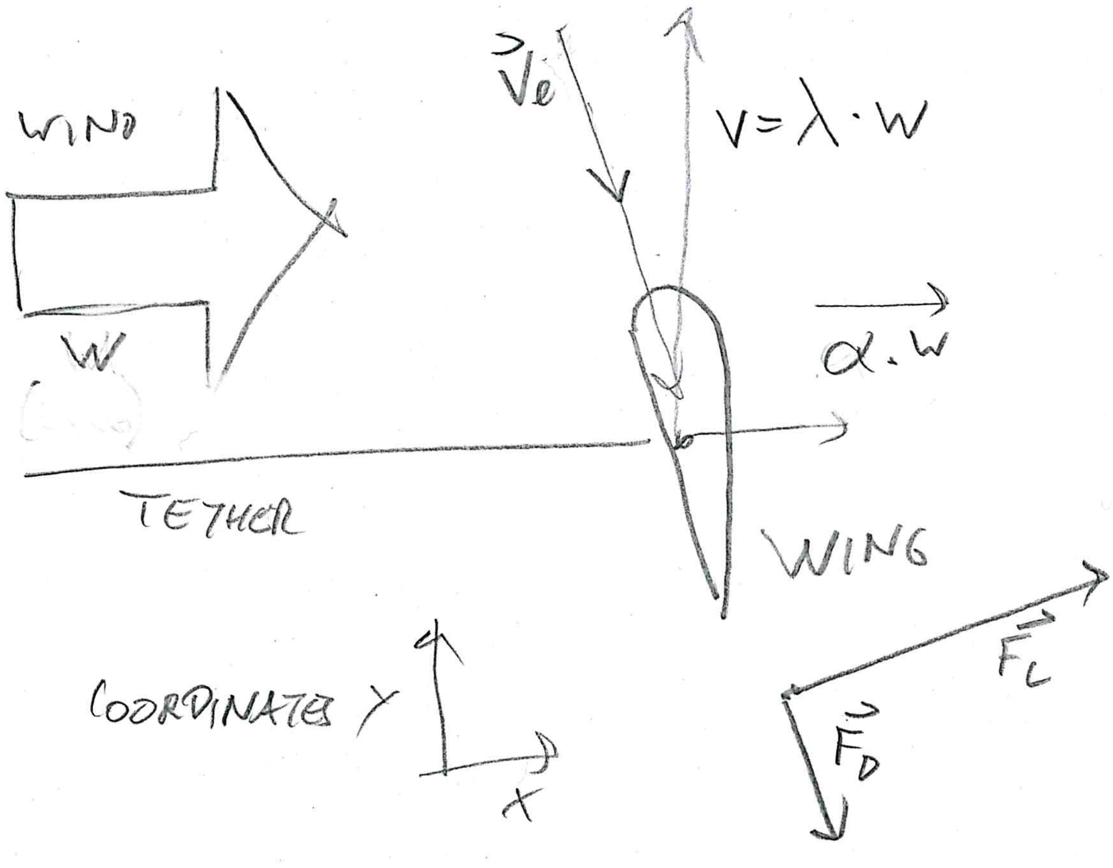
6.2 AIRBORNE WIND ENERGY (AWE)

SEE SLIDES

6.3 LET US DERIVE LOYD'S FORMULA IN PUMPING MODE

REGARD KITE/AIRFOIL UNDER IDEALIZED CONDITIONS

- TETHER PARALLEL TO WIND
- NO GRAVITY, STEADY WIND $W \equiv u_\infty$
- STEADY CROSSWIND FLIGHT WITH DOWNWIND COMPONENT AND C_L & C_D GIVEN
- ROLLOUT SPEED : $\alpha \cdot W$
- "TIP SPEED RATIO" λ
- WING AREA A



WIND &

MOTION VECTOR IN X-Y-FRAME

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$$\vec{w} = \begin{bmatrix} w \\ 0 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} \alpha \cdot w \\ \lambda \cdot w \end{bmatrix}$$

