

# Stochastic Model Predictive Control for Smart Grid Applications

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# Control of Residential PV Battery Systems under Uncertainties

## Introduction

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### Fade-out of fossil-based electricity generation

Less controllable renewable sources

→ Use decentralized flexibilities

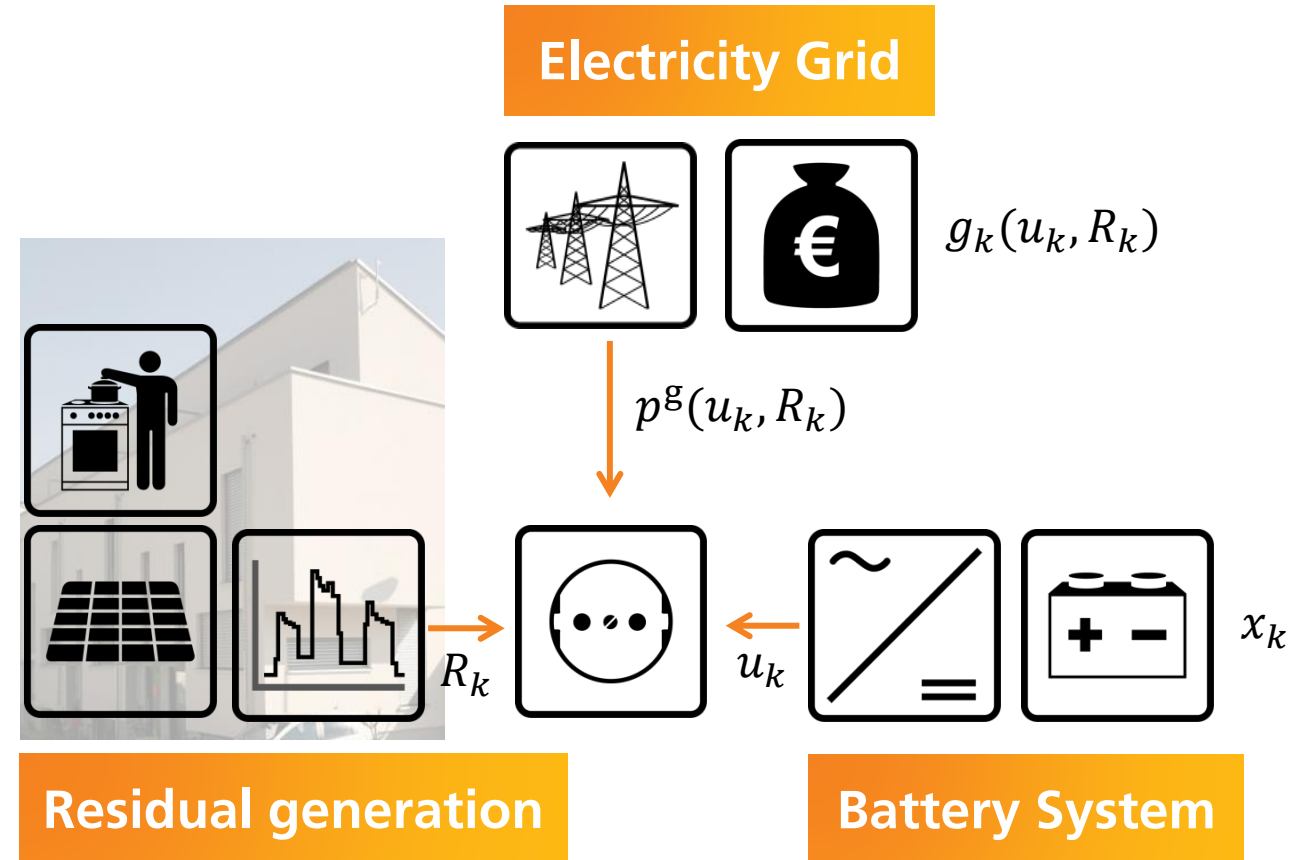
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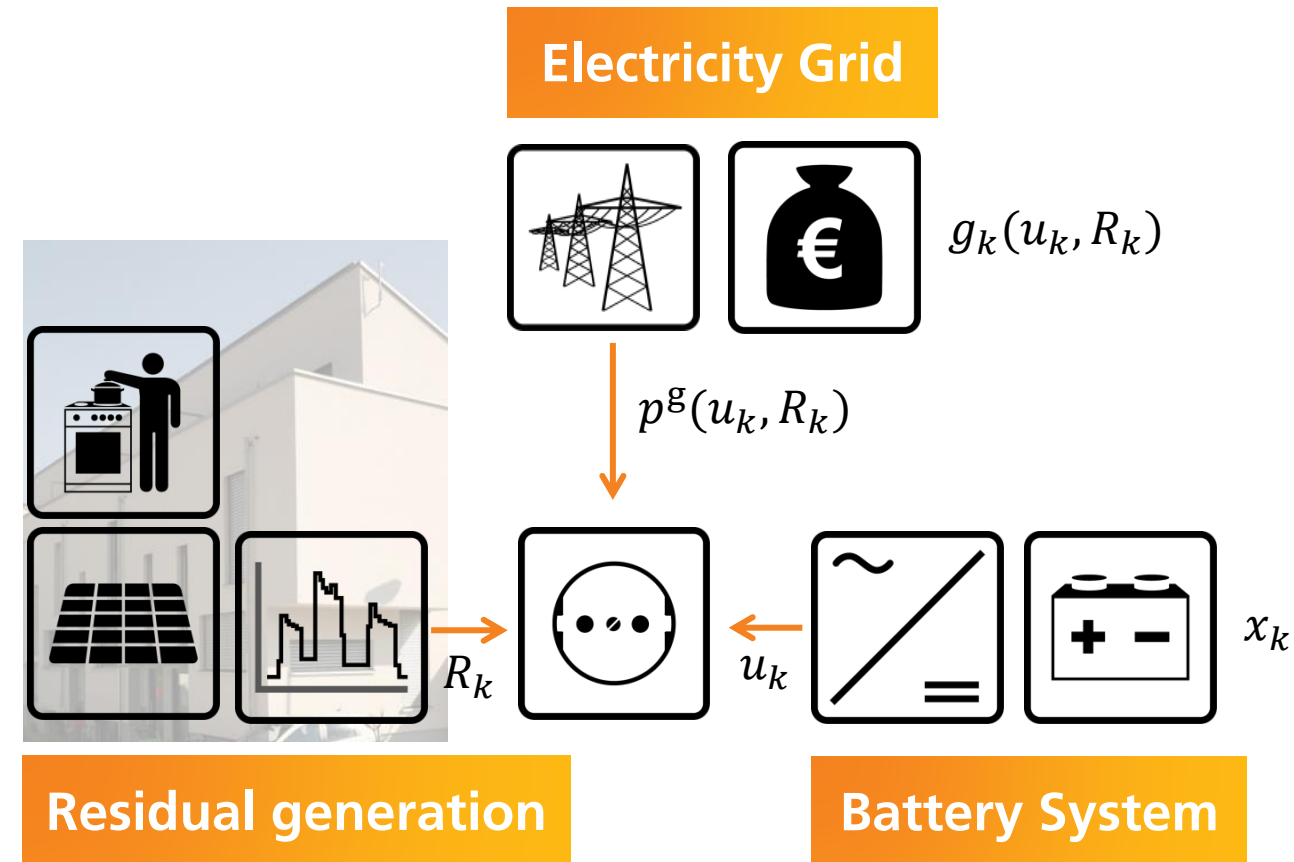
### Fade-out of fossil-based electricity generation

