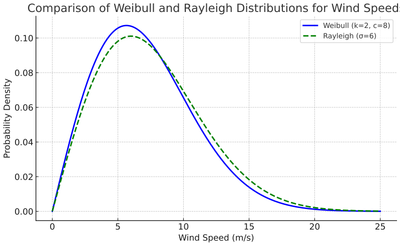
ask yourself the same question, when the reason there's mixing is because there are trees or tall buildings compared to over ocean

Statistics of the wind

occurence vs max. wind speed (miles/hour)



All of the wind speed PDFs we saw were smoothly single-modal (they had one local maximum)...

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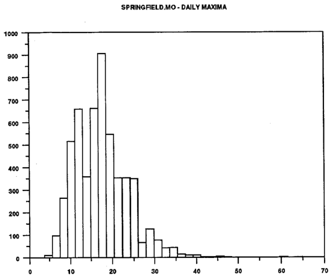

image from: `https://www.itl.nist.gov/div898/winds/pdf_files/b95033.pdf` page 53

All of the wind speed PDFs we saw were smoothly single-modal (they had one local maximum). But, this histogram of recorded max. wind speeds isn't smoothly single-modal. What happened?

- (A) measurement error
- (B) a tornado
- (C) we haven't taken a 'large enough number' of measurents yet.

Statistics of the wind

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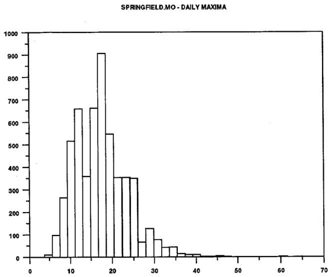

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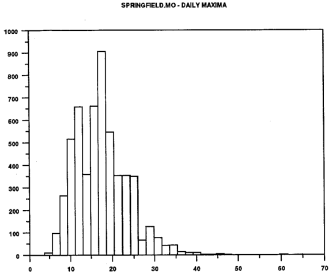

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All of the wind speed PDFs we saw had smooth tails. But, sometimes, histograms have 'blips' on their tails. What's more-likely to be happening here?

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Statistics of the wind

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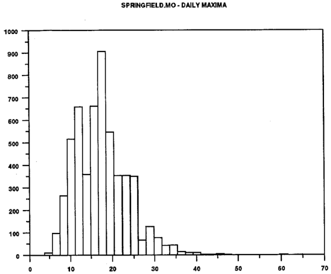

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- (A) measurement error
- (B) a tornado
- (C) we haven't taken a 'large enough number' of measurents yet.
- (D) I don't actually know, but you shouldn't trust the far-tails of PDFs. Statistical distributions are really unreliable at predicting rare events. ←

Actuator Disc Model and Betz' Limit (Momentum Theory)

C_{T} vs. a

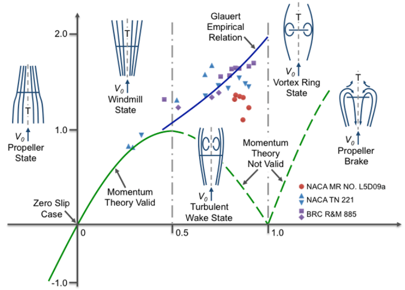

image from `https://www.e-education.psu.edu/aersp583/node/471`

We met this axial induction factor a. What system are we talking about if $a < 0$?

- (A) a fan
- (B) a wind turbine
- (C) a small wind turbine standing in front of a huge wall

Actuator Disc Model and Betz' Limit (Momentum Theory)

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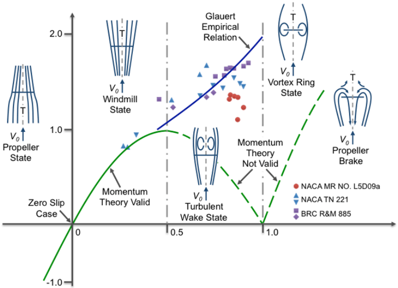

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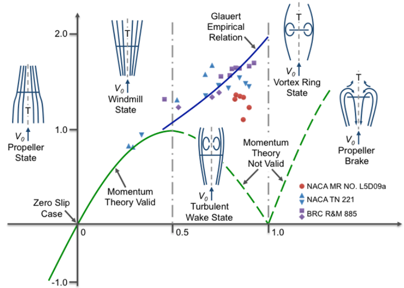

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We met this axial induction factor a. What system are we talking about if $0 < a < \frac{1}{2}$?

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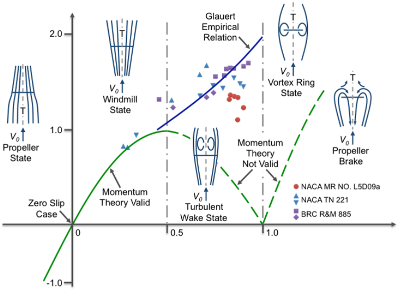
Actuator Disc Model and Betz' Limit (Momentum Theory)

C_T vs. a



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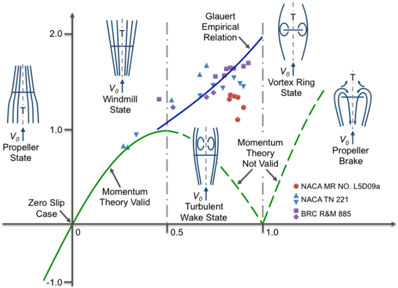

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We met this axial induction factor a. What system are we talking about if $\frac{1}{2} \leq a \leq 1$?

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- (B) a wind turbine
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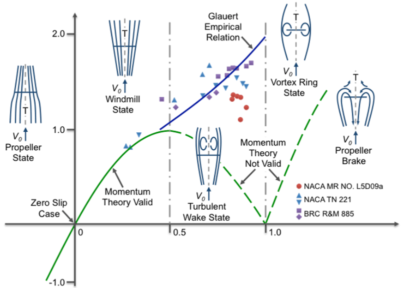
Actuator Disc Model and Betz' Limit (Momentum Theory)

C_{T} vs. a



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standing in front of a huge wall

$\leftarrow$

Actuator Disc Model and Betz' Limit (Momentum Theory)

Remember the assumptions of Bernoulli's theorem:

$$\mathrm{p} + \frac{1}{2}\rho_{\mathrm{air}}u^2 = \text{constant}$$

In which situation are you allowed to use this?

