Surname: First name: Matriculation number:

Wind Energy Systems
Albert-Ludwigs-Universität Freiburg – Summer Semester 2025

Mandatory Self-Assessment of Course Pre-Requisites

Prof. Dr. Moritz Diehl und Rachel Leuthold

Deadline: 10am on Friday 16 May, 2025.
Submission: in the box in front of 102-00-073

The Wind Energy Systems (WES) course is a block-schedule course. As a result, the interaction with the professor (Prof. Dr. Moritz Diehl) during the course will be fast, intense, and focused. This means that a certain amount of understanding of mathematics and physics are necessary so that the instruction time can be used to best advantage.

Therefore, while the majority of the exercises can be completed at your own pace (with optional and ungraded submission), this preparatory assignment - which focuses on the prerequisite information - is mandatory. The total earned exercise score during this assignment must be at least 70-percent of the points available on this sheet (39 points out of the 55 available), in order to be admitted to the final exam.

Please note, that the final exam will be 'closed-book,' which means that ONLY pens, a calculator, and one A4 page (that is, two sides) of notes can be used. The questions will be partially multiple-choice and partially short-answer.

Please submit your solutions to this homework assignment into the box in front of Prof. Diehl's office (Building 102, Room 00-073) by 10am on Friday the 16th of May, 2025, in preparation for the second official class meeting on May 21st. If you have questions, please send them to the course tutor (Rachel) at `rachel.colette.leuthold@imtek.uni-freiburg.de`.

Current information about the course can be found at:
`https://www.syscop.de/teaching/ss2025/wind-energy-systems`.

In this exercise sheet, we want to check that your understanding of the course prerequisites.

1. **Matrix and Vector Operations** **[*15 pt*]**

   For all of the following problems, please consider the following matrices and vectors:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 5 & 7 \\ 8 & 6 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

   We recommend you calculate the tasks by hand first, but feel free to check your results with `Python`/`MATLAB`/`Mathematica`/etc!

   (a) $(\boldsymbol{A}+\boldsymbol{B})\boldsymbol{v} =$ [*1 pt*]

   (b) $\boldsymbol{A}\boldsymbol{v}+\boldsymbol{B}\boldsymbol{v} =$ [*1 pt*]

   (c) $(\boldsymbol{A}+\boldsymbol{B})\boldsymbol{C} =$ [*1 pt*]

(d) $\boldsymbol{A}\boldsymbol{A}^{-1} =$ [*1 pt*]

(e) $\boldsymbol{v}^\top \boldsymbol{v} =$ [*1 pt*]

(f) $\boldsymbol{v}\boldsymbol{v}^\top =$ [*1 pt*]

(g) $\boldsymbol{A}\boldsymbol{B} =$ [*1 pt*]

(h) $\boldsymbol{B}\boldsymbol{A} =$ [*1 pt*]

(i) $\boldsymbol{A}(\boldsymbol{B}\boldsymbol{C}) =$ [*1 pt*]

(j) $(\boldsymbol{A}\boldsymbol{B})\boldsymbol{C} =$ [*1 pt*]

(k) $\boldsymbol{A}^\top =$ [*1 pt*]

(l) $(\boldsymbol{A}\boldsymbol{v})^\top =$ [*1 pt*]

(m) $\boldsymbol{v}^\top \boldsymbol{A}^\top =$ [*1 pt*]

(n) $\boldsymbol{v}^\top \boldsymbol{A}^\top \boldsymbol{A}\boldsymbol{v} =$ [*1 pt*]

(o) $\displaystyle\sum_{i=1}^{2} v_i =$ [*1 pt*]

(p) $\begin{bmatrix} 1 & 1 \end{bmatrix} \boldsymbol{v} =$ [*1 pt*]

(q) $\|\boldsymbol{v}\|_2 =$ [*1 pt*]

2. **Linear System** [*4 pt*]

