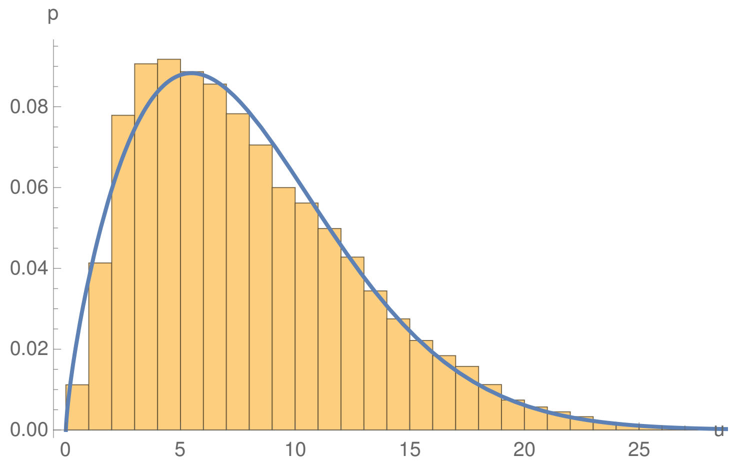
Exercise Sheet 2SOLUTION: Wind Resource

The Wind Resource

1. First, let's consider some real, local wind speeds, to get an idea of its probability distribution.
   (a) To do this, find hourly wind speed data at 100m height for year 2022 at the coordinates of the Rosskopf wind turbines: `https://open-meteo.com/en/docs/historical-weather-api`.
   (b) Open the link and familiarize yourself with the website, see what data is available. Think about other measurements, that can be relevant to wind power output.
   (c) Download the data using the API or as CSV file.
   (d) Open the data in a programming environment, extract the wind speed measurements from the dataset, and remove the entries without valid data points.
   (e) Consider some basic statistical properties of the wind speed dataset. What are:
      i. the average wind speed $\overline{U}$?
         The mean wind speed $\overline{U} \approx 4.1$ m/s.
      ii. the standard deviation σ of the wind speed measurements?
         The standard deviation $\sigma \approx 2.3$ m/s.
   (f) Plot a histogram of the wind speeds, normalized so as to represent the probability density function.



Note: This is an exemplar plot and you graph will look a bit different due to difference in used data set. The blue line is a fit of the Weibull distribution.

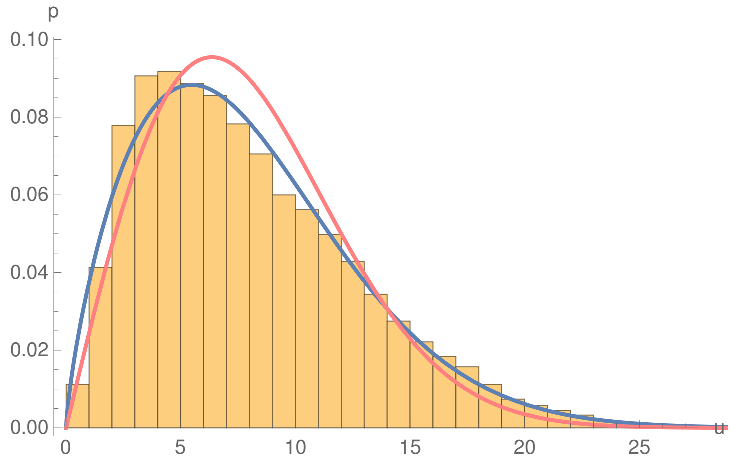
   (g) If you want to quickly get an idea of the mean wind speed corresponding to a particular Weibull distribution, which of the two parameters (shape k or scale λ) should you look at?
      We expect the scale parameter to have a similar order of magnitude to the mean wind velocity $\overline{U}$.
   (h) Estimate the Rayleigh distribution for this dataset, and add a plot of that Rayleigh distribution to your histogram.
      The Rayleigh distribution has only a scale parameter s. The mean of this distribution is given by:

$$\overline{U} = s\left(\frac{\pi}{2}\right)^{\frac{1}{2}},$$

which can be rearranged to give:

$$s = \overline{U}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}$$

In this case, $s = 4.1\sqrt{(2/pi)} \approx 3.27$.



Here, the pink line is the Rayleigh distribution.

(i) By comparing (visually) the histogram and Rayleigh distribution, do you think that the wind at this site is 'highly variable', 'somewhat variable' or 'not variable?' Please explain.