(A) pouring honey onto your toast

(B) using a stabmixer on a thin soup

(C) within a curved (smooth) ventilation duct

(D) when a mass of cold air 'slides' down a mountain (katabatic wind)

(E) flow past the wing of a commercial airliner (cruising Mach 0.85)

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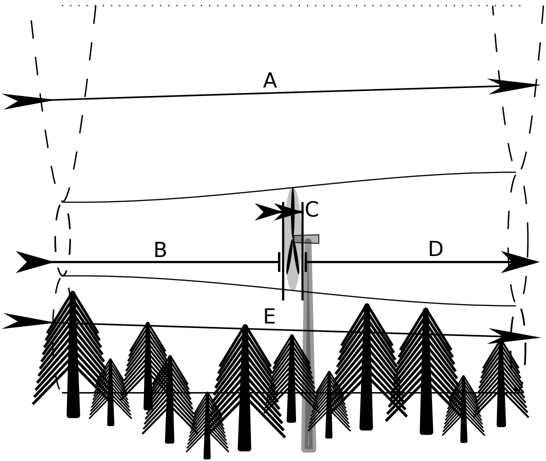
(E) flow past the wing of a commercial airliner (cruising Mach 0.85)

no friction, well below speed-of-sound, no energy added/removed from outside, no height differences!

Actuator Disc Model and Betz' Limit (Momentum Theory)



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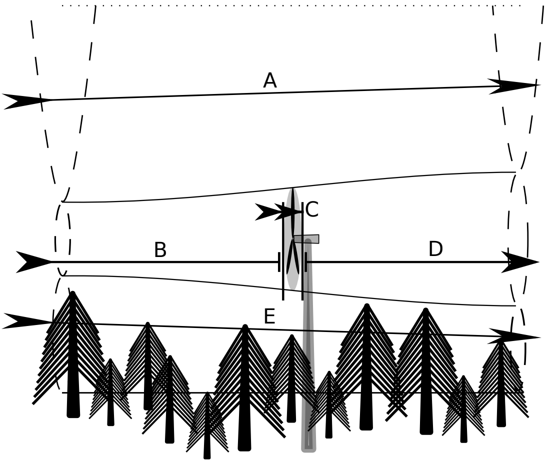

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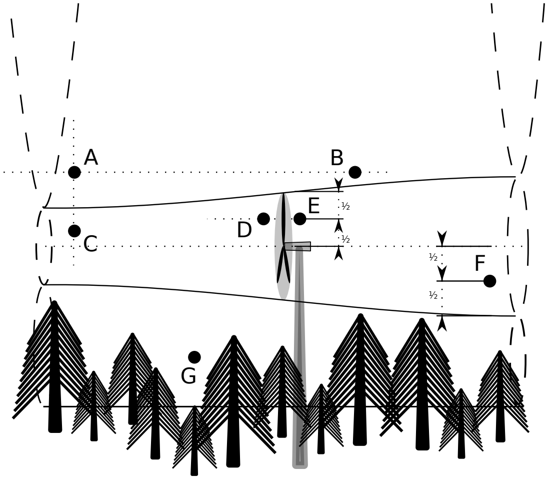
In which situation(s) are you allowed to use this?

(A), (B), (D): no friction, and no energy added/ removed from outside! (and (A) only with streamtube chosen so height difference is small)

Actuator Disc Model and Betz' Limit (Momentum Theory)



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At which locations are the axial wind speed the same?

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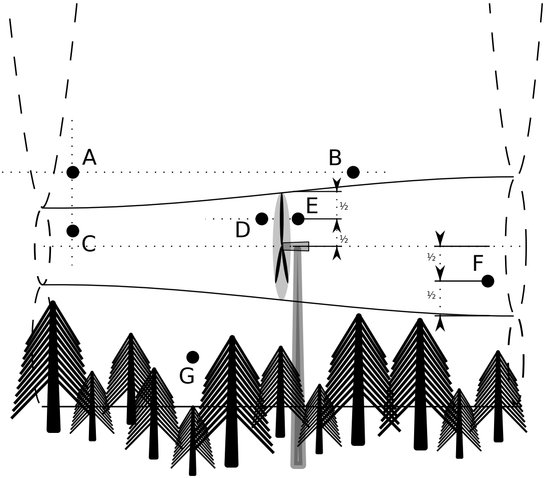

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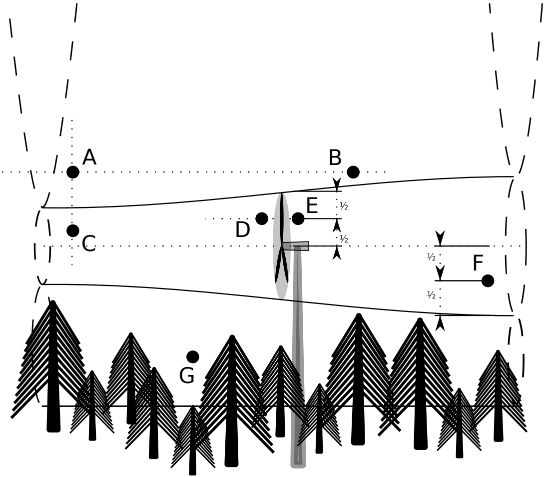
At which locations are the axial wind speed the same?

$u_{\mathrm{D}} = u_{\mathrm{E}}$. (If the velocity profile is fairly flat and/or there's not a lot of altitude difference, then also: $u_{\mathrm{A}} = u_{\mathrm{C}}$.)

Actuator Disc Model and Betz' Limit (Momentum Theory)



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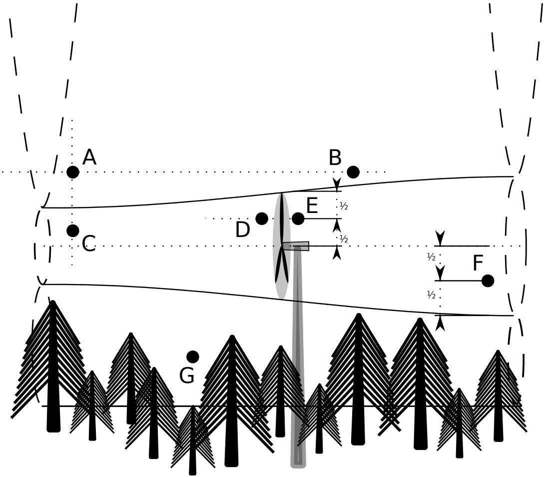