Less controllable renewable sources

→ Use decentralized flexibilities

### Control storage optimally while ...

- mitigating uncertainties in forecasts
- navigating a complex economic environment
- considering efficiencies of the system



# Thesis Goal and Contributions

## Introduction

### Development of a Stochastic Control Algorithm Applied to Smart Grid System

#### Stochastic Control

- Groß, A., Wittwer, C., and Diehl, M. Stochastic model predictive control of photovoltaic battery systems using a probabilistic forecast model. *European Journal of Control* 56 (Nov. 2020), 254–264

#### Stochastic modeling

- Groß, A., Wittwer, C., Lenders, A., Zech, T., and Diehl, M. Using Probabilistic Forecasts in Stochastic Optimization. In *The 16th International Conference on Probabilistic Methods Applied to Power Systems* (2020)
- Groß, A., Lenders, A., Schwenker, F., Braun, D. A., and Fischer, D. Comparison of short-term electrical load forecasting methods for different building types. *Energy Informatics* 4, 3 (Sept. 2021), 13

#### System modeling

- Groß, A., Wille-Haussmann, B., Wittwer, C., Achzet, B., and Diehl, M. Stochastic Nonlinear Model Predictive Control for a Switched Photovoltaic Battery System. *IEEE Transactions on Control Systems Technology* (2022)

#### Applications

- Groß, A., Schumann, J., Marchand, S., Mittelsdorf, M., Kohrs, R., and Diehl, M. Electric Vehicle Charge Management taking into account Grid State and Forecast Uncertainties of Photovoltaic Generation. In *EVS33* (Sept. 2020)