Convert the following system of equations into its equivalent matrix form ($\boldsymbol{Ax} = \boldsymbol{b}$) by defining the matrix $\boldsymbol{A}$ and the vector $\boldsymbol{b}$.

$$\begin{aligned} 3x_1 + 2x_2 + 6x_3 - 5 &= 0, \\ 4x_2 + 0.5x_3 &= 10 \end{aligned} \tag{1}$$

If we define the vector $\boldsymbol{x}$ as $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ then:

(a) $\boldsymbol{A} =$ [*1 pt*]

(b) $\boldsymbol{b} =$ [*1 pt*]

(c) Will you be able to solve (1) to find a unique, numerical solution for all three variables x_1, x_2, and x_3? (This is a yes/no question; you don't need to solve the system.) [*1 pt*]

(d) If a magic oracle also told you that $x_3 = 1$, would you then be able to find a unique, numerical solution for all three variables x_1, x_2, and x_3? (This is a yes/no question; you don't need to solve the system.) [*1 pt*]

3. **Histograms and Probabilities** **[*8 pt*]**

Suppose that we ran an experiment, and recorded the values taken during a trial in the form of a histogram. This histogram is shown in Figure 1.

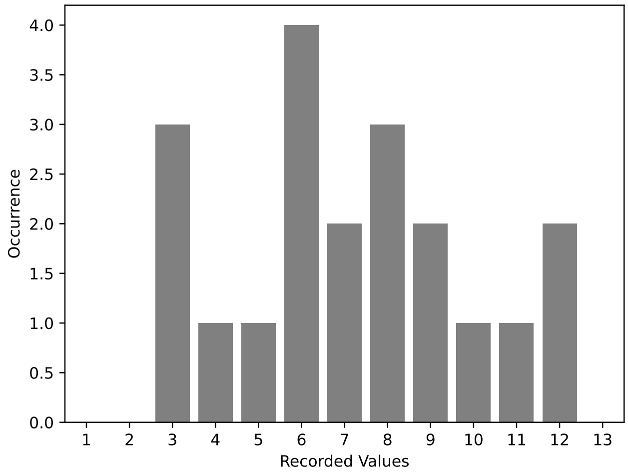

Figure 1: Histogram to be used with Problem 3

(a) How many measurements are recorded in the histogram? [*1 pt*]

(b) What is the experimental 'mean' of the recorded values? [*1 pt*]

(c) What is the experimental 'median' of the recorded values? [*1 pt*]

(d) Which of the following is most likely to be the correct probabilistic model for the data collected in the histogram? (That is, which probability distribution function $P_X(x)$ would you decide to use, when describing the likelihood that our experiment would measure any particular outcome x?) (Hint: if you're having difficulty here, maybe you want to try subproblem 3h first?) [*1 pt*]

i. $P_X(x) = \begin{cases} 1/36 & 0 \leq x < 36 \\ 0 & \text{otherwise} \end{cases}$

ii. $P_X(x) = \begin{cases} 1/36 & 3 \leq x < 12 \\ 0 & \text{otherwise} \end{cases}$

iii. $P_X(x) =$
$\begin{cases} (x-1)/36 & x \in \{2,3,4,5,6,7\} \\ (13-x)/36 & x \in \{8,9,10,11,12\} \\ 0 & \text{otherwise} \end{cases}$

iv. $P_X(x) = \begin{cases} 3/36 & x = 3 \\ 1/36 & x = 4 \\ 1/36 & x = 5 \\ 4/36 & x = 6 \\ 2/36 & x = 7 \\ 3/36 & x = 8 \\ 2/36 & x = 9 \\ 1/36 & x = 10 \\ 1/36 & x = 11 \\ 2/36 & x = 12 \\ 0 & \text{otherwise} \end{cases}$

(e) Suppose that we have a continuous random variable Y, where the probability of measuring any particular outcome y is described by the probability density function $p_Y(y)$. Let's say that this probability density function reads as:

$$p_Y(y) = \begin{cases} 0 & y \leq 1, \\ \frac{y-1}{32} & 1 < y \leq 9, \\ 0 & y > 9. \end{cases}$$