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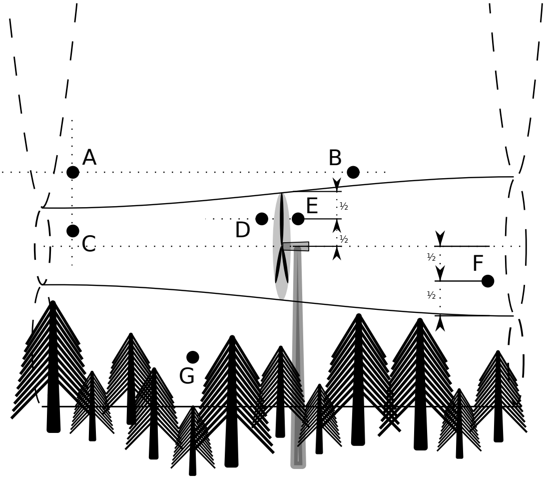
$\mathrm{p_C} = \mathrm{p_F}$ ($= \mathrm{p_A}$ if the altitude difference is not too large)

Actuator Disc Model and Betz' Limit (Momentum Theory)



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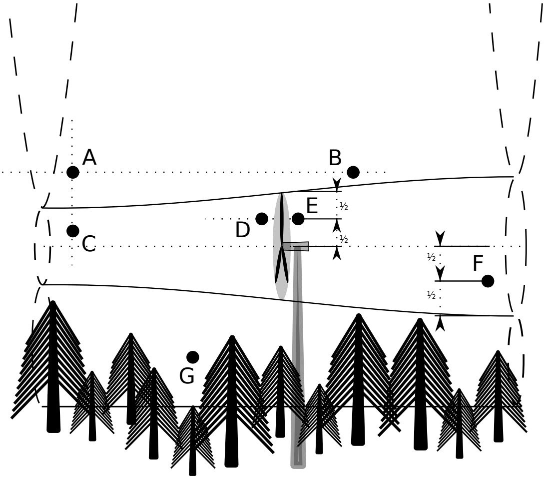
At which location is the axial wind speed the highest?

Actuator Disc Model and Betz' Limit (Momentum Theory)



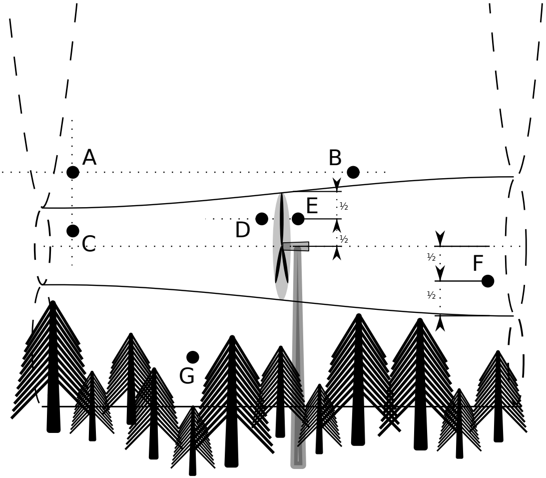
Remember the assumptions of Bernoulli's theorem:

$$\mathrm{p} + \frac{1}{2}\rho_{\mathrm{air}} u^2 = \text{constant}$$

At which location is the axial wind speed the highest?

(B), because there's a very-slight decrease in area of it's streamtube, due to the expanding wind-turbine's streamtube.

Actuator Disc Model and Betz' Limit (Momentum Theory)



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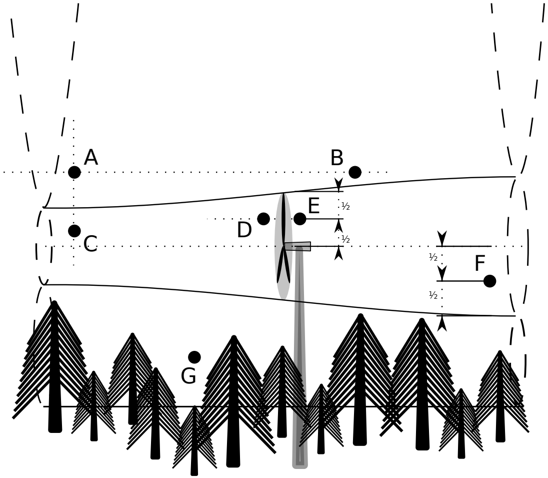
$$p + \frac{1}{2}\rho_{\mathrm{air}} u^2 = \text{constant}$$

At which location is the axial wind speed the lowest?



Remember the assumptions of Bernoulli's theorem:

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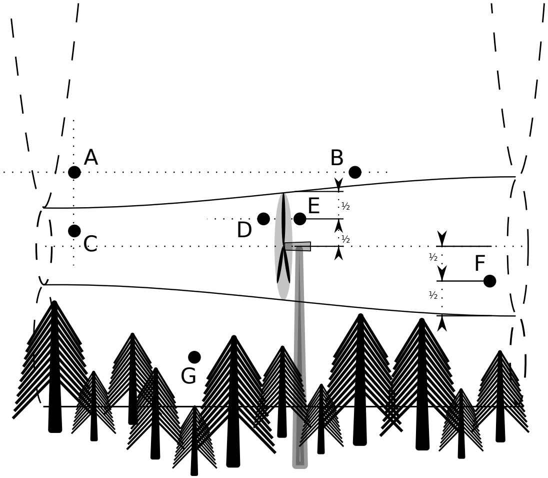
At which location is the axial wind speed the lowest?

(G), but that's because of friction with the trees not Bernoulli

Actuator Disc Model and Betz' Limit (Momentum Theory)



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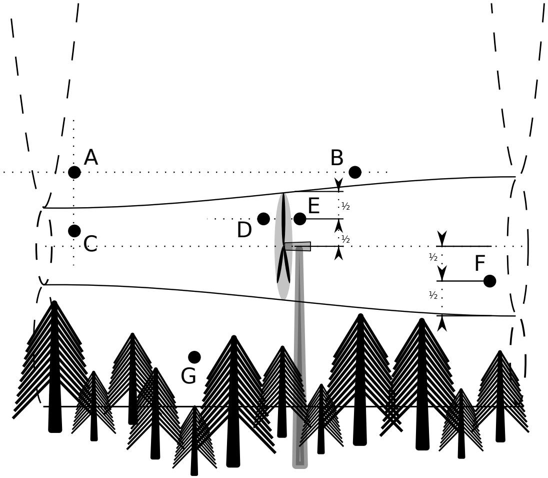
At which location (where we can use Bernoulli) is the axial wind speed the lowest?



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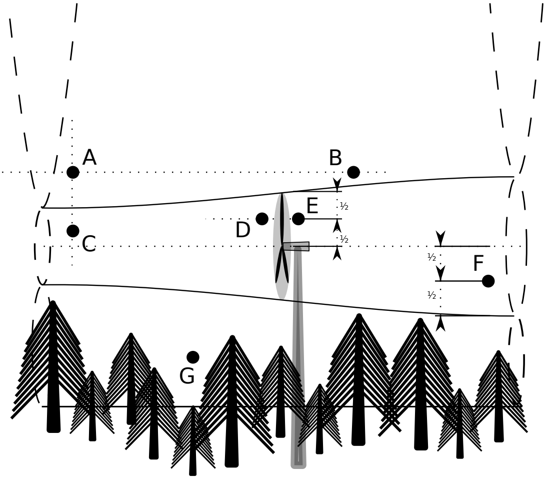
At which location (where we can use Bernoulli) is the axial wind speed the lowest?