# Notation Deterministic MPC

## Stochastic Control

### Deterministic Optimal Control Problem (OCP)

$$\min_{\mathbf{x}, \mathbf{u}} J(\mathbf{x}, \mathbf{u} | \mathbf{R}) = \sum_{k=0}^{N-1} g_k(x_k, u_k, R_k) + g_N(x_N)$$

$$\text{s.t. } x_k \in \mathbb{X} \quad \forall k = 0, \dots, N,$$

$$u_k \in \mathbb{U}(x_k) \quad \forall k = 0, \dots, N-1,$$

$$x_{k+1} = f^x(x_k, u_k, R_k) \quad \forall k = 0, \dots, N-1,$$

$$x_0 = x^{\text{init}}$$

### Where

$x_k$ : State of charge (SoC) of battery system

$u_k$ : Control input i.e., battery power

$R_k = p^{\text{pv}} - p^{\text{load}}$ : Residual generation

$g_k(x_k, u_k, R_k)$ : stage costs (supply price, feed-in tariff)

$g_N(x_N)$ : terminal costs (final battery SoC)

$f^x(x_k, u_k, R_k)$ : state transition (charging / discharging / losses)

# Multi-stage Stochastic MPC

## Stochastic Control

### Define new state variable residual generation

$$R_{k+1} = f^R(R_k, \bar{\mathbf{R}}, \varepsilon_k) \quad \varepsilon_k \sim \mathcal{F}_k$$

# Multi-stage OCP – residual generation model

## Stochastic Modeling

### Define new state variable residual generation

$$R_{k+1} = f^R(R_k, \bar{\mathbf{R}}, \varepsilon_k) \quad \varepsilon_k \sim \mathcal{F}_k$$

### Markov-Chain instead of deterministic forecast

$$R_{k+1} = f^R(R_k, \bar{\mathbf{R}}, \varepsilon_k) = \bar{R}_{k+1} + \tau(R_k - \bar{R}_k) + \varepsilon_k$$

Forecast next step

Persistence term

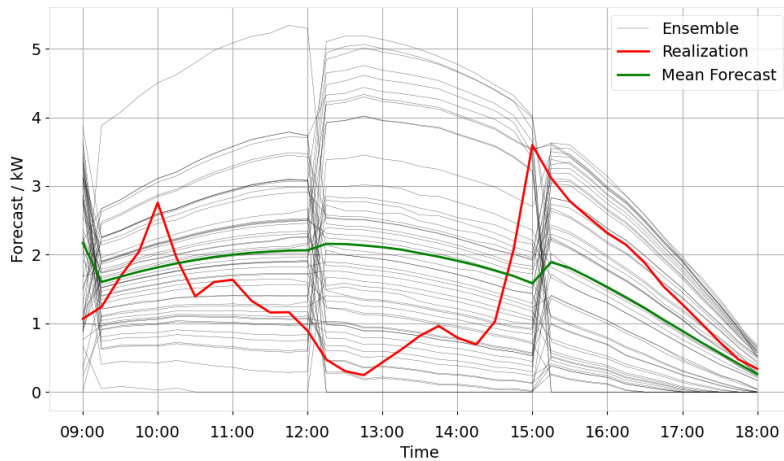
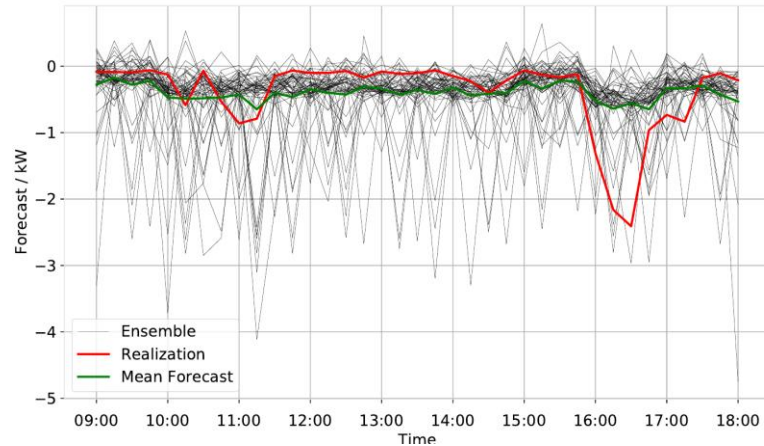
uncertainty



# Use Probabilistic Forecasts to Determine Short-term Model

## Stochastic Modeling

### Probabilistic Forecasts



Parameters can also be obtained from past data (fc + meas)

Markov-chain model parameters

Maximum Likelihood estimation

e.g., Gaussian with width

$$\hat{\sigma} = \sqrt{\frac{1}{N-1} \sum_{k=0}^{N-2} \frac{1}{M} \sum_{m=0}^{M-1} (\bar{R}_{k+1}^m - \rho_{k+1}(\bar{R}_k^m, \hat{\mathbf{R}}))^2}$$

