The acados software package
Fast, embeddable solvers for nonlinear optimal control

Jonathan Frey and Moritz Diehl

Systems Control and Optimization Laboratory (syscop)

MPCRL summer school

October 9, 2023

- ▶ Real world optimal control applications with
    - ▶ fast dynamics,
    - ▶ nonlinear optimal control problem formulations,
    - ▶ strict hardware limitations

  require tailored high-performance algorithms.
- ▶ acados implements such algorithms based on
    - ▶ Sequential Quadratic Programming (SQP)
    - ▶ Real-Time Iteration (RTI)
- ▶ Application projects include
    - ▶ Wind turbines
    - ▶ Drones
    - ▶ Race cars
    - ▶ Driving assistance systems
    - ▶ Electric drives

Continuous-time optimal control problem (OCP):

$$\begin{aligned}
\underset{x(\cdot),z(\cdot),u(\cdot)}{\text{minimize}} \quad & \int_{t=0}^{T} \ell(x(t),z(t),u(t))\,\mathrm{d}t + M(x(T)) \\
\text{subject to} \quad & x(0) = \bar{x}_0, \\
& 0 = f(\dot{x}(t),x(t),z(t),u(t)), \quad t \in [0,T], \\
& 0 \geq g(x(t),z(t),u(t)), \qquad t \in [0,T].
\end{aligned} \tag{1}$$

In MPC, instances of these problems are solved repeatedly, with current state $\bar{x}_0$.

$$\underset{\substack{x_0,\ldots,x_N,\\ u_0,\ldots,u_{N-1},\\ z_0,\ldots,z_{N-1},\\ s_0,\ldots,s_N}}{\text{minimize}} \quad \sum_{k=0}^{N-1} l_k(x_k,u_k,z_k) + M(x_N) + \sum_{k=0}^{N} \rho_k(s_k) \tag{2a}$$

$$\text{subject to} \quad \begin{bmatrix} x_{k+1} \\ z_k \end{bmatrix} = \phi_k(x_k,u_k), \qquad k = 0,\ldots,N-1, \tag{2b}$$

$$0 \geq g_k(x_k,z_k,u_k) - J_{s,k}s_k \quad k = 0,\ldots,N-1, \tag{2c}$$

$$0 \geq g_N(x_N) - J_{s,N}s_N, \tag{2d}$$

$$0 \leq s_k \qquad k = 0,\ldots,N. \tag{2e}$$

- ▶ ϕ_k – discrete time dynamics on $[t_k, t_{k+1}]$ – typically `acados` integrator from ODE or DAE
- ▶ l_k discrete version of Lagrange cost term ℓ on $[t_k, t_{k+1}]$
- ▶ slack variables s_k separate from controls – handled efficiently
- ▶ inequality constraints g_k
- ▶ problem functions can vary stage wise in `C`
- ▶ from high-level interfaces
  - ▶ initial and terminal shooting node handled separately – MHE support
  - ▶ parameters can be varied conveniently
- ▶ more detailed problem formulation can be found *here*.

What is acados?

- ▶ Solvers and interfaces for
  - ▶ OCP structured NLP (2)
  - ▶ Initial value problems for ODEs and DAEs – integrators
- ▶ Exploit block-sparse structure inherent in OCP / MHE formulation $\rightarrow$ specialized solver
- ▶ Successor of the `ACADO Toolkit`
  - ▶ Code generation for all parts of the SQP method
- ▶ Principles of `acados`
  - ▶ efficiency – `BLASFEO`, `HPIPM`, `C`
  - ▶ flexibility – general formulation
  - ▶ modularity – encapsulation
  - ▶ portability – self-contained `C` library with little dependencies
- ▶ Model functions code generation using `CasADi`
- ▶ Problem formulation in high-level interface (`Python`, MATLAB, `Octave`)
- ▶ Generation of problem specific `C` solver based on templates
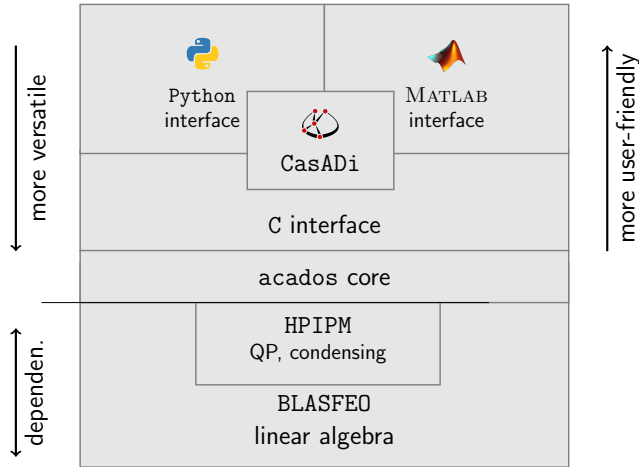
The interplay between the acados dependencies, the 'core' C library and its interfaces.