(F), because the flow keeps slowing down even after the turbine

Actuator Disc Model and Betz' Limit (Momentum Theory)



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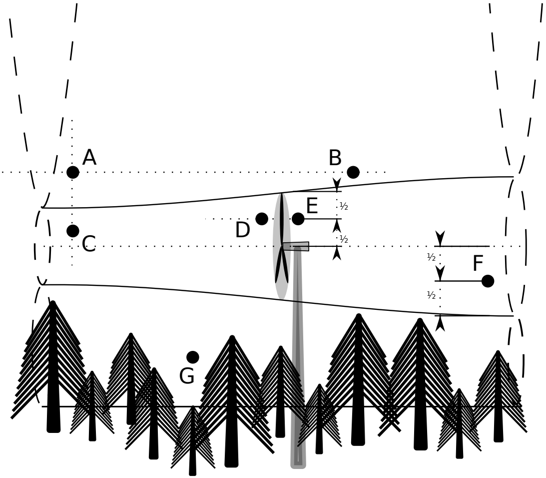
Judged by Bernoulli, at which location is the static pressure p the highest?

Actuator Disc Model and Betz' Limit (Momentum Theory)



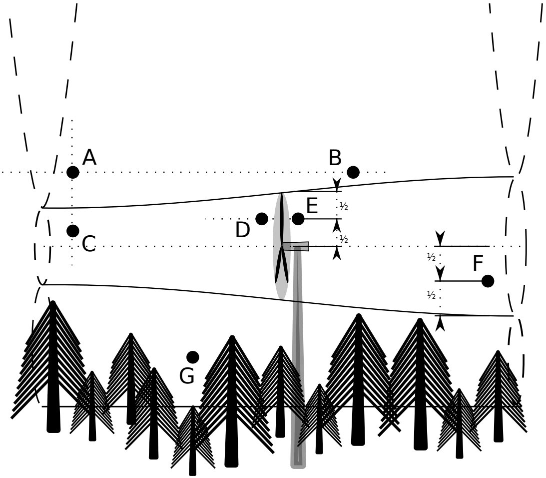
Remember the assumptions of Bernoulli's theorem:

$$\mathrm{p} + \frac{1}{2}\rho_{\mathrm{air}}u^2 = \text{constant}$$

Judged by Bernoulli, at which location is the static pressure p the highest?

(D) because the flow has been slowing since location C without having to drive a turbine

Actuator Disc Model and Betz' Limit (Momentum Theory)

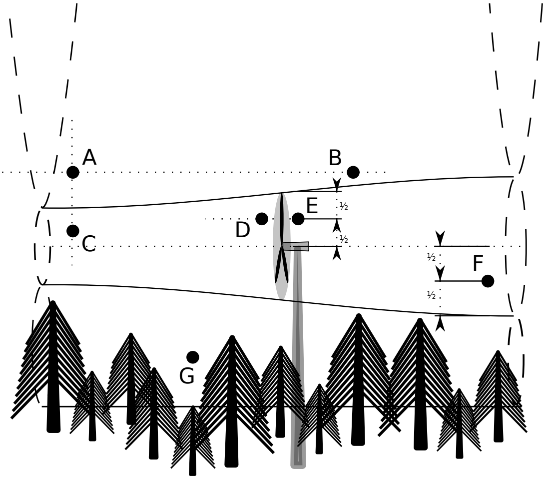


Remember the assumptions of Bernoulli's theorem:

$$\mathrm{p} + \frac{1}{2}\rho_{\mathrm{air}} u^2 = \text{constant}$$

Ignoring altitude effects, at which location is the static pressure p the lowest?

Actuator Disc Model and Betz' Limit (Momentum Theory)



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Ignoring altitude effects, at which location is the static pressure p the lowest?

(E) because we've just powered a turbine!!

Actuator Disc Model and Betz' Limit (Momentum Theory)

C_{T} vs. a

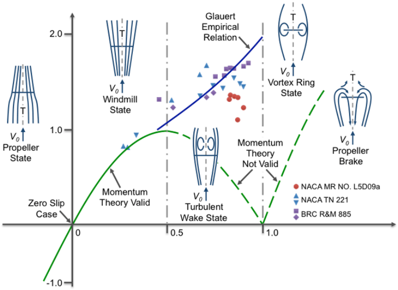
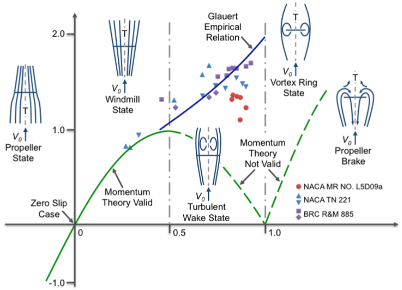


image from `https://www.e-education.psu.edu/aersp583/node/471`

Remembering assumptions, where are we allowed to use Momentum Theory, Rotor Disk Theory, or BEM?

- (A) during wind gusts
- (B) while pitching the blades
- (C) while yawed
- (D) when the wind shear is strong
- (E) when $C_{\mathrm{T}} > 1$
- (F) otherwise

C_T vs. a



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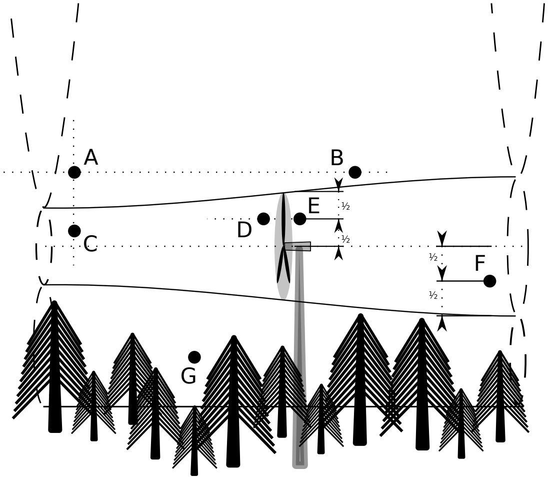
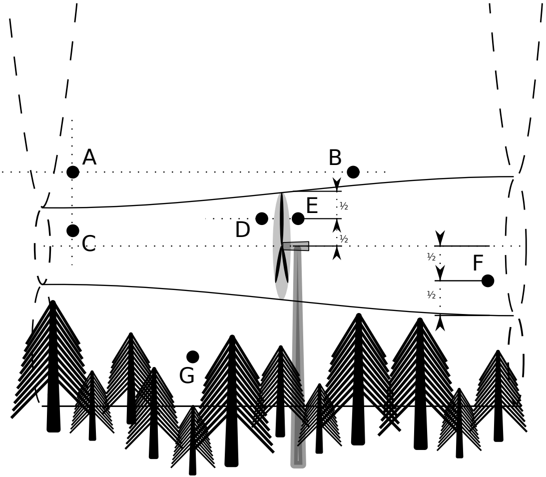
Wake Rotation & Rotor Disc Theory



At which locations does a fluid element have the same angular-velocity about the turbine's axis?

