$$(1) \Leftrightarrow \frac{x^2}{2} + 6c_3 x + 2c_2 \Big|_{x=L} = 0$$

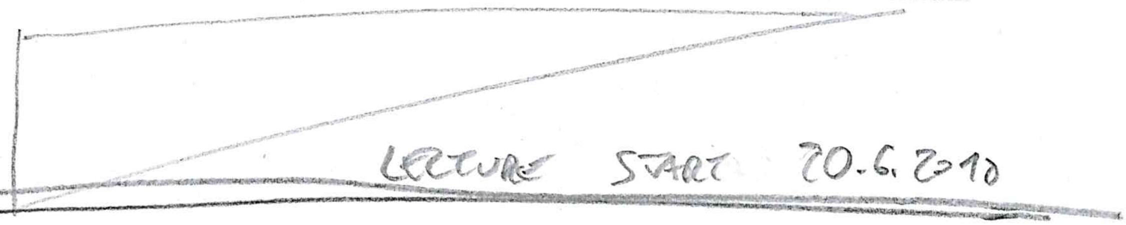
$$(2) \Leftrightarrow x + 6c_3 \Big|_{x=L} = 0 \Leftrightarrow c_3 = -\frac{L}{6}$$

$$(1) \Leftrightarrow \frac{L^2}{2} - L^2 + 2c_2 = 0 \Leftrightarrow c_2 = \frac{1}{4}L^2$$

$3L^2 - 6L^2$

$$w(x) = \frac{q}{EI}\left(\frac{x^4}{24} - \frac{L}{6}x^3 + \frac{1}{4}L^2x^2\right) = \frac{q\,x^2}{EI \cdot 24}\left(x^2 - 4Lx + 6L^2\right)$$

$Q(x) =$



LECTURE START 20.6.2010

4.2.1

MAXIMUM STRESS: $\sigma = E \cdot \varepsilon$ $\qquad \varepsilon = z \cdot \frac{d^2w}{dx^2}$

AT BOUNDARY

d = Half Beam Thickness

$$= z \cdot \frac{M(x)}{EI}$$

$$\sigma_{max} = E \cdot \frac{d \cdot M(x)}{EI} = \frac{d}{I} \cdot M(x)$$

FOR EXAMPLE, IF $\sigma_{max} = 250\,MPa$, $I = \frac{\pi}{4}d^4$ (DISC), $d = 1m$

THE MAXIMUM MOMENT IS $M_{max} = \sigma_{max} \cdot \frac{I}{d} = 250 \cdot 10^6 \frac{\pi}{4}$

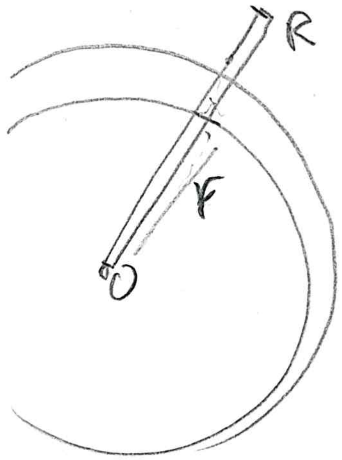
$\approx 2 \cdot 10^8\,Nm$

$= 200m \cdot 1MN$

A HIGHER MOMENT WILL LEAD TO PLASTIC DEFORMATIONS

4.2.2 LOADS AT BLADE ROOT

FOR A BLADE IN AN IDEAL DESIGN,
THE DISTRIBUTED LOAD $q(r)$ IS GIVEN BY $\frac{1}{B}$
THE THRUST OF THE CORRESPONDING ANNULUS



$$dF = \overbrace{4a(1-a)}^{=C_T(a)} \cdot \frac{1}{2}\rho U_\infty^2 \cdot 2\pi r \cdot dr$$
$$= \underbrace{C_T(a)\ \frac{1}{2}\rho U_\infty^2\ 2\pi r}_{=B\cdot q(r)} \cdot dr$$