What is the expected value $E\{Y\}$? *[1 pt]*

(f) If $f(y) = y + 3$, what is the expected value $E\{f(Y)\}$? *[1 pt]*

(g) If $g(y) = y^2$, what is the expected value $E\{g(Y)\}$? *[1 pt]*

(h) If you integrate the probability density function over all possible values of y, what value would you get? That is, what is $\int_{-\infty}^{+\infty} p_Y(y)\mathrm{d}y$? *[1 pt]*

4. **Physics: Forces and Reference Frames** **[*20 pt*]**

Suppose we have a frictionless plane, inclined at an angle ϕ to the horizontal, as sketched in Figure 2. On this plane, there is a block of mass m, that's attached by a spring to some fixed point above the block. When attached to the block, the spring has a length ℓ; before the block was attached, the spring had a length ℓ_0. The spring has a spring constant k, and the acceleration of gravity is g. Please neglect all friction effects, including from the air.

Notice in Figure 2 that there are two orthonormal reference frames defined: one is the 'Earth-fixed' reference frame of $\hat{\boldsymbol{x}}$ (parallel to the ground) and $\hat{\boldsymbol{y}}$ (opposing gravity); and the other is the 'body-fixed' reference frame of $\hat{\boldsymbol{t}}$ (parallel to the surface of the inclined plane) and $\hat{\boldsymbol{n}}$ (perpendicular to the inclined plane). In the following problem, we're going to use the body-fixed reference frame, because this happens to make the problem simpler.

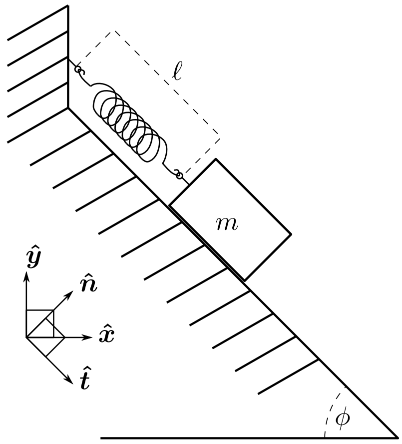

Figure 2: Sketch of the block-and-spring system to be used with Problem 4

(a) Can you find the vertical direction unit vector $\hat{\boldsymbol{y}}$ in terms of the tangential- and normal-direction unit vectors $\hat{\boldsymbol{t}}$ and $\hat{\boldsymbol{n}}$, and the inclination angle ϕ? (Hint: $\hat{\boldsymbol{y}}$ should be a linear combination of $\hat{\boldsymbol{t}}$ and $\hat{\boldsymbol{n}}$.) That is: $\hat{\boldsymbol{y}}(\hat{\boldsymbol{t}}, \hat{\boldsymbol{n}}, \phi) =$ [*2 pt*]

(b) What is the force on the block due to gravity $\boldsymbol{F}_{\mathrm{g}}$, in the direction $\hat{\boldsymbol{y}}$? [*2 pt*]

(c) Please now write this gravitational force $\boldsymbol{F}_{\mathrm{g}}$ as the vector sum of its contributions in the tangential- and normal- directions $\hat{\boldsymbol{t}}$ and $\hat{\boldsymbol{n}}$. [*2 pt*]

(d) If the plane were level with the ground ($\phi = 0°$), what would be the normal force $\boldsymbol{F}_{\mathrm{N}}$ on the block due to the plane's surface? (That is, what non-friction force does the plane have to apply so that the block doesn't fall through the solid surface?) Please write this solution as the vector sum of any contributions in the tangential- and normal- directions $\hat{\boldsymbol{t}}$ and $\hat{\boldsymbol{n}}$. [*2 pt*]

(e) Let's re-consider subproblem 4d when $\phi \neq 0$. In this case, what is the normal force $\boldsymbol{F}_{\mathrm{N}}$ on the block due to the inclinded plane? Please write this solution as the vector sum of any contributions in the tangential- and normal- directions $\hat{\boldsymbol{t}}$ and $\hat{\boldsymbol{n}}$. [*2 pt*]