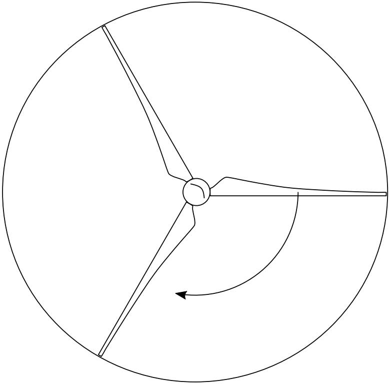
Wake Rotation & Rotor Disc Theory



At which locations does a fluid element have the same angular-velocity about the turbine's axis?

$\omega_\mathrm{E} = \omega_\mathrm{F}$ and
$\omega_\mathrm{A} = \omega_\mathrm{B} = \omega_\mathrm{C}$

Wake Rotation & Rotor Disc Theory



Rotor Disk Theory (and zero-drag BEM) said:

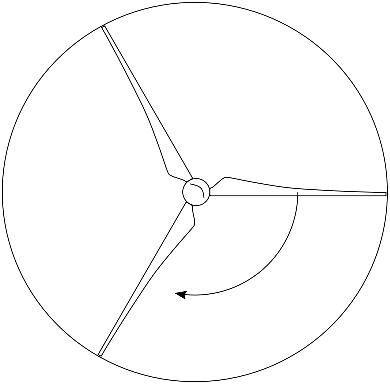
$$a' = \frac{a(1-a)}{\lambda_r^2}$$

When looking along the wind, if the turbine's rotor turns clockwise, what direction is the wake rotated?

(A) clockwise

(B) anti-clockwise

Wake Rotation & Rotor Disc Theory



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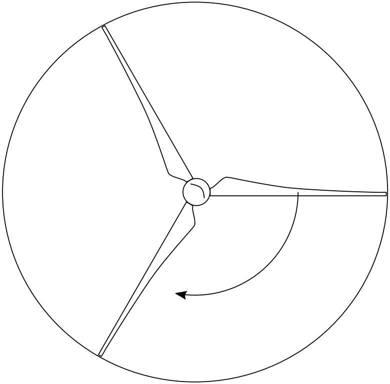
the torque from the flow on the blades is equal and opposite from the torque from the blades on the flow.

Wake Rotation & Rotor Disc Theory



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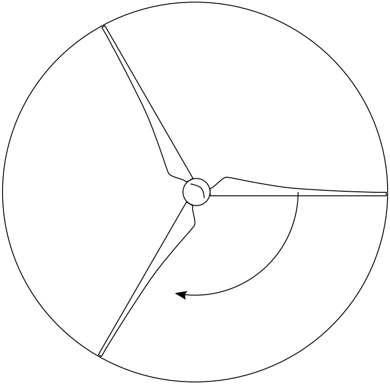
This means that the fluid elements passing over the blade...... get a lot more angular velocity than the fluid elements passing over the blade.......

- (A) tips, middle
- (B) tips, root
- (C) middle, tips
- (D) middle, root
- (E) root, tips
- (F) root, middle

Wake Rotation & Rotor Disc Theory



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(A) tips, middle

(B) tips, root

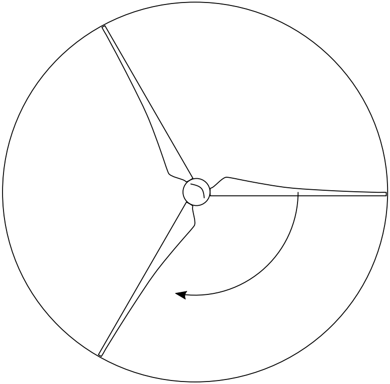
(C) middle, tips

(D) middle, root

(E) root, tips ←

(F) root, middle

Wake Rotation & Rotor Disc Theory



Rotor Disk Theory (and zero-drag BEM) said:

$$a' = \frac{a(1-a)}{\lambda_r^2}$$

In what scenario(s) do we have the most wake rotation? When each annulus also has/makes the most...

- (A) axial velocity
- (B) thrust
- (C) torque
- (D) power

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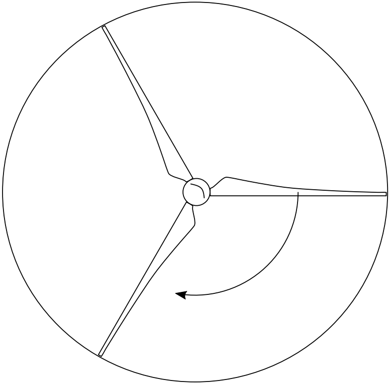

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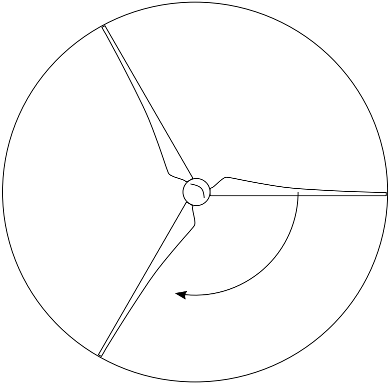

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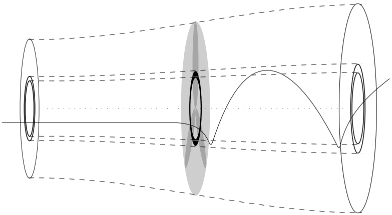
since the annular torque is the annular thrust times the radial distance, these two are the same.

Blade Element Momentum Theory (BEM)



$$B\boldsymbol{F}_{\text{aero}}(r) \cdot \hat{\boldsymbol{x}} = \boldsymbol{F}_{\text{A}}(r)$$
$$B\boldsymbol{F}_{\text{aero}}(r) \cdot \hat{\boldsymbol{t}} = \boldsymbol{F}_{\text{Tan}}(r)$$

When we computed $\boldsymbol{F}_{\text{A}}(r)$ and $\boldsymbol{F}_{\text{Tan}}(r)$ we gave each radial position it's own annulus, and each annulus it's own streamtube. Then, we applied the same steps as with momentum theory and rotor disk theory to that streamtube.