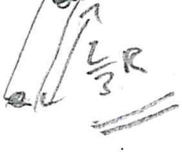
THE BENDING MOMENT AT THE BLADEROOT, $r=0$, CAN
BE COMPUTED BY INTEGRATION

$$M(0) = \int_0^R r \cdot q(r)\, dr = \frac{1}{B} C_T(a) \frac{1}{2}\rho U_\infty^2 \cdot 2\pi \cdot \underbrace{\int_0^R r^2\, dr}_{=:\frac{R^3}{3}}$$

$$= \frac{1}{B} C_T(a)\frac{1}{2}\rho U_\infty^2\ \frac{2}{3}\pi R^3$$

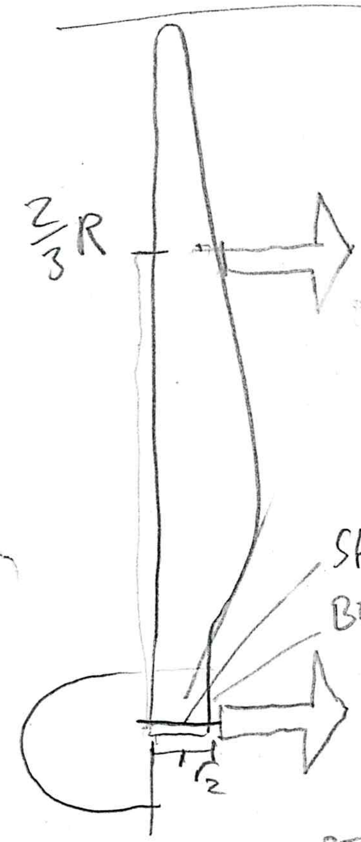
$$= \frac{1}{B}\left(\frac{2}{3}\right) R \cdot \underbrace{\left(C_T(a)\frac{1}{2}\rho U_\infty^2\ \pi R^2\right)}_{=:F_T}$$ TOTAL THRUST OF TURBINE

EASY TO REMEMBER: $\frac{F_T}{B}$ = (TOTAL FORCE ON BLADE)



TIMES
$\frac{2}{3}R$ = (2/3 OF RADIUS)
EQUALS MOMENT $M(0)$

SHEAR FORCE AT BLADE ROOT IS TRIVIALLY GIVEN BY $\frac{F_T}{B}$



REMEMBER $F_T \approx \frac{P}{(1-a)U_\infty}$,

SHEAR FORCE $Q(0) = \frac{F_T}{B}$

BENDING MOMENT

$$M(0) = Q(0) \cdot \frac{2}{3} R$$

AT

E.G. 1 MN

FOR 6 MW AT

$U_\infty = 9 \frac{m}{s}$, $a = \frac{1}{3}$

ROTOR RADIUS R ?

$$P \approx \frac{16}{27} \frac{1}{2} \rho U_\infty^3 \pi R^2$$

$$\Longleftrightarrow R = \sqrt{P / \left(\frac{16}{27} \cdot \underbrace{\frac{1}{2}\rho U_\infty^3}_{500 \frac{W}{m^2}} \pi\right)}$$

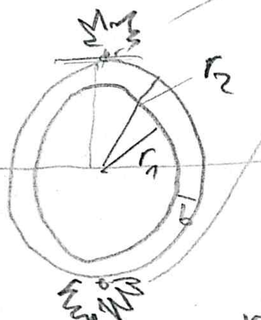
$$\approx \sqrt{\frac{P}{1 kW/m^2}}$$

$$\approx \sqrt{6000 m^2} \approx 75$$

MAXIMUM BENDING STRESS AT BLADE ROOT ?

