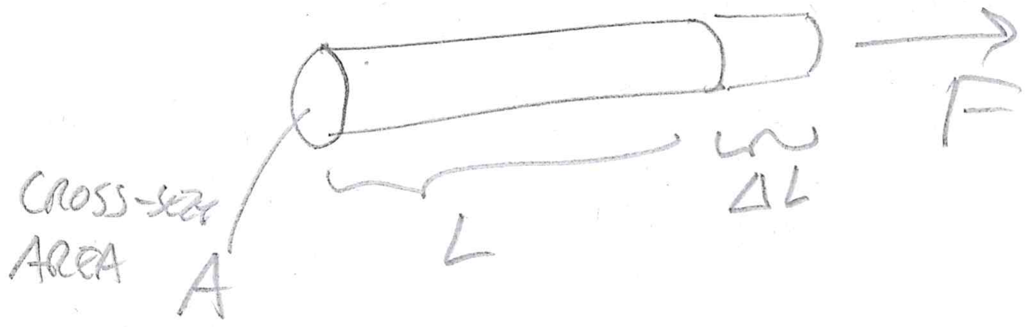
4.2 STRESS AND STRAIN

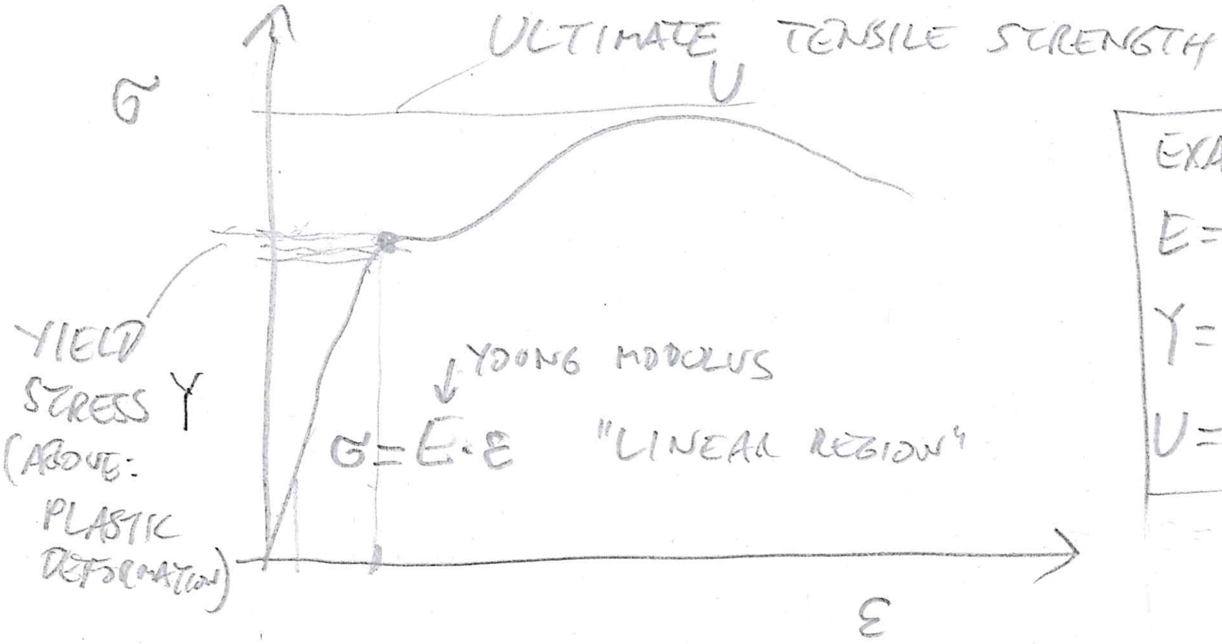
REGARD MATERIAL UNDER TENSION



STRESS : $\sigma = \dfrac{F}{A}$ [Pa]

STRAIN : $\varepsilon = \dfrac{\Delta L}{L}$ [-]

STRESS-STRAIN CURVE



EXAMPLE STEEL:

$E = 200$ GPa

$Y = 250$ MPa

$U = 500$ MPa

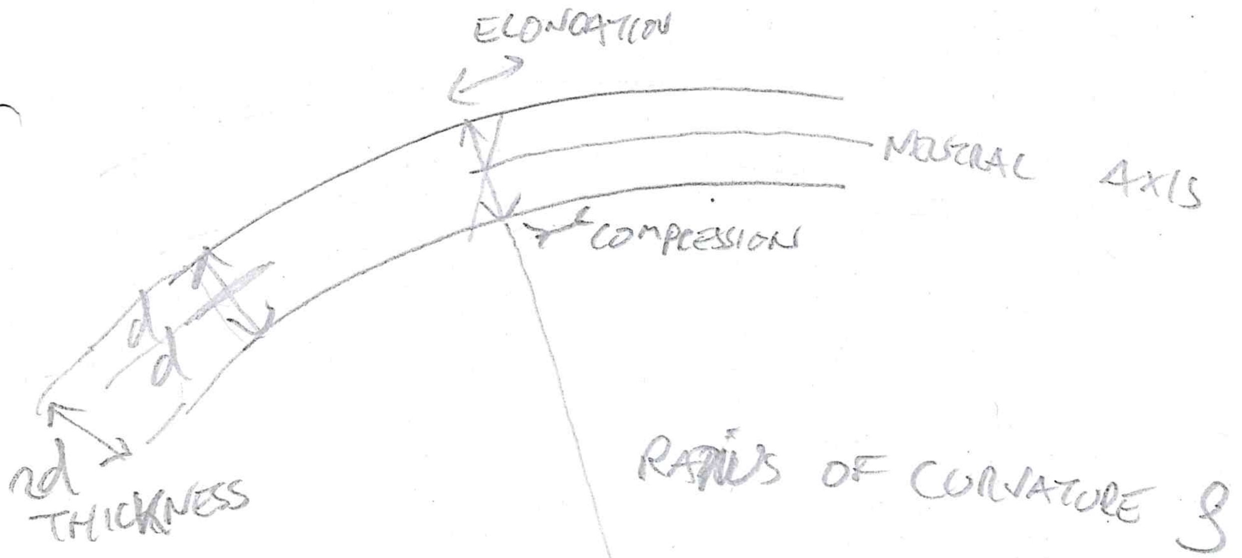
AT WHICH STRAIN DOES STEEL START TO DEFORM PLASTICALLY / PERMANENTLY

$$\sigma_Y = E \cdot \varepsilon_Y \qquad \sigma_Y = Y$$

$$\sigma_Y = \frac{Y}{E} = \frac{250\ \text{MPa}}{200 \cdot 10^3\ \text{MPa}} = 1{,}25 \cdot 10^{-3}$$

$$= 0.125\ \text{‰}$$

WHEN DOES A BEAM START TO DEFORM?



$$\varepsilon_Y = \frac{d}{\rho}$$

AT WHAT BENDING MOMENT WOULD DEFORMATION START

4.3 STATIC BEAM BENDING

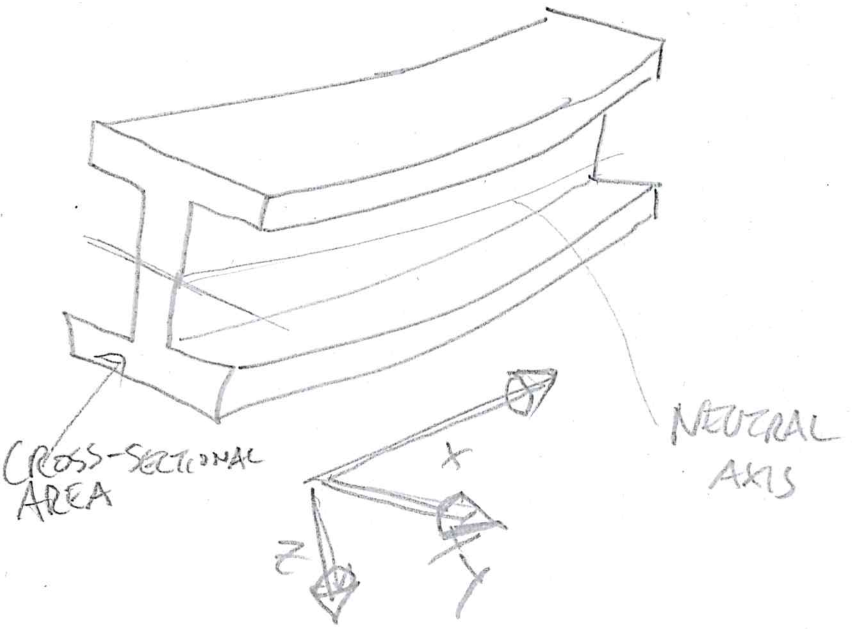
(EULER-BERNOULLI BEAM THEORY)

