Modeling and System Identification – Microexam 3

Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg January 30, 2019, 14:10-14:55, Freiburg

Surname: First Name: Matriculation number:

Subject: Programme: Bachelor ☐ Master ☐ Lehramt ☐ others ☐ Signature:

Please fill in your name above and tick exactly ONE box for the right answer of each question below.

1. Consider the ODEs $\ddot{a} = c_1\dot{a} + c_2 a + \dot{s}$ and $\ddot{s} = c_3\dot{a} + c_4\dot{s} + s$ and the output model $y = c_5 s + c_6\dot{a}$. Specify matrices A and C such that $\dot{x} = Ax$ and $y = Cx$ where we define the state as $x = (a, \dot{a}, s, \dot{s})^\top \in \mathbb{R}^4$ (2 points).

2. Which expression describes the Kalman filter Innovation Update Step of the state estimate? $\hat{x}_{[k|k]} = \cdots$

(a) ☐ $\hat{x}_{[k\|k-1]} + P_{[k\|k-1]} \cdot C_k^\top V_k (y_k - C_k \hat{x}_{[k\|k-1]})^{-1}$	(b) ☐ $\hat{x}_{[k\|k-1]} + P_{[k\|k]} \cdot C_k^\top V_k^{-1} (y_{k-1} - C_k \hat{x}_{[k\|k-1]})$
(c) ☐ $\hat{x}_{[k-1\|k]} + P_{[k-1\|k]} \cdot C_{k-1}^\top V_k^{-1} (y_{k-1} - C_{k-1} \hat{x}_{[k-1\|k]})$	(d) ☐ $\hat{x}_{[k\|k-1]} + P_{[k\|k]} \cdot C_k^\top V_k^{-1} (y_k - C_k \hat{x}_{[k\|k-1]})$

3. Consider the scalar ARX model $y(k) = \theta_1 y(k-1) + \theta_2 u(k-1) + w_k$ where $w_k \sim \mathcal{N}(0, \sigma^2)$. Given measurements $y(k)$ and controls $u(k)$, $k = 0, \ldots, N$, specify functions $f_k(\theta)$ and weighing factors c_k (that account for the noise variance) such that the parameter estimate $\theta^* = [\theta_1^*, \theta_2^*]^\top$ is given by $\theta^* = \arg\min_{\theta \in \mathbb{R}^2} \sum_{k=1}^N c_k \|f_k(\theta)\|_2^2$

4. Consider the optimization problem from the previous question. It can be reformulated as $\theta^* = \arg\min_{\theta \in \mathbb{R}^2} \|\tilde{y} - \Phi\theta\|_W^2$.
State the dimensions of $\tilde{y}, \Phi$ and W (1 point).

Specify $\tilde{y}, \Phi$ and W (1 point).

5. Which of the following is **not** an assumption of the standard Kalman filter?

(a) ☐ Measurement and state noise have zero mean.	(b) ☐ The model needs to be linear.
(c) ☐ The model needs to be time invariant.	(d) ☐ Measurement and state noise are Gaussian.

6. Which of the following steps is **not** part of the Kalman filter algorithm?

(a) ☐ initialization	(b) ☐ normalization
(c) ☐ innovation update	(d) ☐ prediction

7. Which of the following formulas is associated with the covariance prediction step $P_{[k|k-1]}$ of the Kalman filter, if $x_{k+1} = A_k x_k + w_k$, where w_k is i.i.d. zero mean noise with covariance W_k? $P_{[k|k-1]} = \ldots$

(a) ☐ $A_{k-1} \cdot P_{[k\|k]} \cdot A_{k-1}^\top + W_{k-1}$	(b) ☐ $A_{k-1}^\top \cdot P_{[k\|k-1]} \cdot A_{k-1} + W_{k-1}$
(c) ☐ $\left(A_k^\top \cdot P_{[k-1\|k-1]} \cdot A_k + W_k\right)^{-1}$	(d) ☐ $A_{k-1} \cdot P_{[k-1\|k-1]} \cdot A_{k-1}^\top + W_{k-1}$

Points on page (max. 9)

8. Let $R(\theta) = \Phi\theta - y$ and $f(\theta) = \frac{1}{2}\|R(\theta)\|_2^2$. Compute the difference between the exact Hessian and the Gauss-Newton Hessian approximation. $\nabla^2 f(\theta) - B_{\mathrm{GN}}(\theta) = \dots$

9. Given the residual function $R(\theta) \in \mathbb{R}^N$ and its Jacobian $J(\theta) \in \mathbb{R}^{N\times d}$ where N is the number of measurements and d is the number of parameters, we compute a parameter estimate θ^* by solving $\theta^* = \arg\min_{\theta\in\mathbb{R}^d} \frac{1}{2}\|R(\theta)\|_2^2$. How can you compute an estimate of the parameter covariance Σ_{θ^*}?