REGARD ANNULAR CROSS SECTION WITH

$$I = \frac{\pi}{4}(r_2^4 - r_1^4) \approx \pi r_2^3 b$$



$r_1 = r_2 - b$

$r_1^4 \approx r_2^4 - 4 r_2^3 b$

$r_2^4 - r_1^4 \approx 4 r_2^3 b$

SUCH THAT $\frac{I}{r_2} \approx \pi r_2^2 b$

AND MAXIMUM STRESS GIVEN BY

$$\sigma_{max} = \frac{M(0)}{(I/r_2)} = \frac{2}{3} \cdot R \cdot \frac{F_T}{B} \frac{1}{\pi r_2^2 b}$$

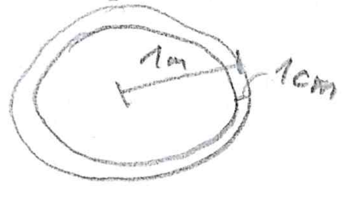
$$b = \frac{2}{3} R \frac{F_T}{B} \cdot \frac{1}{\pi r_2^2} \frac{1}{\sigma_{max}} -$$

AT

FOR $r_2 = 1m$,

$\sigma_{max} = 250$ MPa

GET $b = 2cm$

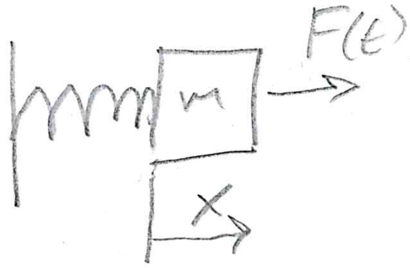


4.3 OSCILLATIONS AND EIGENMODES

4.3.1 INTRO:

SPRING-MASS-DAMPER SYSTEM

$$m\ddot{x} + \beta\dot{x} + \underbrace{kx}_{} = F(t)$$



x(t) DISPLACEMENT   m : MASS

F(t) EXTERNAL FORCE

kx(t) SPRING FORCE

β (VISCOUS/LINEAR) DAMPING

FOR $F(t) = F_0 \cdot e^{j\omega t}$ (TAKE REAL PART IF DESIRED)

SOLUTION IS GIVEN BY

$$x(t) = X_0 \cdot e^{j\omega t} \quad \text{SUCH THAT}$$

$$\dot{x} = j\omega X_0 e^{j\omega t}$$

$$\ddot{x} = -\omega^2 X_0 e^{j\omega t}$$

$$m(-\omega^2)X_0 e^{j\omega t} + \beta(j\omega) X_0 e^{j\omega t} + k X_0 e^{j\omega t} = F_0 e^{j\omega t}$$

$$\Leftrightarrow X_0 \cdot (\underbrace{k - m\omega^2}_{\text{REAL}} + \underbrace{j\beta\omega}_{\text{IMAGINARY}}) = F_0$$

X_0 IS COMPLEX NUMBER WITH MAGNITUDE

$$|X_0| = \frac{F_0}{\sqrt{(k-m\omega^2)^2 + \beta^2\omega^2}}$$

MAXIMA APPROXIMATELY TAKEN AT NATURAL RESONANT "EIGEN FREQUENCY" ω_{NR} WITH

$$k - m\omega_{NR}^2 = 0 \iff \omega_{NR} = \sqrt{k/m}$$

HOW MUCH CAN F_0 BE AMPLIFIED?