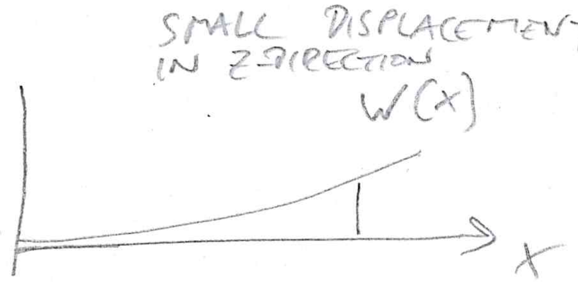
HOOKE'S LAW :

$$\underset{\text{STRESS [Pa]}}{\sigma} = \underset{\text{[Pa]}}{E} \cdot \underset{\text{[\%]}}{\varepsilon}$$

E ← YOUNG'S MODULUS

ε ← STRAIN (DEFORMATION)



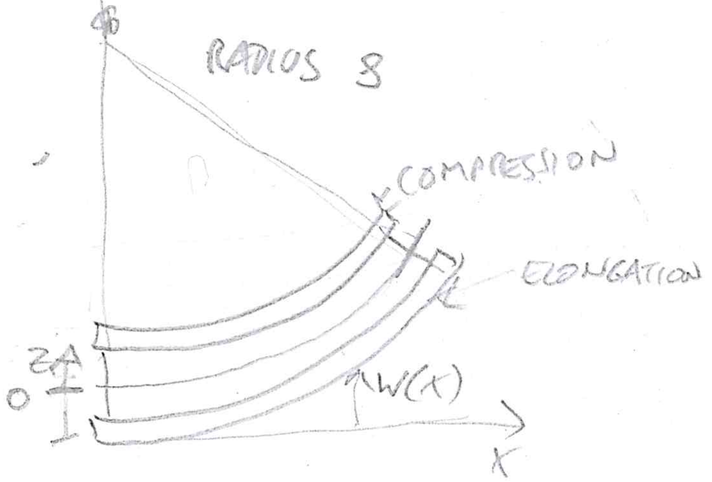


STRAIN

$$\varepsilon = \frac{z}{\rho}$$

z ← DISTANCE FROM NEUTRAL AXIS

ρ ← RADIUS OF CURVATURE



$$\varepsilon = z \cdot \frac{d^2 w(x)}{d x^2}$$

BENDING MOMENT

$$M(x) = \int z \cdot \sigma \cdot dA$$

$$= \int z^2 E \cdot \frac{d^2 w(x)}{d x^2} \, dA$$

$$= E \cdot \frac{d^2 w(x)}{d x^2} \cdot \boxed{\int z^2 dA} =: I$$

(OBSERVATION FIRST DUE TO DA VINCI, BUT EULER-BERNOULLI REDISCOVERED IT AND MADE IT USEFUL FOR COMPUTATIONS

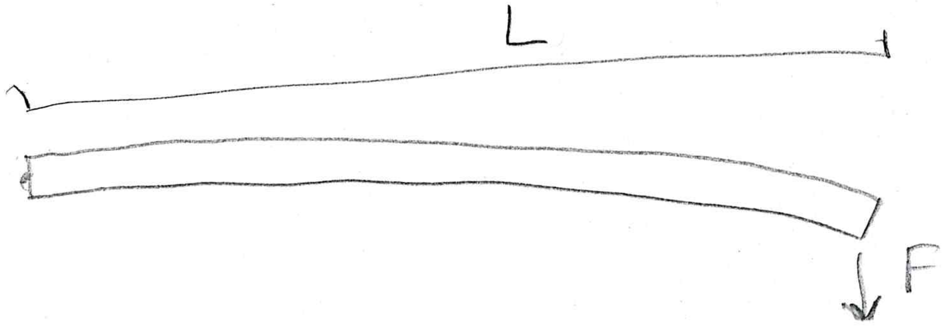
$$I := \int z^2 \, dA$$ "SECOND MOMENT OF AREA"