▶ BLASFEO: Basic Linear Algebra for Embedded Optimization
▶ HPIPM: High-Performance Interior Point Method

	Active-Set	Interior-Point	First-Order
dense	qpOASES, DAQP	HPIPM	
sparse	[PRESAS]	CVXGEN, OOQP	FiOrdOs, OSQP
OCP structure	qpDUNES, [ASIPM]	HPMPC, HPIPM, [ASIPM], [FORCES]	

Table: Overview: QP solver types and their way to handle sparsity.
underline: available in acados + support in Simulink
gray: not interfaced in acados, [proprietary]

efficient condensing from HPIPM:

▶ condensing: OCP structured → dense, expand solution
▶ partial condensing: OCP structured with horizon N → OCP structured with horizon $N_2 < N$, expand solution, $N_2 \,\hat{=}\,$ qp_solver_cond_N

- solve Initial Value Problems (IVP) for
  - Ordinary Differential Equations (ODE)
  - Differential-Algebraic Equations (DAE)
  - + sensitivity propagation (derivative of result with respect to initial state, control input)
- integrators in `Python`, are `'ERK'`, `'IRK'`, `'IRK_GNSF'`
- size of Butcher table: `sim_method_num_stages`
- time step is divided into `sim_method_num_steps` intervals, use the integration method on each interval
- ERK: explicit Runge-Kutta
  - integration order $\text{sim\_method\_num\_stages} = 1, 2, 4$
- IRK: implicit Runge-Kutta
  - Gauss-Legendre Butcher tableaus
    - integration order $2 \cdot \text{sim\_method\_num\_stages}$, A-stable, but not L-stable
  - Gauss-Radau IIA
    - integration order is $2 \cdot \text{sim\_method\_num\_stages} - 1$, L-stable
- IRK_GNSF: structure-exploiting implicit Runge-Kutta method
  - Butcher tableaus as IRK
  - Detecting and Exploiting Generalized Nonlinear Static Feedback Structures in DAE Systems for MPC, J. Frey, R. Quirynen, D. Kouzoupis, G. Frison, J. Geisler, A. Schild, M. Diehl, ECC 2019

- ▶ Continuous time formulation
  - ▶ many discretization options
- ▶ Model functions provided as `CasADi` expressions
- ▶ Problem description via `AcadosModel`, `AcadosOcp`
- ▶ Solver classes `AcadosOcpSolver`, `AcadosSimSolver`

- ► Continuous time formulation
    - ► many discretization options
- ► Model functions provided as `CasADi` expressions
- ► Problem description via `AcadosModel`, `AcadosOcp`
- ► Solver classes `AcadosOcpSolver`, `AcadosSimSolver`
- ► Template workflow
    - ► Model function and derivatives generated using `CasADi`
    - ► C code to set up the OCP solver using the C interface
    - ► `Makefile` to compile everything into a shared library
    - ► Shared library is loaded in `Python` and used via a wrapper
    - ► Generated solver can be used in alternative wrapper and embedded framework, C++, ROS

Important Ressources

- `https://docs.acados.org/`
  - Python API - documents all options in template interface: `https://docs.acados.org/python_api`
  - Installation instructions `https://docs.acados.org/installation`
- acados MATLAB problem formulation PDF: `https://github.com/acados/acados/blob/master/docs/problem_formulation/problem_formulation_ocp_mex.pdf`
- latest acados publication:
  acados – a modular open-source framework for fast embedded optimal control R. Verschueren, G. Frison, D. Kouzoupis, J. Frey, N. van Duijkeren, A. Zanelli, B. Novoselnik, T. Albin, R. Quirynen, M. Diehl Mathematical Programming Computation 2021
- acados forum `https://discourse.acados.org`
- Github examples `https://github.com/acados/acados/tree/master/examples`

- ▶ Cost Discretization within IRK and Gauss-Newton Hessian propagation (GNRK integrator)
- ▶ fast implementation of zero-order robust optimization (zoRO)

▶ `acados` installed in the Docker container

▶ Great success!

Problem Shooting – OCP

- ▶ check OCP solver status, 0 – Success, other values defined in `types.h`
- ▶ `ocp_solver.print_statistics()`
    - ▶ KKT residuals: `stat` stationarity – Lagrange gradient, `eq`: equality constraints, `ineq`: inequality constraints, `comp`: complementarity
    - ▶ `qp_stat`: status of the QP solver, should be 0
    - ▶ `qp_iter`: number of iterations in QP solver
- ▶ initialization: `set()` – x, u, multipliers
- ▶ infeasibility: introduce slacks (soft constraints)
- ▶ check failing QP: `get_from_qp_in()`
- ▶ try different Hessian approximations: Definiteness & Exactness
    - ▶ `ocp.solver_options.hessian_approx = 'GAUSS_NEWTON'` or `'EXACT'`
    - ▶ Option to turn off exact hessian contributions from cost, constraints or dynamics
    - ▶ Set numerical approximation for cost hessian
    - ▶ add a Levenberg Marquardt term, `ocp.solver_options.levenberg_marquardt`
- ▶ iterates don't converge:
    - ▶ reduce the step size, `ocp_opts.set('step_size', alpha)` with `alpha` < 1.
    - ▶ globalization (preliminary implementation) of a merit function based backtracking:
        - ▶ `ocp.solver_options.globalization = 'FIXED_STEP'` or `'MERIT_BACKTRACKING'`
        - ▶ Second order correction: set `globalization_use_SOC` to 1
        - ▶ Armijo condition: set `line_search_use_sufficient_descent` to 1.