Exercises for Course on State-Space Control Systems (SSC)
Albert-Ludwigs-Universität Freiburg – Summer Term 2019

Exercise 2: Eigenvalues and Stability, Lyapunov Stability

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Theoretical Exercises

1. Consider again the inverted pendulum, example (5.4) in *Feedback Systems* where you can assume that $u = 0$. Check if the equilibrium points $(0, 0)$ and $(\pi, 0)$ are (asymptotically) stable by linearizing at the equilibrium and then examining the eigenvalues. Compare with the phase portrait from last week's exercise sheet.

2. Consider the ODE

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \begin{bmatrix} k & 1 \\ -4k & -3 \end{bmatrix} x$$

   with parameter k. Compute the eigenvalues λ_1, λ_2 of the system matrix and plot the real part of λ_1, λ_2 as a function of k.

   For $k = 0.5$ compute the eigenvectors (numerically using Matlab) and use them to diagonalize the system. Compare the diagonal entries to the eigenvalues.

3. Exercise 5.4 (Lyapunov functions) from *Feedback systems*.

Matlab

The second order FitzHugh-Nagumo equations are given by

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 10\left(V - \frac{V^3}{3} - R + I_{\text{in}}\right),$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = 0.8\,(-R + 1.25V + 1.50)$$

They describe a simplified model the spike generation in neurons. V is the membrane potential, R is a recovery variable, I_{in} is the magnitude of the stimulus current. For more details have a look at this article[1].

1. How many equilibria are the for a fixed value of I_{in}. Plot the equilibrium as a function of I_{in}.

2. Check if the equilibrium points are (asymptotically) stable by linearizing around the equilibrium points and examining the eigenvalues.

3. For $I_{\text{in}} \in \{0, 1.5, 2.5\}$ use `ode45` to simulate the system using initial values close to the equilibrium point and plot V as a function of t.

4. What do you observe?

[1] Note that they use slightly different parameters in the article.