SPRING FORCE: $F^{SPRING} = k \cdot X$

REGARD

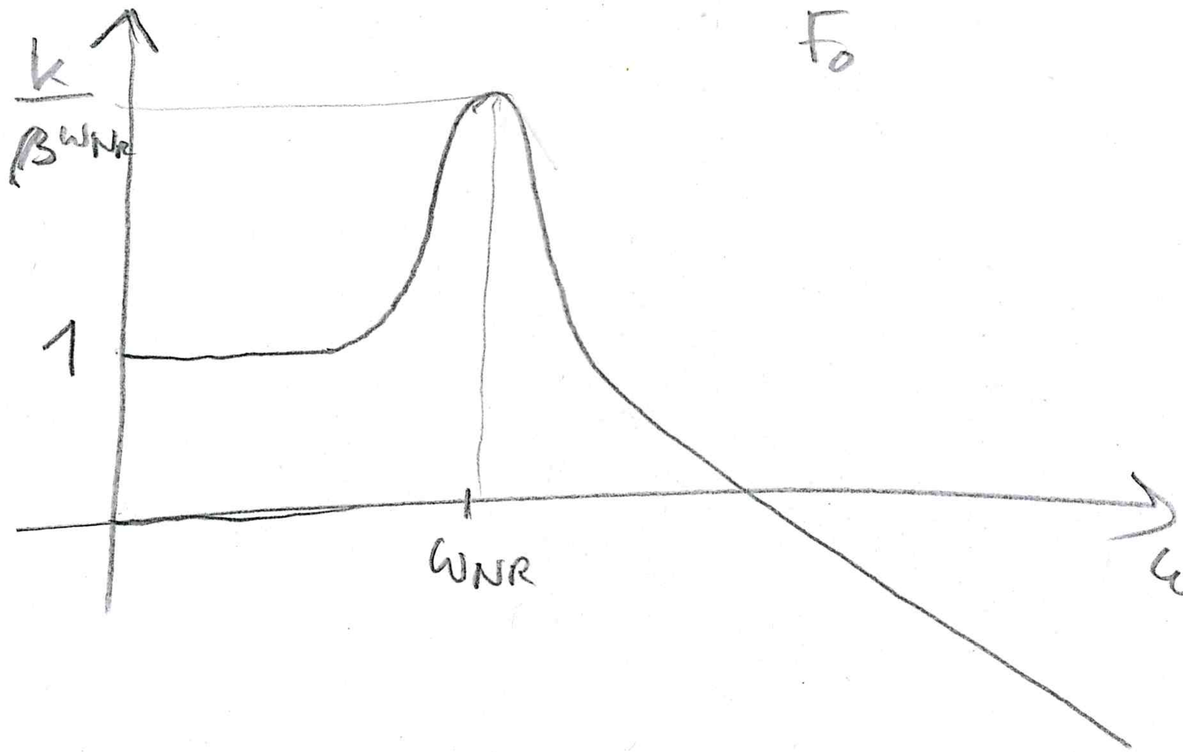
$$|F_0^{SPRING}| = k|X_0| = \frac{F_0}{\sqrt{\left(1-\left(\frac{\omega}{\omega_{NR}}\right)^2\right)^2 + \frac{\beta^2}{k^2}\omega^2}}$$

AT $\omega = \omega_{NR}$ GET

$$\frac{|F_0^{SPRING}|}{F_0} = \frac{k}{\beta\,\omega_{NR}}$$

i.e., THE SMALLER THE DAMPING, THE HIGHER THE AMPLIFICATION

BODE DIAGRAM OF $\frac{|F_0^{SPRING}(\omega)|}{F_0}$



AMPLIFICATION FACTORS CAN BE 5 - 10, SO RESONANCE SHALL TYPICALLY BE AVOIDED!

AT VERY LOW FREQUENCIES, SPRING FORCE EQUALS APPLIED FORCE, I.E. STATIC ANALYSIS IS SUFFICIENT (CF. SECTION 4.2)

4.3.2 EIGENMODES

FOR SPRING-MASS-(DAMPER) SYSTEMS WITH MORE THAN ONE DEGREE OF FREEDOM, THE DISPLACEMENT CAN BE DESCRIBED BY A VECTOR $w(t) \in \mathbb{R}^n$ AND THE EQUATION OF MOTION BECOMES

$$\underbrace{M}_{\text{MASS MATRIX}} \cdot \ddot{w} + D \cdot \dot{w} + \underbrace{K}_{\text{STIFFNESS MATRIX}} \cdot w = F(t), \quad M \in \mathbb{R}^{4 \times 4}, \; K \in \mathbb{R}^{4 \times}$$