

Exercise Sheet 3: Blade Element Momentum Method

In this problem, we want to see how the Blade Element Momentum (BEM) method gives the total thrust on a wind turbine.

To do this, consider an infinitesimally thin annulus (with radius r) sliced from a three-bladed ($B = 3$) wind turbine rotor of radius R . Assume for the following problem that tip losses can be neglected. We will also again use $\mu = r/R$ the normalized radial position of the annulus.

The effective velocity at the rotor annulus is called $\mathbf{W}(r) = W(\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{t}})$, where $\hat{\mathbf{x}}$ points along the axis of rotation in the downwind direction, and $\hat{\mathbf{t}}$ points tangentially in the direction of rotation. Assume that the problem is axially symmetric so that all the blades behave identically.

We will use a demonstration turbine called 'Turbine A.' Turbine A is defined by the following parameters: tip speed ratio $\lambda = 7$, the local chord solidity $\sigma(r) = 8/(441\mu)$, the rotor radius $R = 50\text{m}$, the effective velocity angle $\phi = 5\text{deg}$, and the 2D lift and drag coefficients $c_l = 1$ and $c_d = 0.01$. Turbine A is running in a freestream wind of $u_\infty = 12\text{m/s}$ with air density $\rho = 1.225\text{kg/m}^3$.

1. Geometry

- (a) What is the area dA of the annulus, if the annulus has a thickness of dr ?
- (b) Assume that the rotor is in a uniform flow field with a freestream wind \mathbf{u}_∞ that is aligned with the rotor axis. What is the freestream dynamic pressure q_∞ ?
- (c) Find the magnitude of the effective velocity W in terms of some parameters of the wind turbine system: the freestream velocity $u_\infty = \|\mathbf{u}_\infty\|_2$, the tip speed ratio λ , the annulus radius r , rotor radius R , and the induction factors.
- (d) What is the effective dynamic pressure $q_e(r)$ based on the magnitude of the effective wind velocity?
- (e) Let's define the chord solidity $\sigma(r)$ as:

$$\sigma(r) = \frac{B}{2\pi\mu} \frac{c}{R}.$$

If $c(r)$ is the chord length of the blade at the annulus, what is the area dS of the blade section at the annulus?

2. Momentum expressions

- (a) What is $dT(r)$ (dF_A in the lecture), the change in axial momentum in the flow due to that annulus, in terms of axial a and tangential a' induction factors?
- (b) What is $dQ(r)$, the change in angular momentum in the flow due to that annulus, in terms of axial a and tangential a' induction factors?

3. Blade element expressions

- (a) If you know that the blade section experiences lift ($d\mathbf{L}$) and drag ($d\mathbf{D}$) forces, what is the thrust $dT(r)$ on the blade section (for one blade)?
- (b) Under the same conditions, what is the torque $dQ(r)$ on the blade element?
- (c) Use the 2D lift and drag coefficients c_l and c_d to write your blade element thrust and torque expressions in terms of the defining parameters: B , u_∞ , λ , r , R , a , a' , ϕ , and σ .

4. The full rotor thrust

- (a) You happen to learn that the induction factors can be approximated as:

$$a(\mu) \approx (0.8 + 28\mu) \cdot 10^{-2}, \quad a'(\mu) \approx (0.3 + 0.6/\mu + 2.9\mu) \cdot 10^{-3}$$

What is the thrust distribution over μ on one blade?

- (b) For Turbine A, what is the thrust on the whole rotor?
(c) For Turbine A, What is the thrust coefficient C_T for the full rotor?