EFFECTIVE WIND $\vec{v}_e = \vec{w} - \vec{v} = \begin{bmatrix} (1-\alpha) w \\ -\lambda w \end{bmatrix}$

WITH $V_e := \|V_e\|_2 = w \cdot \sqrt{(1-\alpha)^2 + \lambda^2}$

DRAG FORCE

$$\vec{F}_D = \frac{1}{2} \rho A \cdot \|V_e\|^2 \cdot C_D \cdot \frac{\vec{V}_e}{\|V_e\|_2} =$$

$$= \frac{1}{2} \rho A V_e^2 \cdot C_D \cdot \begin{bmatrix} 1-\alpha \\ -\lambda \end{bmatrix} \frac{1}{\sqrt{(1-\alpha)^2 + \lambda^2}}$$

LIFT FORCE, PERPENDICULAR, IN DIRECTION

$$\begin{bmatrix} \lambda \\ (1-\alpha) \end{bmatrix} \frac{1}{\sqrt{(1-\alpha)^2 + \lambda^2}} \quad , \quad \text{i.e.}$$

$$\vec{F}_L = \frac{1}{2} \rho A V_e^2 C_L \cdot \begin{bmatrix} \lambda \\ (1-\alpha) \end{bmatrix} \frac{1}{\sqrt{(1-\alpha)^2 + \lambda^2}}$$

STEADY STATE : NO ACCELERATION,
I.E. NO FORCE
IN Y-DIRECTION

$$\vec{F}_L + \vec{F}_D = \frac{1}{2}\rho A v_e^2 \frac{1}{\sqrt{(1-\alpha)^2+\lambda^2}} \begin{bmatrix} C_D(1-\alpha) + C_L \cdot \lambda \\ -C_D\lambda + C_L(1-\alpha) \end{bmatrix}$$

$$\overset{!}{=} \begin{bmatrix} * \\ 0 \end{bmatrix} =: \begin{bmatrix} F_T \\ 0 \end{bmatrix} \leftarrow \text{TETHER FORCE}$$

$$\Leftrightarrow \lambda C_D = (1-\alpha) C_L \Leftrightarrow \boxed{\lambda = \frac{C_L}{C_D} \cdot (1-\alpha)}$$

GENERATED POWER :

ROLL-OUT-SPEED $\alpha \cdot w$ TIMES F_T

$$P = \alpha \cdot w \cdot F_T$$

$$= \alpha \cdot w \cdot \frac{1}{2}\rho A w^2 \frac{\left(\frac{C_L}{C_D}\right)^2 (1-\alpha)^2 \cdot k_1 \cdot \left[C_D(1-\alpha) + \frac{C_L^2}{C_D}(1-\alpha)\right]}{\frac{C_L}{C_D}(1-\alpha) \cdot k_1}$$

USE $\sqrt{(1-\alpha)^2+\lambda^2}$

$$= (1-\alpha)\sqrt{1+\frac{C_L^2}{C_D^2}}$$

$$= \frac{C_L}{C_D} \cdot (1-\alpha) \cdot \underbrace{\sqrt{1+\frac{C_D^2}{C_L^2}}}_{=: k_1}$$

$$= \frac{1}{2}\rho A w^3 \alpha \cdot (1-\alpha)^2 \cdot \left[\frac{C_L^3}{C_D^2} + C_L\right] = \rightarrow$$

$$P = \frac{1}{2}\rho A w^3 \, \alpha(1-\alpha)^2 \frac{C_L^3}{C_D^2}\left[1 + \frac{C_D^2}{C_L^2}\right]$$