Going from far-upstream to far-downstream, if $C_{\text{T}} > 0$, this individual streamtube will:

- (A) get wider
- (B) stay the same width
- (C) get thinner

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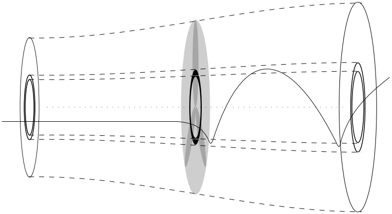

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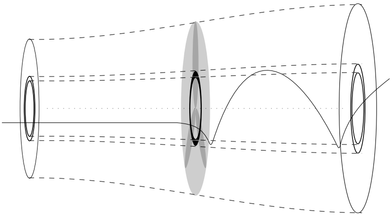

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Why do we only have induction factors for two dimensions (a and a')? (That is, why have we assumed that radial-direction induction is negligible?)

Ⓐ because radial-induction is negligible

Ⓑ because the equations for radial-induction are too complicated

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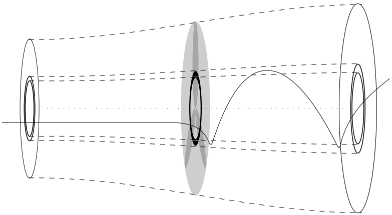

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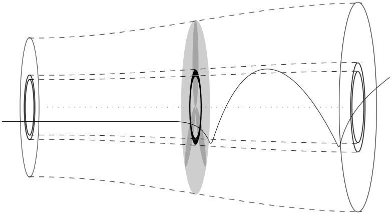

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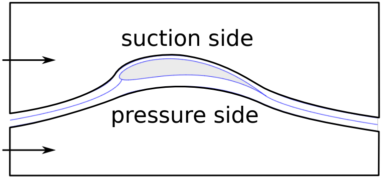

So, The pressure will try to 'escape' over the tips of the blades, damaging the assumptions of our BEM approach. Specifically: the (neglected) radial-induction is not actually small...

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(C) near the tips

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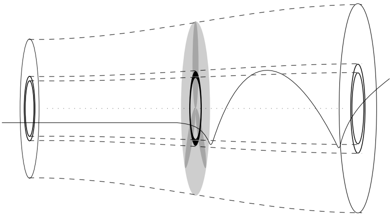

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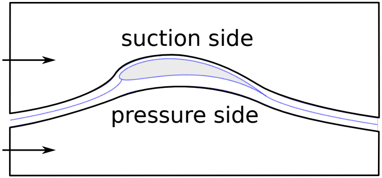

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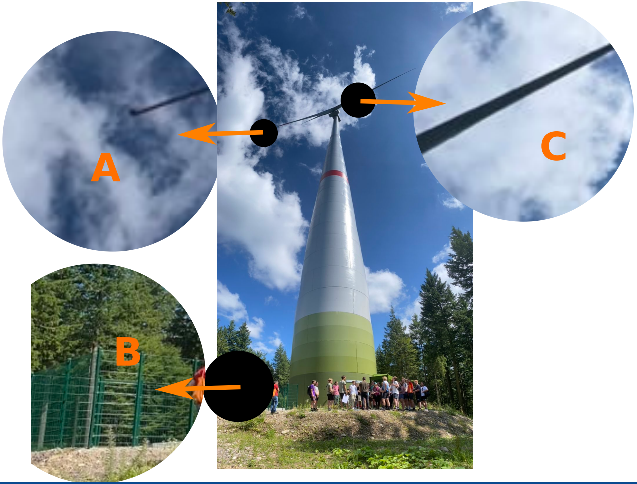

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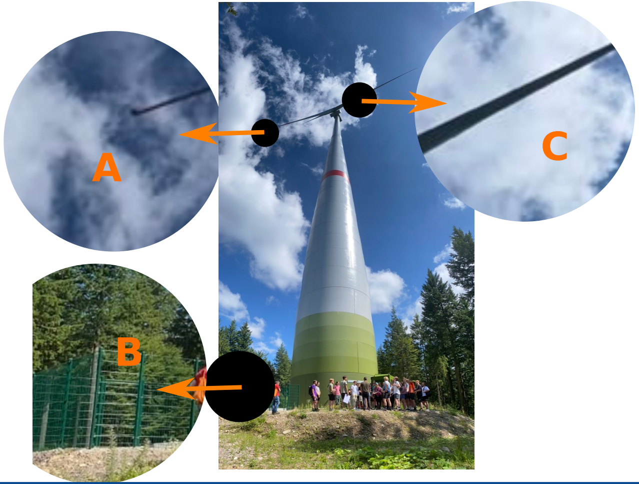
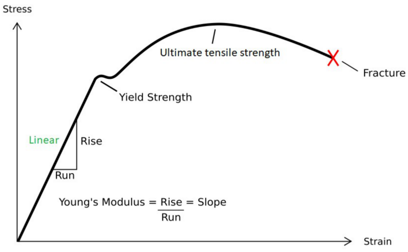


(A) winglets!

Stress and strain



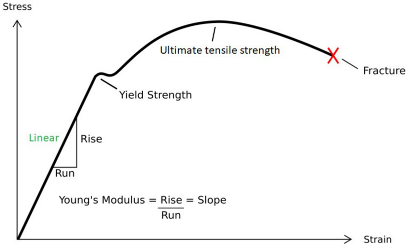
What does it mean, that the x-axis of this plot is the strain and the y-axis of this plot is the stress?

(A) we always start our calculations assuming a certain amount of deformation, with the stress being a dependent variable

(B) even if the beam isn't deforming, doing "more" to the beam will always make the internal forces increase further

(C) even if the internal forces don't change, doing "more" to the beam will always make the beam deform further

(D) eh, we're just doing what the first guy did.



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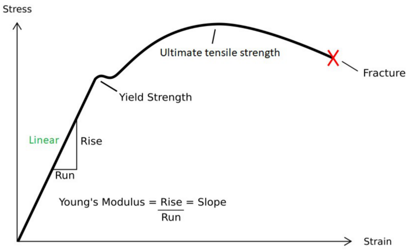

If you're looking through your cutlery drawer and you find a fork with the prongs bent out of shape, what must have happened to it? That fork was once stressed beyond...

