

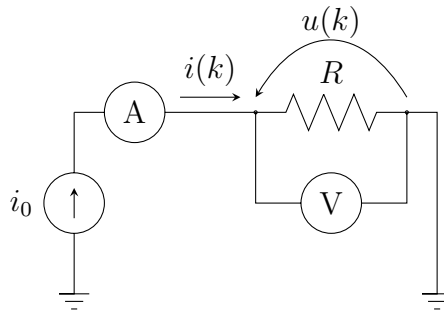
Exercise 1: Resistance Estimation Example
(to be returned before October 30th, 8:30)

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In this exercise you investigate some important facts from statistics in numerical experiments.

Exercise Tasks

1. We consider the following experimental setup:



Imagine you are sitting in a class of 200 electrical engineering students and you want to estimate the value of R using Ohm's law. Since the value of the current i_0 flowing through the resistor is not known exactly, an ammeter is used to measure the current $i(k)$ and a voltmeter to measure $u(k)$. Every student is taking 1000 measurements. The measurement number is represented by k . We assume that the measurements are noisy:

$$i(k) = i_0 + n_i(k) \quad \text{and} \quad u(k) = u_0 + n_u(k)$$

where $u_0 = 10 \text{ V}$ is the true values of the voltage across the resistor, $i_0 = 5 \text{ A}$ is the true value of the current flowing through the resistor and $n_i(k)$ and $n_u(k)$ are the values of the noise. Please consider the data-set with all measurements of all students provided in the repository.

Let us now investigate the behaviour of the three different estimators which were introduced in the lecture:

$$\hat{R}_{\text{SA}}(N) = \frac{1}{N} \sum_{k=1}^N \frac{u(k)}{i(k)} \quad \hat{R}_{\text{LS}}(N) = \frac{\frac{1}{N} \sum_{k=1}^N u(k)i(k)}{\frac{1}{N} \sum_{k=1}^N i(k)^2} \quad \hat{R}_{\text{EV}}(N) = \frac{\frac{1}{N} \sum_{k=1}^N u(k)}{\frac{1}{N} \sum_{k=1}^N i(k)}$$

We will write code to simulate the behavior of these estimators. For each of the three estimators, carry out the following tasks.

- (a) **CODE:** Compute the result of the function $\hat{R}_*(N)$, for $N = 1, \dots, N_{\text{max}}$ using your personal measurements (student 1 or experiment 1). Do this for each estimator ($*$ can be either SA, LS or EV). Plot the three curves in one plot.
PAPER: Do the estimators converge for $N \rightarrow \infty$? (5 points)

- (b) CODE: It is good practice to analyze the results of several experiments to cancel noise. Luckily, you get the datasets of all other students. Plot the function $\hat{R}_*(N)$, $N = 1, \dots, N_{\max}$ for each estimator (* can be either SA, LS or EV). To see the stochastic variations, plot all these functions in one graph per estimator.
PAPER: Do you see any difference to the plot from task (b)? (2 points)
- (c) CODE: Compute the mean of $\hat{R}_*(N)$ over all experiments (all 200 students) and plot it for N from 1 to N_{\max} . (1 point)
- (d) CODE: Plot a histogram containing all values of $\hat{R}_*(N_{\max})$ and PAPER: discuss what makes the difference. (2 points)

This sheet gives in total 10 points.