# Multi-stage Stochastic MPC

## Stochastic Control

### Define new state variable residual generation

$$R_{k+1} = f^R(R_k, \bar{\mathbf{R}}, \varepsilon_k) \quad \varepsilon_k \sim \mathcal{F}_k$$

### Summarize

$$y_k = (x_k, R_k)^T \in \mathbb{Y} := \mathbb{X} \times \mathbb{R}$$

### With combined state transition

$$y_{k+1} = F(y_k, u_k, \varepsilon_k) := \begin{pmatrix} f^x(x_k, u_k, R_k) \\ f^R(R_k, \bar{\mathbf{R}}, \varepsilon_k) \end{pmatrix}$$

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Define policies  $\mu(y)$  in set of policies

$$\mathbb{M} := \{\mu : \mu(y) \in \mathbb{U} \wedge F(y, \mu(y), \varepsilon) \in \mathbb{Y} \quad \forall y \in \mathbb{Y}, \forall \varepsilon\}$$

Iteratively yielding state trajectories

$$y_{k+1}(\mu, \hat{y}, \varepsilon) := F(y_k(\mu, \hat{y}, \varepsilon), \mu_k(y(\mu, \hat{y}, \varepsilon)), \varepsilon_k)$$

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Arriving at the stochastic multi-stage OCP for the optimal policies

$$\min_{\mu \in \mathbb{M}^N} \mathbb{E}_{\varepsilon \sim \mathcal{F}} \left[ \sum_{k=0}^{N-1} g_k(\mu_k(y_k(\mu, \hat{y}, \varepsilon)), y_k(\mu, \hat{y}, \varepsilon)) + g_N(y_N(\mu, \hat{y}, \varepsilon)) \right]$$

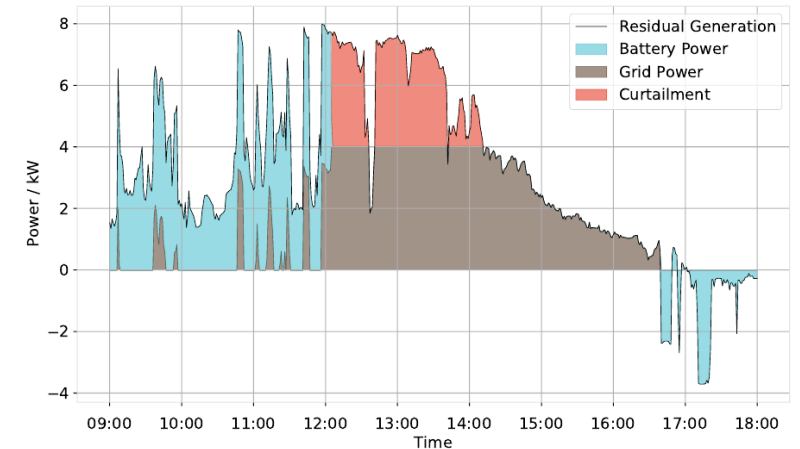
# Sample Case: PV Battery System with curtailment limit

## System Modeling

### Standard control: Charge battery with excess PV

Maximizes self-sufficiency

Feed-in is limited to 50% of nominal power → curtailment

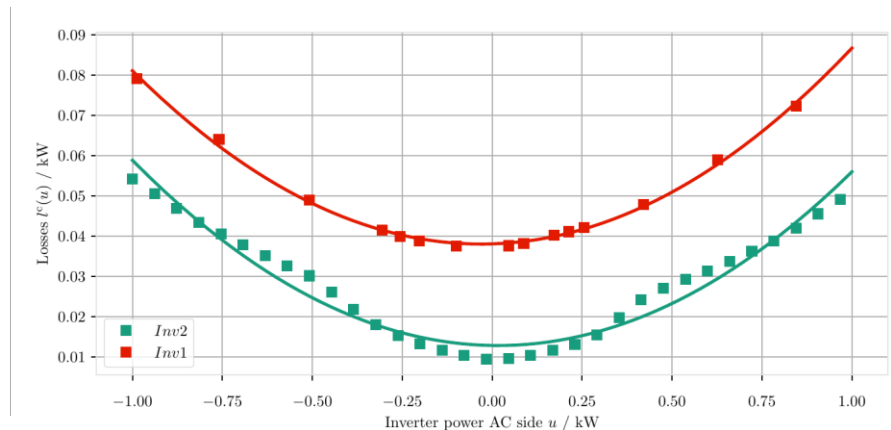
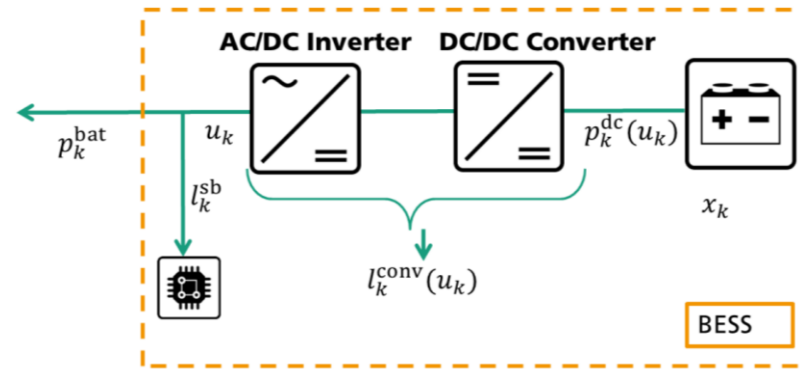


