Exercises for Course on State-Space Control Systems (SSC)
Albert-Ludwigs-Universität Freiburg – Summer Term 2019

Exercise 12: Tracking MPC and Disturbance Modeling

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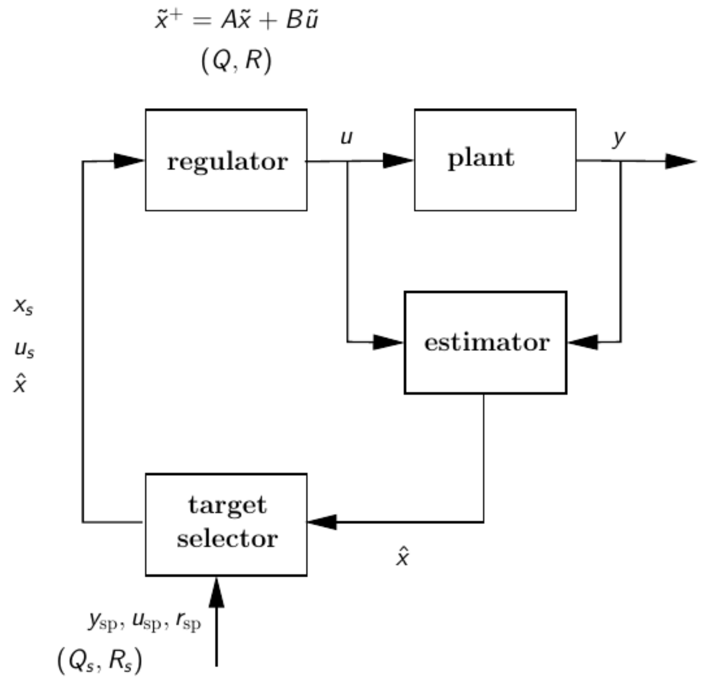
In this exercise, we consider setpoint tracking and rejecting nonzero disturbances to achieve offset-free MPC. We consider a linear system of the form

$$x_{k+1} = Ax_k + Bu_k \tag{1a}$$
$$y_k = Cx_k \tag{1b}$$

Our goal is to reach a steady-state output that satisfies $r_{\text{sp}} = HCy_{\text{s}}$ where r_{sp} denotes the setpoint. Note that we make the simplifying assumption that there are no additional inequality constraints.

This is achieved with the following setup:



Please download the Matlab template from the course webpage and fill in the corresponding gaps.

1. In the function `getSteadyState` set up the steady-state target problem in order to solve it with `fmincon`. The steady-state target problem is defined as follows:

$$(x_{\text{s}}, u_{\text{s}}) = \arg\min_{x,u} \quad \frac{1}{2}\|u - u_{\text{sp}}\|^2_{R_{\text{s}}} + \frac{1}{2}\|Cx - y_{\text{sp}}\|^2_{Q_{\text{s}}}$$
$$\text{s.t.} \quad \begin{bmatrix} I - A & -B \\ HC & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ r_{\text{sp}} \end{bmatrix}$$

We have $y_{\text{sp}} = [2, 1]^\top$, $H = [1, 0]$ and $r_{\text{sp}} = 2$, $u_{\text{sp}} = 0$. Note the we actually do not need to recompute $(x_{\text{s}}, u_{\text{s}})$ if the setpoint does not change.

2. In the function `getControl` define the regulator problem in order to solve it with `fmincon`. The regulator problem, which is defined in terms of the *deviation variables* $\tilde{x}_k = x_k - x_{\text{s}}$ and $\tilde{u}_k = u_k - u_{\text{s}}$,

is given as:

$$\begin{aligned}\min_{\substack{\tilde{x}_0,\dots,\tilde{x}_{N-1},\\ \tilde{u}_0,\dots,\tilde{u}_{N-1}}} \quad & \sum_{k=0}^{N-1} \frac{1}{2}\|\tilde{x}_k\|_Q^2 + \frac{1}{2}\|\tilde{u}_k\|_R^2 \\ \text{s.t.} \quad & \tilde{x}_{k+1} = A\tilde{x}_k + B\tilde{u}_k, \ k = 0,\dots,N-2, \\ & \tilde{x}_0 = \hat{x} - x_\text{s}.\end{aligned}$$

where $\hat{x}$ is our current estimate of the system state.

3. In the function `estimate` use the provided functions `update_kf` and `predict_kf` in order to compute the state estimate.

4. Simulate the system in closed-loop and plot the true and estimated state trajectory, as well as the outputs and the setpoint. What do you observe?

5. If you check out the provided Matlab code for the plant, you see that our system model (1) is incorrect and that there is a constant disturbance $d_k = d_\text{const}$ added to the state equations. To account for this nonzero disturbance, we adapt the steady-state target problem and the estimation problem accordingly. The regulator problem is unchanged.

   The augmented system including disturbance modeling is given as:

$$\begin{bmatrix} x_{k+1} \\ d_{k+1} \end{bmatrix} = \underbrace{\begin{bmatrix} A & B_d \\ 0 & \mathbb{I} \end{bmatrix}}_{\tilde{A}} \begin{bmatrix} x_k \\ d_k \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\tilde{B}} u_k$$

$$y_k = \underbrace{\begin{bmatrix} C & C_d \end{bmatrix}}_{\tilde{C}} \begin{bmatrix} x_k \\ d_k \end{bmatrix}$$

   where we will assume $A_d = \mathbb{I}$ and $C_d = 0$, i.e. the disturbances only affect the state equation.

   The steady-state target problem is now:

$$\begin{aligned}(x_\text{s}, u_\text{s}) \quad = \quad \min_{x,u} \quad & \frac{1}{2}\|u - u_\text{sp}\|_{R_\text{s}}^2 + \frac{1}{2}\|Cx + C_d\hat{d} - y_\text{sp}\|_{Q_\text{s}}^2 \\ \text{s.t.} \quad & \begin{bmatrix} I - A & -B \\ HC & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} B_d\hat{d} \\ r_\text{sp} - HC_d\hat{d} \end{bmatrix}\end{aligned}$$

   where $\hat{d}$ is the current estimate of the disturbance. Within the estimator, we need to estimate both x and d by providing the corresponding the matrices $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, as well as covariance matrices of appropriate size.

6. Plot the results obtained with the augmented system in the same figure as the previous results and compare.