Exercises for Course on Modeling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2022-2023

Exercise 0: General Information and Preliminaries

Prof. Dr. Moritz Diehl, Katrin Baumgärtner, Jakob Harzer, Yizhen Wang, Rashmi Dabir

The lecture course on Modelling and System Identification (MSI) shall enable the students to create models that help to predict the behaviour of systems. Here, we are particularly interested in dynamic system models, i.e. systems evolving in time. With good system models, one can not only predict the future (like in weather forecasting), but also control or optimize the behaviour of a technical system, via feedback control or smart input design. Having a good model gives us access to powerful engineering tools. This course focuses on the process to obtain such models. It builds on knowledge from three fields: Systems Theory, Statistics, and Optimization. We will recall necessary concepts from these three fields on demand during the course, and ask a few basic questions on these fields already in this first sheet.

Organization of the Course

The course is based on two pillars, lectures and exercises, and accompanied by written material for self-study. The exercise sheets include both pen-and-paper exercises as well as programming exercises using Python. During exercise sessions, we will help you to solve the exercises and discuss exercise solutions. Course language is English and all course communication is via this course homepage:

`https://www.syscop.de/teaching/ws2022/modelling-and-system-identification`

Lectures The lectures take place on Mondays, 8:00 - 10:00 AM and Wednesdays, 9:00-10:00 AM, in Building 101, HS 036. Lecture recordings from past years for self-study as well as a detailed course plan are uploaded to the course webpage. Please check regularly, the schedule might be subject to changes during the semester.

Exercises Exercise sheets are uploaded to the course webpage on Mondays. You have one week to work on the sheet and you might work in groups of three (!).

Your solutions to the programming exercises have to be handed in via Ilias. The pen-and-paper solutions are collected during the Monday lecture.

Exercise sessions take place on Wednesdays. During the exercise session the corrected exercises are handed out and discussed. Afterwards there is room for questions on the current exercise sheet.

Written material Written material that accompanies the lecture course comprises two scripts and one book:

- The latest version of the script for this course can always be found on the course homepage.
- A script by Johan Schoukens (Vrije Universiteit Brussel, Belgium) that will be made available on the course homepage but can also be found at `http://syscop.de/files/2015ws/msi/Schoukens_sysid_2013.pdf`.
- The textbook *Ljung, L. (1999). System Identification: Theory for the User. Prentice Hall.* This book is available in the faculty library as a hard copy and a main reference for this course.

Final Evaluation, Exercises and Microexams The final grade of the course is based solely on a final written exam at the end of the semester. The final is a closed book exam, only pencil, paper, and a calculator, and two double-sided A4 pages of self-chosen formulae are allowed.

Each exercise sheet gives a maximum of 10 points. Three online microexams written during some of the lecture slots give a maximum of 10 exercise points each. In order to pass the exercises accompanying the course (Studienleistung), one has to obtain at least 50% of the maximum exercise points in each of the three blocks:

- Exercises 1 - 3 + Microexam 1 (total 40 points)
- Exercises 4 - 6 + Microexam 2 (total 40 points)
- Exercises 7 - 10 + Microexam 3 (total 40 points + 10 Bonus Points)

Linear Algebra Basics A voluntary mini-exercise to refresh the knowledge of some matrix properties.

1. ON PAPER: Consider the following linear system :

$$\begin{cases} a + \ \ b + 5c = 15 \\ 2a + \ \ b + \ \ c = 10 \\ \qquad 2b + 3c = -50 \end{cases}$$

   Please bring it to the form $\Phi\theta = y$, where $\theta = [a,\ b,\ c]^\top$

   (a) Specify the matrix Φ and the vector y, and give their dimensions.
   (b) Compute the rank of Φ. Is it invertible? What does this mean for the solution $\theta = \Phi^{-1}y$?
   (c) What other properties does Φ have, i.e. is it symmetric, orthogonal, positive semi definite, positive definite, negative semi definite, negative definite?

2. ON PAPER: Consider the function $f(x) : \mathbb{R}^2 \mapsto \mathbb{R}^2$ with $x = [x_1,\ x_2]^\top \in \mathbb{R}^2$

$$f(x) = \begin{bmatrix} 5\log(x_1) - x_2^2 \\ 2x_1 + \mathrm{e}^{3x_2} \end{bmatrix}$$

   and its Jacobian matrix $J \in \mathbb{R}^{2\times 2}$, defined as

$$J(x) = \frac{\partial f}{\partial x}(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1}(x) & \frac{\partial f}{\partial x_2}(x) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x) & \frac{\partial f_1}{\partial x_2}(x) \\ \frac{\partial f_2}{\partial x_1}(x) & \frac{\partial f_2}{\partial x_2}(x) \end{bmatrix}.$$

   Given the point $\bar{x} = [1,\ 2]^\top$, is $J(\bar{x})$ invertible?

3. ON PAPER: Show that for any $A \in \mathbb{R}^{n\times m}$ holds that $A^\top A$ is symmetric and PSD.
   (Hint: matrix B is PSD, if for any $x \in \mathbb{R}^m$ holds that $x^\top Bx \geq 0$).