### Inverter modeling

Losses due to conversion: non-linear

Standby losses reduced

when converter is switched off



# Sample Case Optimal Policies

## System Modeling

## Policy evaluation maps measurement to control

### Legend

Yellow: Discharging

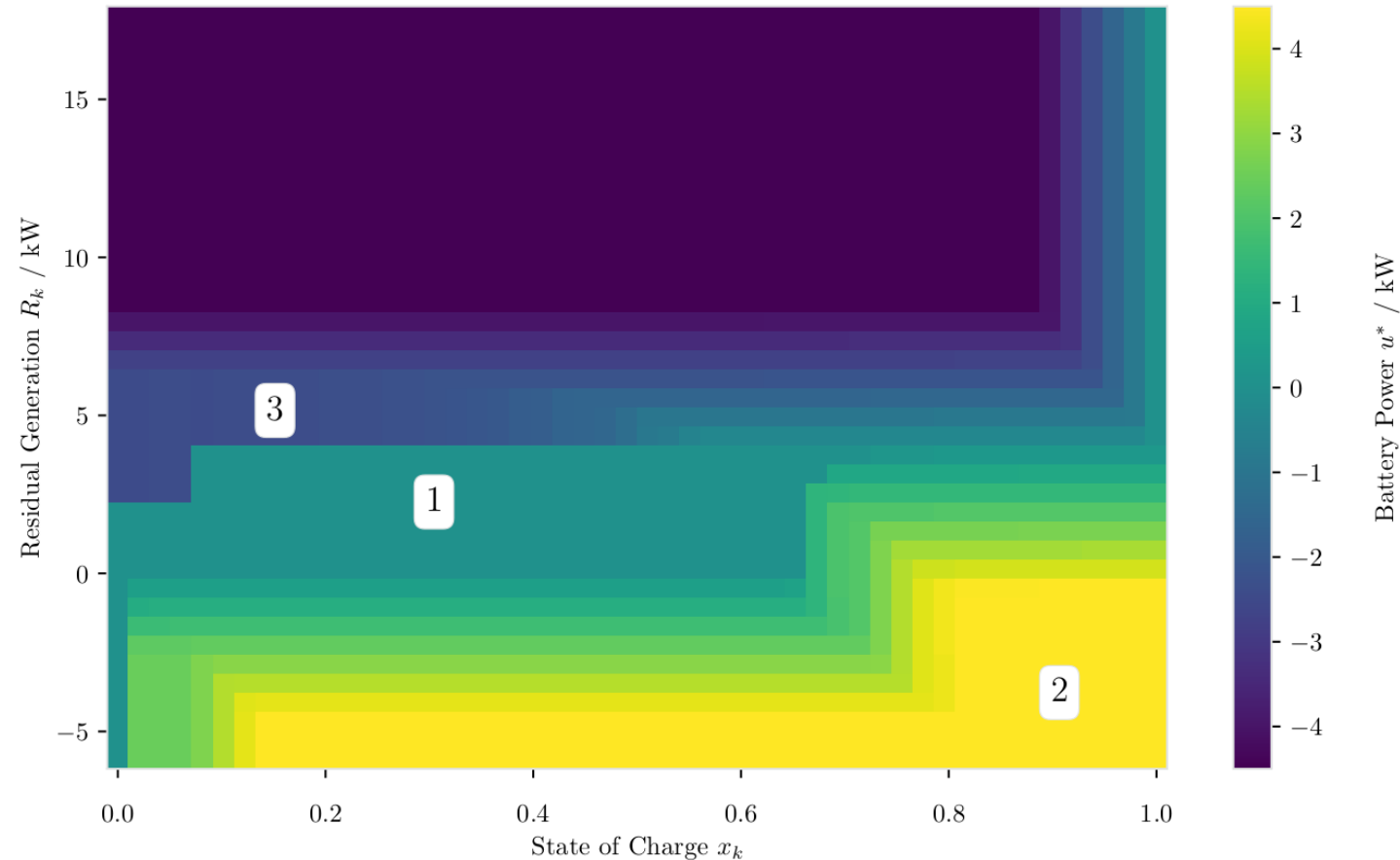
Dark Blue: Charging