(a) ☐ $\frac{\|R(\theta^*)\|_2^2}{N-d}(J(\theta^*)^\top J(\theta^*))^{-1}$	(b) ☐ $\|R(\theta^*)\|_2^2(J(\theta^*)^\top J(\theta^*))^{-1}$
(c) ☐ $\frac{R(\theta^*)^2}{N-d}(J(\theta^*)^\top J(\theta^*))^{-1}$	(d) ☐ $J(\theta^*)^\top J(\theta^*)$

10. Consider $f(\theta) = \frac{1}{2}\|R(\theta)\|_2^2$ with $R(\theta) \in \mathbb{R}^N$ and $J(\theta) = \nabla_\theta R(\theta)^\top$. What is the definition of the Hessian of $f(\theta)$? $\nabla^2 f(\theta) = \cdots$

(a) ☐ $J(\theta)^\top R(\theta)$	(b) ☐ $J(\theta)^\top J(\theta) + \sum_{i=1}^N \nabla_\theta J_i(\theta)$
(c) ☐ $J(\theta)^\top J(\theta) + \sum_{i=1}^N \nabla^2 R_i(\theta) R_i(\theta)$	(d) ☐ $(J(\theta)^\top J(\theta))$

11. Which of the following models generally leads to a convex estimation problem?

(a) ☐ Output-Error	(b) ☐ LIP, additive noise	(c) ☐ Input-Output-Error	(d) ☐ Equation-Error

12. An unconstrained minimization problem with strictly convex objective always has …

(a) ☐ a local maximum.	(b) ☐ a unique global maximum
(c) ☐ multiple local minima.	(d) ☐ a unique global minimum.

13. Given measurements $u(k)$ and $y(k)$, $k = 1, \dots, N$, we try to identify a model by solving the following optimization problem: $\min\limits_{\theta\in\mathbb{R}^2} \sum_{k=3}^N (y(k) - \theta_1 y(k-1) - \theta_2 u(k-2))^2$. What model assumptions do we make?

(a) ☐ IIR model with Gaussian equation errors	(b) ☐ FIR model with Gaussian equation errors
(c) ☐ FIR model with non-Gaussian equation errors	(d) ☐ IIR model with non-Gaussian equation errors

14. Which numerical integration method is preferable as a good compromise of computational effort and accuracy?

(a) ☐ RK4	(b) ☐ Euler	(c) ☐ Triangulation	(d) ☐ Finite Differences

15. In which way does the Extended Kalman Filter (EKF) *extend* the regular Kalman Filter algorithm? In contrast to the regular Kalman Filter, the EKF can be applied to …

(a) ☐ nonlinear systems.	(b) ☐ systems with large state space dimension.
(c) ☐ time-invariant systems.	(d) ☐ systems that are perturbed by non-Gaussian noise.

16. State one shortcoming of the Extended Kalman Filter.

17. Consider the Extended Kalman Filter system model $x_{k+1} = f(x_k) + w_k$, $y_k = g(x_k) + v_k$ with state vector $x_k \in \mathbb{R}^3$, output vector $y_k \in \mathbb{R}^2$, and noise terms w_k, v_k. We assume that f and g are nonlinear. Specify matrices A_k and C_k such that the above system can be reformulated in the form that is assumed by the regular Kalman Filter, i.e. $x_{k+1} = A_k x_k + b_k + w_k$, $y_k = C_k x_k + v_k$?

18. Which statement is **not** true about Moving Horizon Estimation (MHE) with horizon length N? Here KF denotes the regular Kalman Filter, EKF denotes the extended Kalman Filter.

(a) ☐ MHE is computationally cheaper than EKF.	(b) ☐ MHE can be applied to nonlinear systems.
(c) ☐ Computing the MHE estimate at time N is as expensive as computing the MHE estimate at time $2N$.	(d) ☐ MHE is equivalent to KF in the unconstrained linear case.

Points on page (max. 11)	