(f) If the attachment of the block to the end of the spring has the effect of stretching the spring (ie., $\ell \geq \ell_0$), and the spring is exactly parallel to the plane's surface, what is the force on the block due to the spring $\boldsymbol{F}_\mathrm{k}$? Please write this solution as the vector sum of any contributions in the tangential- and normal- directions $\hat{\boldsymbol{t}}$ and $\hat{\boldsymbol{n}}$. *[2 pt]*

(g) Assuming that these three above-described forces - $\boldsymbol{F}_\mathrm{g}$ gravitational force, $\boldsymbol{F}_\mathrm{N}$ normal force, and $\boldsymbol{F}_\mathrm{k}$ spring force - are the only forces acting on the block. What is the resultant force $\boldsymbol{F}_\Sigma$ on the block? *[2 pt]*

(h) Do you expect this resultant force to be a scalar or a vector quantity? *[2 pt]*

(i) What is the acceleration $\boldsymbol{a}$ of the block? Please write this solution as the vector sum of any contributions in the tangential- and normal- directions $\hat{\boldsymbol{t}}$ and $\hat{\boldsymbol{n}}$.

[2 pt]

(j) Suppose you learn that you can find the total energy of this system by summing the kinetic energy associated with the block's motion, the potential energy associated with the block's position, and the potential energy associated with the spring's extension. Do you expect the output of this sum to be a scalar or a vector quantity? *[2 pt]*

5. **Physics: Moments and SI Units** **[*8 pt*]**

Suppose we have a seesaw made of some massless but infinitely-strong material. On one side of the frictionless pivot, at a distance ℓ_e from the pivot, stands an elephant with mass m_e. On the other side of the pivot, are two identical bunnies, each of mass m_b. The bunnies are placed, so that the distance between the pivot and the first bunny (ℓ_b) is the same as the distance between the first bunny and the second bunny (also ℓ_b). This situation is sketched in Figure 3. In all of these problems, we're going to assume that the seesaw is in perfect equilibrium, and the well-behaved animals do not move!

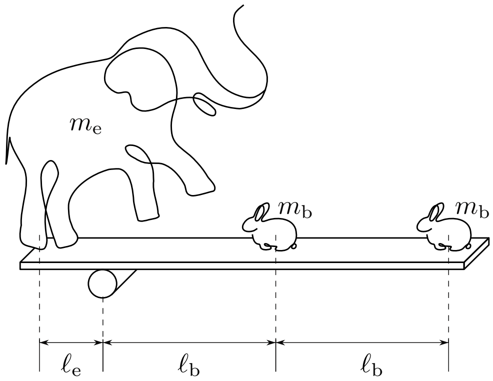

Figure 3: Sketch of the seesaw system to be used with Problem 5

(a) Let's consider the side of the see-saw with the elephant. If the acceleration of gravity g acts perpendicular to the seesaw, what is the torque about the pivot due to the presence of the elephant? (Please make sure you indicate somehow the direction of the torque.) [*2 pt*]

(b) Now, let's move to the other side of the see-saw. With the same assumptions, what is the torque about the pivot due to the presence of the bunnies? (Please make sure you indicate somehow the direction of the torque.) [*2 pt*]

(c) Strictly assuming that that seesaw is in perfect equilibrium and perpendicular to the direction of gravity, what equation would relate the various masses and lengths? [*2 pt*]

(d) Now, suppose we learn that the elephant has a mass of $m_\mathrm{e} = 1.5$ tonnes (with each tonne being a thousand kilograms), and each bunny has a mass of $m_\mathrm{b} = 500$g. If the elephant is standing at a distance of $\ell_\mathrm{e} = 20$cm from the pivot, what must be the distance between the pivot and the farthest bunny ($2\ell_\mathrm{b}$), in meters? [*2 pt*]