- (A) the Young's modulus
- (B) the yield stress
- (C) the ultimate stress
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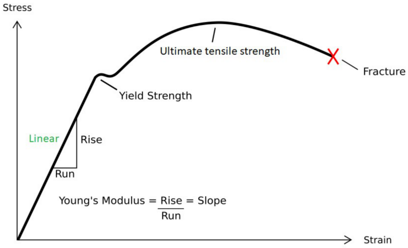

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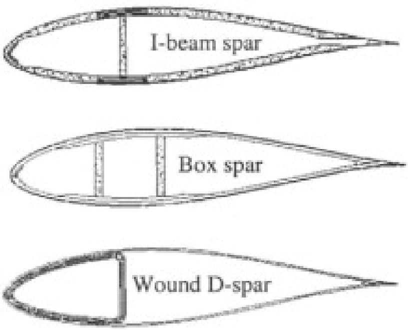
(Static) beam bending (Euler-Bernoulli theory)



$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) = q(x)$$

figure from:

`https://ars.els-cdn.com/content/image/1-s2.0-S1566136903801255-gr2.jpg`

In some cases, we approximated Euler-Bernoulli instead as:

$$\frac{\mathrm{d}^4(EIw)}{\mathrm{d}x^4} = q(x)$$

In the case of which utensil is that simplification most reasonable?

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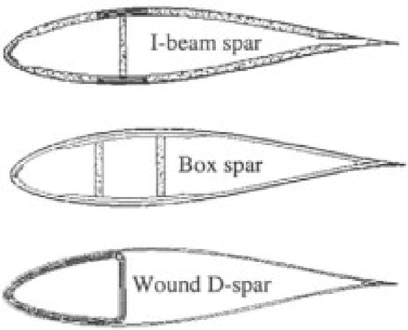

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if the knife handle is almost as thin as the knife blade and made form the same material, then (B).

otherwise, (D)

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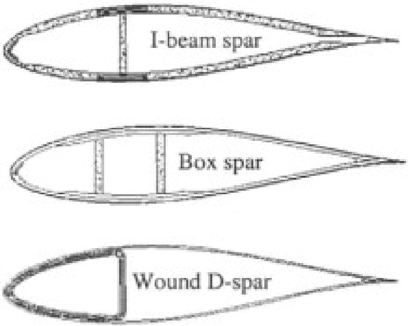

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) = q(x)$$

When trying to solve any ordinary differential equation that looks like

$$\frac{\mathrm{d}^N}{\mathrm{d}x^N} f(x) = q(x),$$

how many boundary conditions do we need?

(A) 1

(B) 4

(C) N

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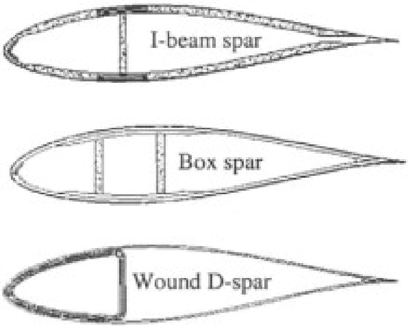

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To find the coefficients c_{N-1}, ... c_0:

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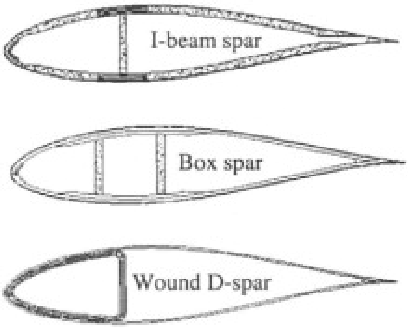
$$\frac{\mathrm{d}^{N-2}}{\mathrm{d}x^{N-2}} f(x) = \int \left(\int q(x)\mathrm{d}x + c_{N-1}\right) c_{N-2}$$

$$\ldots$$

(Static) beam bending (Euler-Bernoulli theory)



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The reason that (usually) only the prongs of the forks and the necks of the spoons that get deformed, is because:

(A) the fork-prongs and spoon-necks are the thinnest parts of the utensil with $I_{\text{(the scooping direction)}} \propto$ (width)^3 (height)^1.

(B) the fork-prongs and spoon-necks are the thinnest parts of the utensile, with $I_{\text{(the across-handle direction)}} \propto$ (width)^1 (height)^3.

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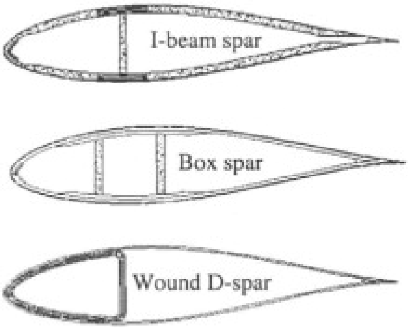

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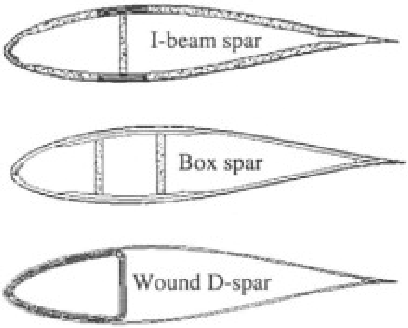

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2w}{\mathrm{d}x^2}\right) = q(x)$$

The blades are usually built around an (internal) structural beam profile. Which part of the Euler-Bernoulli beam theory is this internal structure affecting?

- (A) the Young's modulus E
- (B) the second moment of area I
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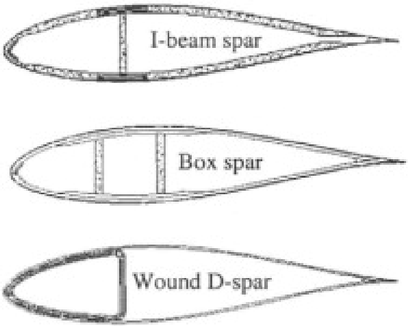

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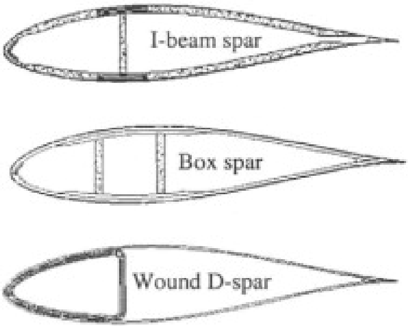

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We can make our beams out of wood, aluminum, steel, fiberglass, or carbon-fiber. Which part of the Euler-Bernoulli beam theory is this internal structure affecting?