This is an exemplar answer for the plots above. The answer to the dataset used in the exercise itself can be different. An answer here would be 'somewhat variable.'

Because the Weibull plot is more concentrated for low wind speeds, we see that the Weibull shape parameter k must be smaller than the shape parameter when the Rayleigh distribution is put into Weibull form. (That is, 2.)

And, small k values indicate that the site has 'greater variability about the mean.' (see Burton et al. pg. 13).

But, since the Weibull plot is only concentrated slightly more towards the lower wind speed than the Rayleigh distribution, we shouldn't jump to extreme conclusions.

(j) Where is the probability density suggested by this plot likely to be accurate, and where not? Please explain.

'Enough' historical data for reliability has only been agglomerated in the center parts of the distribution. The high speed probabilities should not be considered reliable.

(k) Because wind power grows with the cube of the wind speed, the average of cubed wind speeds is important for wind turbine siting decisions.

i. Compute the average $\overline{U^3}$ of U_i^3, and the cubed average speed $\overline{U}^3$. Which number is higher?

$$\overline{U^3} \approx 151, \qquad \overline{U}^3 \approx 71.$$

We can see that $\overline{U^3} > \overline{U}^3$.

2. ased on the wind data for a wind turbine at the height of 100m you decide that it would be more feasible to construct a wind turbine with a height of 80m. Now you are asked to present wind profiles and probability distribution for this height. How would you go about it?

(a) or a quick assessment, you assume a logarithmic wind profile and using the two existing values you can use it to estimate the unknown wind speed. But where to find it?

Check the website from above and see for what other heights there is a wind speed

(b) Now after finding the second wind speed profile, how to estimate the new u_{80}?

Given a wind speed u_{z_1} at an altitude z_1 and two constants a and z_0, wind speed u_{z_2} at altitude z_2 can be found using the following equations:

$$u(z_2) = u(z_1)\frac{\ln\left((z_2 - a)/z_0\right)}{\ln\left((z_1 - a)/z_0\right)}$$

The logarithmic wind profile uses two constants (here, a and z_0) to find the wind at a given altitude:

$$u(z) = a\log\left(\frac{z}{z_0}\right)$$

With two datapoints we can find a and z_0:

$$u_{10} = a\log\left(\frac{10}{z_0}\right), \qquad u_{100} = a\log\left(\frac{100}{z_0}\right)$$

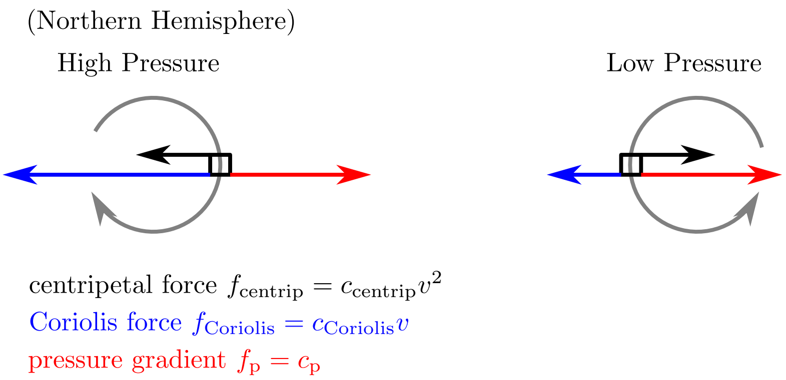


Figure 1: The force balance acting on the fluid elements in a gradient wind must lead to a resultant (net) force acting radially inwards. (The shown vectors are not-to-scale.)

If we solve this set of equations, we find a and z_0. We can then plug those values into the general expression to find u_{80}:

$$u_{80} = u_{100} \frac{\ln\left((80 - a)/z_0\right)}{\ln\left((100 - a)/z_0\right)}$$

(c) **Bonus!** In addition, if you like, list pros and cons of the larger turbine with higher hub height, and decide on your recommendation to the developer.

The main advantage of a larger/higher turbine is that more power could be extracted. If the larger radius is R_{100} and the smaller radius is R_{60}, we see that we can extract a factor of $(u_{100}/u_{80})^3(R_{100}/R_{60})^2$ more power.

On the other hand, u_{100}/u_{60} is not so large (approximately 1.1). So, while the developer may want to use a larger turbine, a higher hub-height might not be worth it...

