Rehearsal Questions for the course "Numerical Optimization"

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1. What is an optimization problem? Objective, degrees of freedom, constraints. Feasible set? Standard form of NLP.
2. Definition of global and local minimum.
3. State a condition under which there exists at least one minimizers.
4. Types of optimization problems: Linear / Quadratic programming (LP/QP), convex, smooth, mixed-integer, ...
5. When is a function convex? Gives the definition and a characterization when it is twice differentiable. Definition. If it is twice differentiable?
6. When is a set convex? Definition.
7. What is a "stationary" point in the the case of unconstrained smooth optimization?
8. How are gradient and Hessian of a function $f : \mathbb{R}^n \to \mathbb{R}$ defined?
9. What are the first order necessary conditions for optimality (FONC) in the case of unconstrained optimization problems?
10. What are the second order necessary conditions for optimality (SONC) in the case of unconstrained optimization problems?
11. What are the second order sufficient conditions for optimality (SOSC) in the case of unconstrained optimization problems?
12. Basic idea of iterative descent methods?
13. Definition of local convergence rates: q/r-linear, superlinear, quadratic?
14. What is a locally convergent algorithm? What is a globally convergent algorithm? What does the term "globalization" usually mean for optimizers?
15. What is the Armijo condition? Why is it used in line-search algorithms?
16. Why is satisfaction of Armijo condition alone not sufficient to guarantee convergence towards stationary points? Give a counterexample.
17. What is backtracking?
18. Define the *the steepest descent method*. What is the local convergence rate?
19. What is Newton's method for solution of nonlinear equations $F(x) = 0$? How does it iterate, what is the motivation for it. How does it converge locally?
20. How works Newton's method for unconstrained optimization?
21. What are *Newton-type / approximate Newton* methods?
22. What is the idea behind Quasi-Newton methods?
23. What is the secant condition? How is it motivated?

24. What is the BFGS formula? Under which condition does it preserve positive definiteness?
25. Prove that the latter condition is necessary for any update formula satisfying the secant condition to stay positive definite.
26. What is a linear least squares problem (unconstrained)? What is a a nonlinear least squares problem (unconstrained)?
27. How does the Gauss-Newton method iterate? When is it applicable?
28. When does the Gauss-Newton method perform well? What local convergence rate does it have?
29. Statistical motivation of least squares terms in estimation problems?
30. What are the differences and similarities between line search and trust region methods?
31. List two ways to compute derivatives with help of computers.
32. What errors occur when computing derivatives with finite differences? Do you know a rule of thumb of how large to choose the perturbation?
33. What is the idea behind Automatic Differentiation (AD)? What is its main advantage?
34. Can AD be applied to compute second order derivatives?
35. There are two ways of AD. Describe briefly. What are the advantages / disadvantages of the two?
36. Write a nonlinear program (NLP) in its standard form. How is the lagrangian function defined?
37. What is the constraint qualification (CQ)? What is the linear independence constraint qualification (LICQ) at some point $\bar{x}$?
38. What are the Karush-Kuhn-Tucker (KKT) conditions for optimality? Why is it useful?
39. What are the first order necessary conditions for optimality (FONC) (constrained)?
40. What are the second order necessary conditions for optimality (SONC) (constrained)?
41. What are the second order sufficient conditions for optimality (SOSC) (constrained)?
42. What is the "active set of constraints"?
43. Give a standard form of a QP.
44. When is a QP convex?
45. What is the main idea of an active set strategy?
46. What is the main idea behind an SQP method (for inequality constrained problems)?
47. What is the L_1-penalty method for equality-constrained problems? Under which condition is it "exact", i.e. has the same local minima as the original NLP?

48. How works Newton's method for equality constrained optimization?
49. What local convergence rate does an SQP method with exact Hessian usually have?
50. What is the basic idea of interior point methods? Gives two views this class of method.
51. What is the Lagrangian dual function of a general NLP?
52. What is the dual problem of a general NLP?
53. What is weak duality? To which problems does it apply?
54. What is strong duality? Under which sufficient conditions does it apply?
55. What is a semidefinite program (SDP)? Give a standard form.
56. How would you reformulate the following eigenvalue optimization problem into an SDP for A_0, A_1, A_2 three symmetric matrices?

$$\begin{array}{ll} \underset{x \in \mathbb{R}^2}{\text{minimize}} & \lambda_{\max}(A_0 + x_1 A_1 + x_2 A_2). \end{array} \tag{1}$$