## Exemplary rules for a sunny morning

**Charge Battery before curtailment (3)**

**Do not Charge at low powers (1)**

**Supply Load from battery (2)**



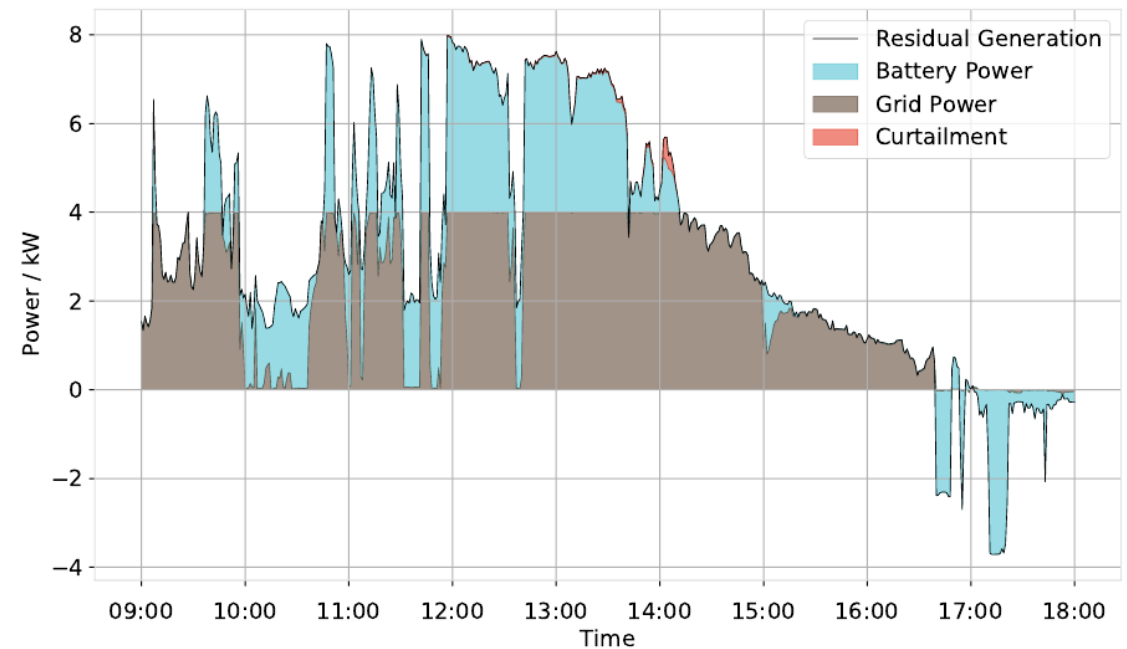
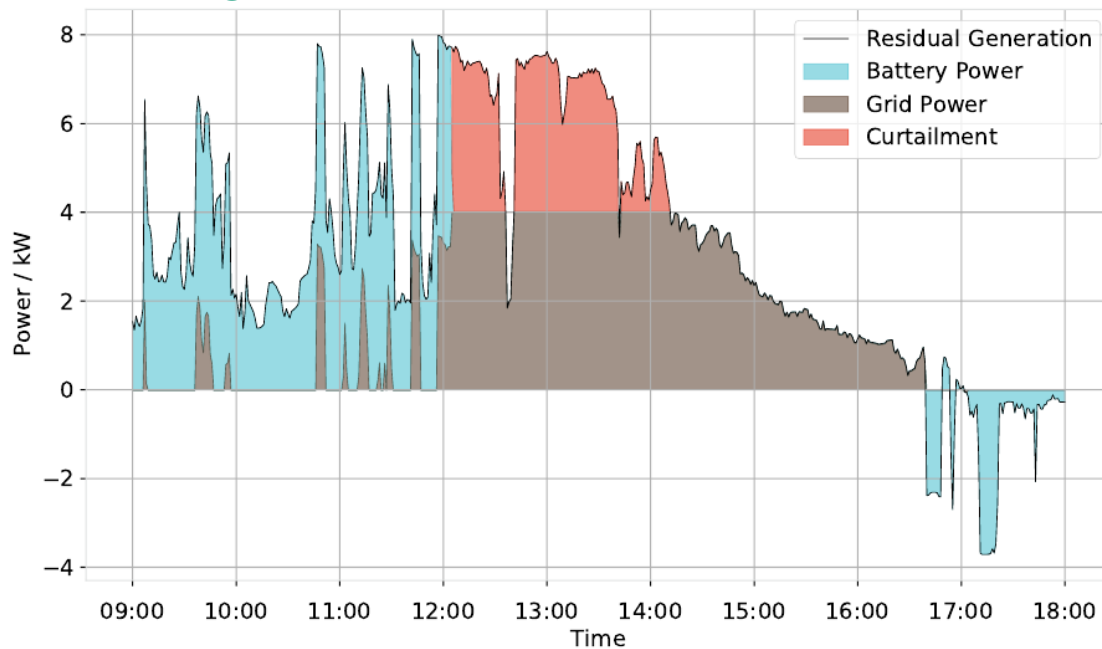
- Policy with 8 kWp PV, 4 kW feed-in limit, 9 kWh battery

