Exercises for Course on Modeling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2022-2023

Exercise 1: Resistance Estimation Example

(to be returned before October 31st, 8:00)

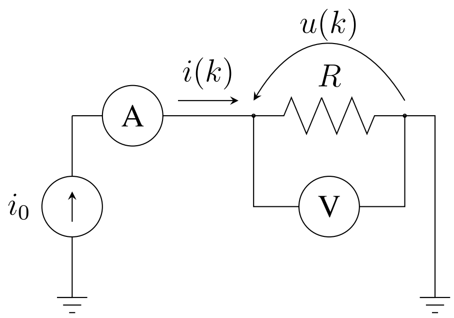
Prof. Dr. Moritz Diehl, Katrin Baumgärtner, Jakob Harzer, Yizhen Wang, Rashmi Dabir

In this exercise you investigate some important facts from statistics in numerical experiments.

Pen-and-paper exercises can be uploaded on the Ilias course page as a pdf or handed in during the lecture.

Exercise Tasks

1. We consider the following experimental setup:



Imagine you are sitting in a class of 200 electrical engineering students and you want to estimate the value of R using Ohm's law. Since the value of the current i_0 flowing through the resistor is not known exactly, an ammeter is used to measure the current $i(k)$ and a voltmeter to measure $u(k)$. Every student is taking 1000 measurements. The measurement number is represented by k. We assume that the measurements are noisy:

$$i(k) = i_0 + n_i(k) \qquad \text{and} \qquad u(k) = u_0 + n_u(k)$$

where $u_0 = 10\,\mathrm{V}$ is the true values of the voltage across the resistor, $i_0 = 5\,\mathrm{A}$ is the true value of the current flowing through the resistor and $n_i(k)$ and $n_u(k)$ are the values of the noise.
Please consider the data-set with all measurements of all students provided on the course website.

Let us now investigate the behaviour of the three different estimators which were introduced in the lecture:

$$\hat{R}_{\mathrm{SA}}(N) = \frac{1}{N}\sum_{k=1}^{N}\frac{u(k)}{i(k)} \qquad \hat{R}_{\mathrm{LS}}(N) = \frac{\frac{1}{N}\sum_{k=1}^{N} u(k)i(k)}{\frac{1}{N}\sum_{k=1}^{n} i(k)^2} \qquad \hat{R}_{\mathrm{EV}}(N) = \frac{\frac{1}{N}\sum_{k=1}^{N} u(k)}{\frac{1}{N}\sum_{k=1}^{N} i(k)}$$

We will write code to simulate the behavior of these estimators. For each of the three estimators, carry out the following tasks.

(a) CODE: Compute the result of the function $\hat{R}_*(N)$, for $N = 1, \ldots, N_{\max}$ using your personal measurements (student 1 or experiment 1). Do this for each estimator ($*$ can be either SA, LS or EV). Plot the three curves in one plot.
PAPER: Do the estimators converge for $N \to \infty$? (5 points)

(b) CODE: It is good practice to analyze the results of several experiments to cancel noise. Luckily, you get the datasets of all other students. Plot the function $\hat{R}_*(N)$, $N = 1, \ldots, N_{\max}$ for each estimator ($*$ can be either SA, LS or EV). To see the stochastic variations, plot all these functions in one graph per estimator.
PAPER: Do you see any difference to the plot from task (b)? (2 points)

(c) CODE: Compute the mean of $\hat{R}_*(N)$ over all experiments (all 200 students) and plot it for N from 1 to $N_{\max}$. (1 point)

(d) CODE: Plot a histogram containing all values of $\hat{R}_*(N_{\max})$ and PAPER: discuss what makes the difference. (2 points)

This sheet gives in total 10 points.