Types of optimization problems

Regard the following optimization problem:

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & \frac{1}{2} x^\top Q x + c^\top x \\
\text{s.t.} \quad & Ax + b = 0, \\
& Cx + d \geq 0,
\end{aligned}
$$

where $Q \in \mathbb{R}^{n \times n}$, $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{p \times n}$, $b \in \mathbb{R}^p$, $C \in \mathbb{R}^{q \times n}$, $d \in \mathbb{R}^q$. Which of the following types of optimization problems does it belong to?

Choose all that apply.

(a) Linear program (LP)

(b) Quadratic program (QP)

(c) Nonlinear program (NLP)

(d) Nondifferentiable optimization problems

Convexity of sets

Which of the following sets are convex? ($b \in \mathbb{R}^m$, $A \in \mathbb{R}^{m \times n}$) Choose all that apply.

(a) $\{x \in \mathbb{R}^n \mid \|Ax\|_2^2 \leq 10\}$

(b) $\{x \in \mathbb{R}^n \mid \|x\|_2^2 = 10 \ \& \ Ax \leq b\}$

(c) $\{x \in \mathbb{R}^n \mid \|Ax + b\|_1 \geq 5\}$

(d) $\{X \in \mathbb{S}^n \mid X \succeq 0\}$
(where $\mathbb{S}^n = \{Q \in \mathbb{R}^{n \times n} \mid Q = Q^\top\}$)

Convexity of functions

Which of the following functions are convex? $(x, c \in \mathbb{R}^n,\ A \in \mathbb{R}^{m \times n}\ )$
Choose all that apply.

(a) $f_1(x) = \exp(\|x\|_2^2)$

(b) $f_2(x) = \max(\|x\|_2, x^\top x) + \|Ax\|_2$

(c) $f_3(x) = \|Ax\|_2 + \log(c^\top x)$

(d) $f_4(x) = \sin(\|x\|_2)$

Convexity of optimization problems

Which of the following optimization problems are convex? Choose all that apply.

(a) $$\min_{x, y \in \mathbb{R}} \quad 3x^2 + \exp y \quad \text{s.t.} \quad \begin{aligned} x + 3y &\leq 0, \\ y + 10 &\geq 0. \end{aligned}$$

(b) $$\min_{x \in \mathbb{R}} \quad 7x^4 \quad \text{s.t.} \quad x^2 - 2 = 0.$$

(c) $$\min_{x, y \in \mathbb{R}} \quad x^2 + 4y^2 \quad \text{s.t.} \quad x^2 + y^2 - 1 \geq 0.$$

(d) $$\min_{x \in \mathbb{R}} \frac{1}{x} \quad \text{s.t.} \quad 1 \leq x \leq 10.$$

Newton's method

Regard the following equation system:

$$\sin(x) - y = 0,$$
$$x^2 + y^2 - 1 = 0.$$

We summarize it as $F(w) = 0$, where $w = (x, y)$ and $F : \mathbb{R}^2 \to \mathbb{R}^2$. We want to solve this root finding problem using (exact) Newton's method. Our current iterate is $w_k = (0, 1)$ (i.e., $x_k = 0$, $y_k = 1$.) Use Newton's method to find the next iterate $w_{k+1} = (x_{k+1}, y_{k+1})$.
As answer, please enter the value of x_{k+1}:

$x_{k+1} = \ldots?$

If necessary, round the value to two decimal digits after the decimal separator by simply dropping the superfluous digits (e.g. "3.149" becomes "3.14").

Optimization using CasADi

Regard the following optimization problem:

$$\begin{aligned} \min_{w \in \mathbb{R}^3} \quad & 2w_1^2 + w_1 w_3 + 2w_3^2 + 3w_2 - \log(w_3 + 1) \\ \text{s.t.} \quad & -2w_1^2 - \frac{1}{2}w_2^2 + 3 \geq 0, \\ & 1 \leq w_3 \leq 4, \end{aligned}$$

where $w = (w_1, w_2, w_3)$. Use CasADi and the solver IPOPT to find the minimizer $w^* = (w_1^*, w_2^*, w_3^*)$ of this problem.
As answer, please enter the value of w_2^*:

$w_2^* = \ldots ?$

If necessary, round the value to two decimal digits after the decimal separator by simply dropping the superfluous digits (e.g. "3.149" becomes "3.14").

Globalization

Regard the following optimization problem:

$$\min_{x \in \mathbb{R}} \quad \sqrt{1+x^2}.$$

We want to solve this problem using a globalized version of Newton's method. Currently we are at the iterate $x_k = 2$, such that the full Newton step would be $p_k = -10$. Our next iterate is $x_{k+1} = x_k + t_k p_k$.
Use the backtracking algorithm to find the value of t_k such that it fulfills Armijo's sufficient decrease condition. The algorithm parameters are $\beta = \frac{1}{2}$, $\gamma = \frac{1}{10}$ and $t_{\max} = 1$. You are allowed to use a calculator.
Having found t_k, the value of the next iterate is $x_{k+1} = \ldots$?

If necessary, round the value to two decimal digits after the decimal separator by simply dropping the superfluous digits (e.g. "3.149" becomes "3.14").