# Sample Case: Operation on one day

## System Modeling

Curtailment can be reduced

More PV-energy can be integrated into the grid without grid reinforcements



# Simulation Setup & Parameter Tuning

## Perform a simulation on 4 load / PV combinations

1 kWh battery, 1 kWp PV installation per 1 MWh yearly load

Calculating costs of yearly operation using method X

→ Defining exploited potential

$$\Pi^{\text{exp}}(X) = \frac{C^{\text{el}}(\text{Standard}) - C^{\text{el}}(X)}{C^{\text{el}}(\text{Standard}) - C^{\text{el}}(\text{Ideal})}$$

## Compared methods

- Deterministic MPC
- Two-stage Stochastic MPC
- Multi-Stage Stochastic MPC (SDP)
- Heuristic

	Site 1	Site 2	Site 3	Site 4
Energy [kWh]	2225	4662	5700	6507
Night Fraction [%]	40.6	47.6	42.5	55.6
Specific PV Yield [kWh / kWp]	1108	1102	1184	1183
Energy above 50% [kWh / kWp]	107	121	160	165

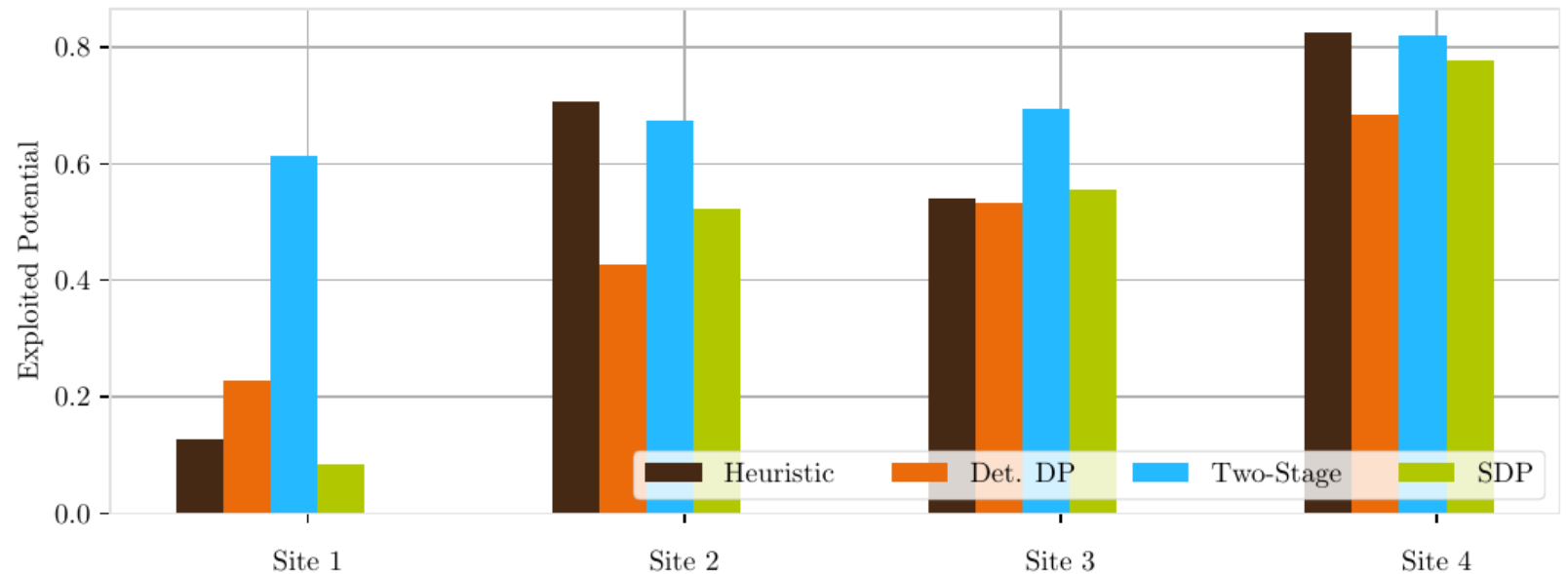


# Results of yearlong simulations for 4 households

SDP (green) performs better than deterministic MPC

Simple heuristic better in many cases

Two-stage stochastic MPC performs best in 3/4 cases

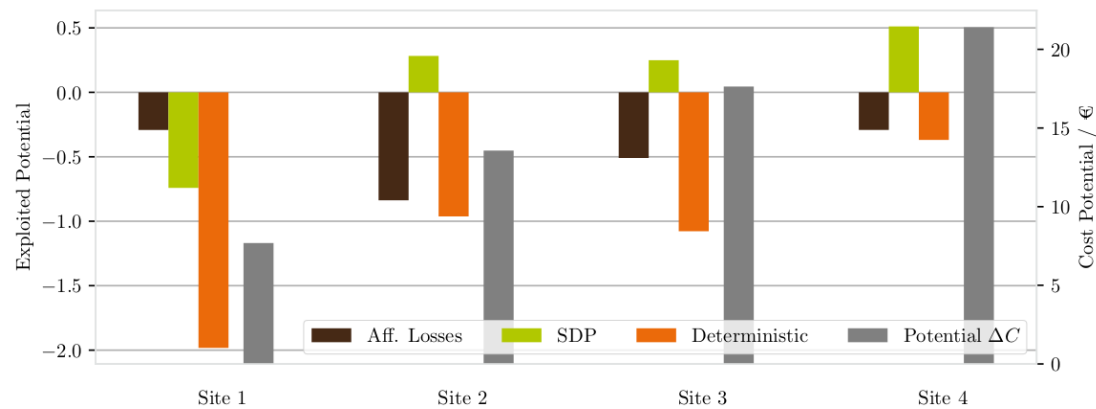


# Results from two other application cases

## Case 2: Switchable Inverter

No feed-in limit, pure self-sufficiency