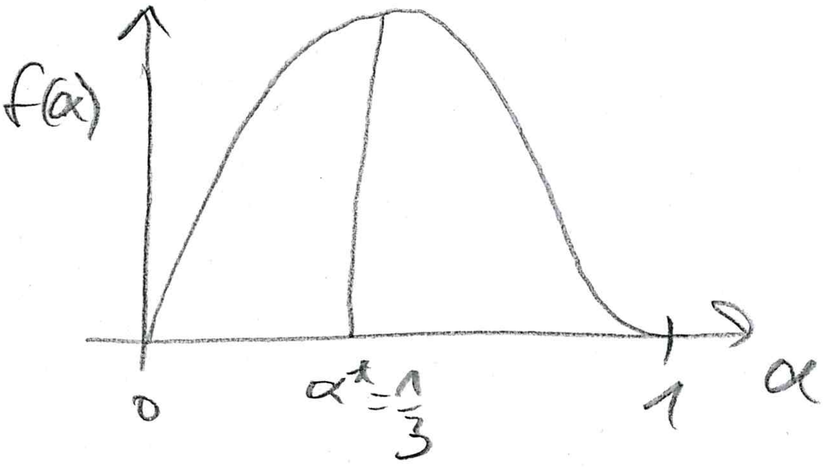
$$= \frac{1}{2}\rho A w^3 \frac{C_L^3}{C_D^2}\left(1+\frac{C_D^2}{C_L^2}\right) \cdot \alpha(1-\alpha)^2$$

MAXIMUM IS REACHED. IF $\underbrace{\alpha(1-\alpha)^2}_{=:f(\alpha)}$ IS MAXIMIZED:

$$f(\alpha) = \alpha(1-\alpha)^2$$

$$f'(\alpha) = (1-\alpha)^2 - 2\alpha(1-\alpha) \overset{!}{=} 0 \Leftrightarrow 1-\alpha = 2\alpha$$

$$\Leftrightarrow \alpha^* = \frac{1}{3}$$



$$f(\alpha^*) = \frac{1}{3}\left(\frac{2}{3}\right)^2 = \frac{4}{27}$$

LOYDS FORMULA:

$$\boxed{P = \frac{1}{2}\rho A w^3 \cdot \frac{4}{27} \cdot \frac{C_L^3}{C_D^2}\underbrace{\left(1+\frac{C_D^2}{C_L^2}\right)}_{\approx 1}}$$

REGARD E.G. $C_L = 1$, $C_D = 0.05$, $w = 10\ m/s$, $\rho = 1.2\ kg/m^3$ AND GET

$$\frac{P}{A} = \underbrace{\frac{1}{2}\rho w^3}_{=600\ \frac{W}{m^2}} \cdot \underbrace{\frac{4}{27} C_L \left(\frac{C_L}{C_D}\right)^2 \left(1 + \frac{C_D^2}{C_L^2}\right)}_{=:\zeta}$$

"HARVESTING FACTOR" "ZETA"

$$\zeta = \frac{4}{27} \cdot 400 \left(1 + \frac{1}{400}\right)$$

$$\approx 59$$

$$\frac{P}{A} = 36\ \frac{kW}{m^2}$$

NOTE $\lambda = \frac{C_L}{C_D} \cdot (1-\alpha) = 20 \cdot \frac{2}{3} = 13$

AND $v_e = w \cdot \lambda \cdot \sqrt{1 + \frac{(1-\alpha)^2}{\lambda^2}} \approx 130\ \frac{m}{s}$

$\approx 470\ km/h$

IF ONE WANTS TO LIMIT λ, ONE HAS TO INCREASE ROLL-OUT-SPEED α

E.G. TO GET $\lambda = 10$ WE HAVE DUE TO $\lambda = \frac{C_L}{C_D}(1-\alpha)$

$$\alpha = 1 - \lambda \cdot \frac{C_D}{C_L} = 1 - 10/20 = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8} = \frac{4}{32} < \frac{4}{27}$$

GET ONLY 84% OF LOYD'S LIMIT

FOR $\lambda = 6$ WOULD GET $\alpha = 1 - \frac{6}{20} = \frac{14}{20} = \frac{7}{10}$

$$f\left(\frac{7}{10}\right) = \frac{7\,(3)^2}{1000} = \frac{63}{1000} \approx 42\% \cdot f(\alpha^*)$$