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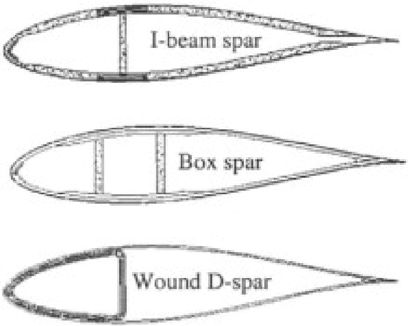

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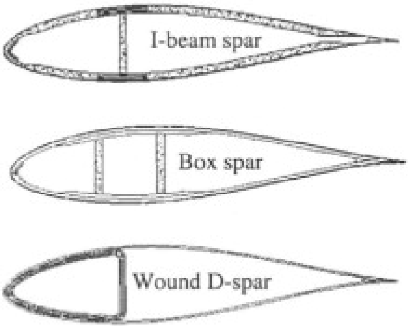

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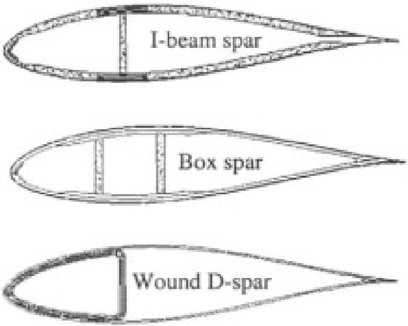

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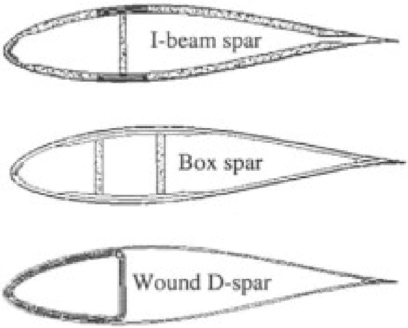

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When we talk about load-distribution $q(x)$ along the blade, what is x?

(A) the axial distance from the rotor center-of-rotation ($x = 0$ at the blade root, and $x =$ whatever downstream position the blade-tips are at

(B) the radial distance from the blade's socket ($x = 0$ at root and $x = R - r_{\mathrm{root}}$ at blade-tip).

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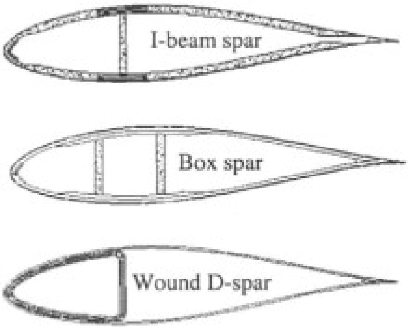

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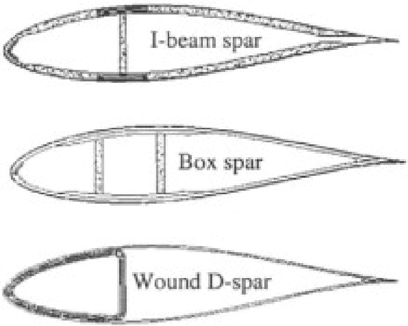

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If we increase $a(r)$, does the axial-direction deflection on the blade tip increase or decrease?

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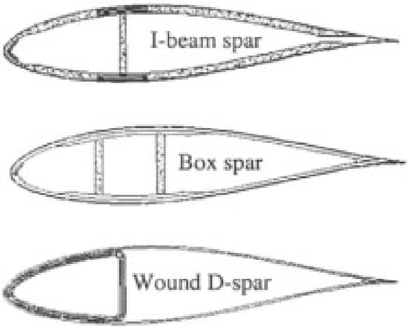
(Static) beam bending (Euler-Bernoulli theory)



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because increasing a means increasing the thrust-force per annulus, which means increasing $q(r)$.

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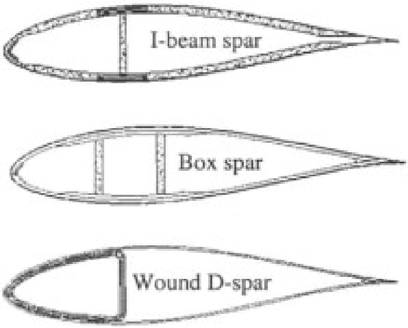

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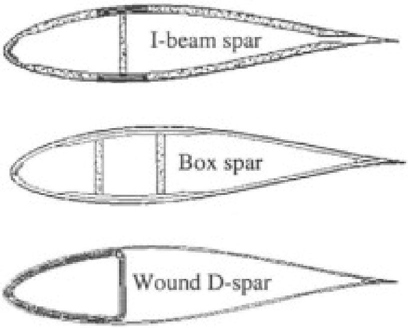
When applying Euler-Bernoulli to our blades, what assumption are we most-likely to violate?

(A) we neglect gravity

(B) we assume that E and I are constant

(C) we assume that deflections are small

(Static) beam bending (Euler-Bernoulli theory)



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because, the first two can be treated by choosing $q(x)$ correctly and by integrating correctly, but the last is in-built into the equations

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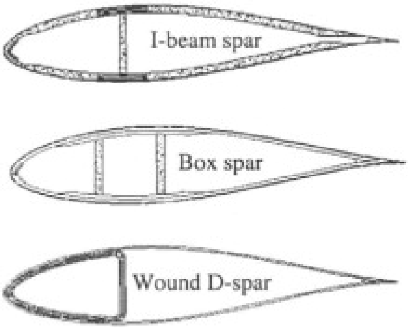

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- (B) the blade middle
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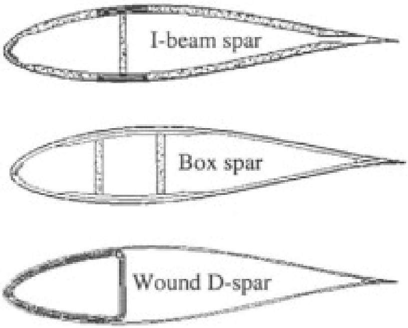

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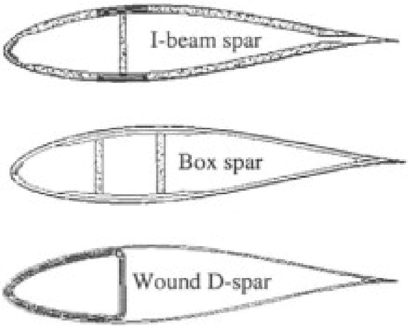

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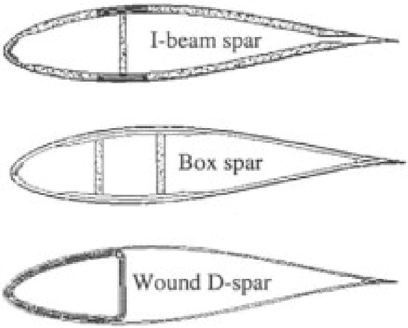

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