Focus on operation at high inverter efficiency

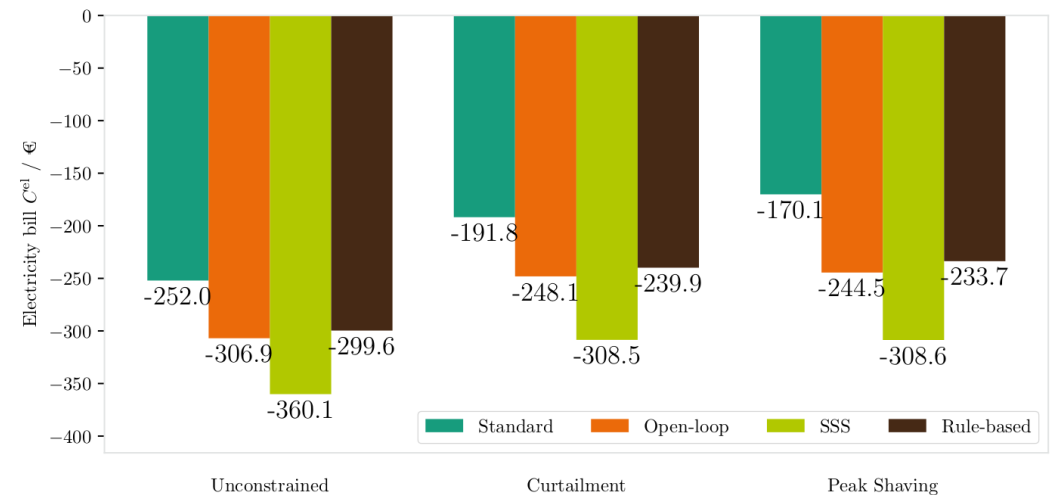


Considering switching and nonlinear losses increases self-sufficiency and reduces costs.

## Case 3: Electric Vehicle Charging

Reach high PV-coverage of charging process

Affected by charge limits, PV power, user requests



SDP shows best results

# Summary of Contributions

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## Forecasts and Stochastic Modeling

Stochastic Markov-chain model for residual generation

Maximum likelihood parameter estimation in data

## System Modeling for Optimization

Nonlinear losses of power converters

On-Off switches with downtime

Integration in SDP algorithm

## Simulation Studies

Studied three application cases: PV-Battery with and without feed-in limit, residential charging of electric vehicles

Lower cost and more renewable energy harvested with stochastic modeling

## Software Integration in Real-life Systems

Implemented SDP algorithm in C++ with interfaces to shell, python, and Java

Field tests for PV-battery system (with Varta Storage) and EV-charging (with other team at ISE) showing proof of concept

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