Consider that a tower costs somewhere around 20 percent of the total wind turbine cost. That is, the tower makes up a significant portion of the wind turbine cost. If the tower is significantly more expensive due to its larger length (material cost, fabrication cost, transportation cost) a taller turbine may cost much more without giving much of a return.

3. Regard a high-pressure region in the northern hemisphere at a latitude of $\phi = 50^o$. We have learnt that geostrophic wind - as well as its refinement, the gradient wind - is parallel to the isobars, and grows with the gradient of the pressure.

(a) In what direction (as seen from above) does the air flow around the high pressure region described: clockwise or counterclockwise?

An anticyclone will rotate clockwise in the northern hemisphere.

(b) The pressure gradient at a specific location A on the boundary of the high-pressure region is 5 Pa/km. What would be the geostrophic wind at this location?

The geostrophic wind lies parallel to the isobars, with a magnitude proportional to the pressure gradient:

$$v = -\frac{\partial p}{\partial x} \frac{1}{2\rho \sin\phi \omega_0}$$

Here, $\rho = 1.225\text{kg/m}^3$ is the density of the air, $\phi = 0.87$ is the latitude, and $\omega_0 = 2\pi/((24)(3600\text{s})) = 7.3{\cdot}10^{-5}\text{rad/s}$ is the Earth's rotation frequency.

Notice that $\frac{\partial p}{\partial x} = 5 \cdot 10^{-3}\text{Pa/m}$, in SI units. This gives: $v \approx -37\text{m/s}$.

(c) Would the gradient wind be faster or slower than the geostrophic wind at this location?

For curved isobars, the sum of the Coriolis and pressure forces must correspond to the centripetal force due to the isobar's radius-of-curvature.

Using specific forces (force per unit mass), the force balance around a pressure center ($s \to -1$ for a high-pressure center in the Northern Hemisphere) becomes:

$$s f_{\text{centrip}} = f_{\text{p}} - f_{\text{Coriolis}} \quad \Rightarrow \quad s c_{\text{centrip}} V_{\text{G}}^2 + c_{\text{Coriolis}} V_{\text{G}} - c_{\text{p}} = 0, \tag{1}$$

with:

$$c_{\text{centrip}} = \frac{1}{\mathsf{r}}, \quad c_{\text{Coriolis}} = 2\sin\varphi\omega_0, \quad c_{\text{p}} = \frac{\partial \text{p}}{\partial x}\frac{1}{\rho}. \tag{2}$$

Notice that we've dropped the negative sign in order to make determining the direction of the force easier.

In the script, we solved for V_{G} with the quadratic formula.

Here, we instead want to first find the geostrophic wind V_{GEO}, and insert it into our quadratic above. The geostrophic wind is the special case of the gradient wind when the radius is infinite ($c_{\text{centrip}} = 0$). That means that the geostrophic wind is:

$$c_{\text{Coriolis}} V_{\text{GEO}} - c_{\text{p}} = 0, \quad \Rightarrow \quad c_{\text{p}} = c_{\text{Coriolis}} V_{\text{GEO}} = c_{\text{Coriolis}} \xi V_{\text{G}}, \tag{3}$$

if $\xi = \frac{V_{\text{GEO}}}{V_{\text{G}}}$ [-].

Now, our quadratic becomes:

$$\frac{s}{R} V_{\text{G}}^2 = c_{\text{Coriolis}} \left(V_{\text{GEO} - V_{\text{G}}} \right) = c_{\text{Coriolis}} V_{\text{G}} \left(\xi - 1 \right) \quad \Rightarrow \quad \xi = 1 + \frac{s}{R c_{\text{Coriolis}}} V_{\text{G}}. \tag{4}$$

So, we see that $\xi > 0$ if $s = (+1)$ (which is the case for flows rotating counter-clockwise), but $\xi < 1$ if $s = (-1)$ (which is case for flows rotating clockwise about their pressure center).

And since flows rotate clockwise in the northern hemisphere about high-pressure zones, the geostrophic wind must be slower than the gradient wind.

TLDR: For flows around high-pressure centers, the percieved centrifugal force (the fictitious force - exactly equal and opposite to the centripetal resultant force - that you use to represent the force you experience of being flung to the outside of a curve if the reference frame in which you're solving the problem is rotating) 'helps' the pressure gradient, rather than the Coriolis force. Therefore, in this circumstance, the gradient wind will be faster than the geostrophic wind.