$$M(x) = E \cdot I \cdot \frac{d^2 w}{dx^2}$$

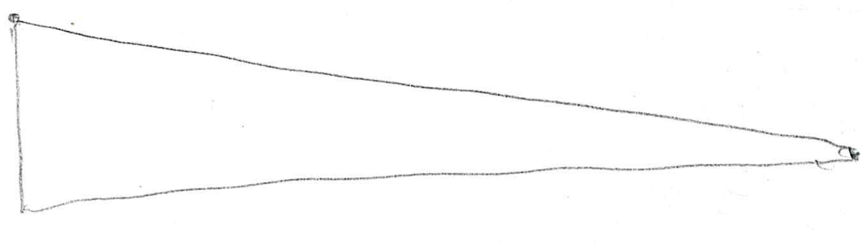
shear force $$Q(x) = \frac{dM}{dx} = \frac{d}{dx}\left(EI \frac{d^2 w}{dx^2}\right)$$

STATIC BEAM EQUATION: $$\frac{d^2}{dx^2}\left(E(x) I(x) \frac{d^2 w}{dx^2}\right) = q(x)$$ ← DISTRIBUTED LOAD

EX: CANTILEVER BEAM WITH END LOAD (& CONSTANT CROSS SECTION)



Moment: $M(x) = F(L - x)$



$$Q(x) = \frac{dM}{dx} = -F$$

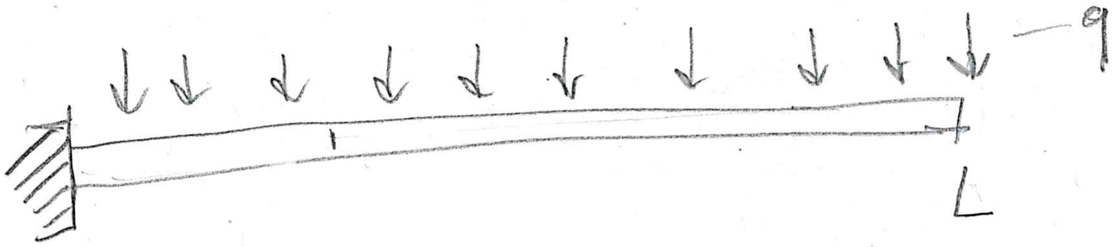
$$\frac{dQ}{dx} = 0$$

$$M(x) = EI \frac{d^2 w}{dx^2} \overset{!}{=} F(L-x) \iff w(x) = \frac{1}{EI}\left(FL \frac{x^2}{2} - F\frac{x^3}{6} + c_0 + c_1 x\right)$$

AT $x=0$: $w(0)=0$, $\frac{dw}{dx}=0 \Rightarrow c_0 = 0, \; c_1 = 0$

$$w(x) = \frac{F}{EI}\frac{x^2}{2}\left(L - \frac{x}{3}\right) = \frac{F}{6 \cdot EI} x^2 (3L - x)$$

EX 2: CANTILEVER BEAM WITH UNIFORMLY DISTRIBUTED LOAD



(E, I CONST).

$$\frac{q}{EI} = \frac{d^4 w}{dx^4} \iff w(x) = \frac{q}{EI}\left(\frac{x^4}{24} + c_3 x^3 + c_2 x^2 + c_1 x + c_0\right)$$

BOUNDARY CONDITIONS: $w(0) = 0$, $\frac{dw}{dx}(0) = 0$ $\Rightarrow$ $c_0 = 0$, $c_1 = 0$

(1) $\frac{d^2 w}{dx^2}(L) = 0$

(2) $\frac{d^3 w}{dx^3}(L) = 0$ (NO POINT FORCE AT END = NO SHEAR FORC

$$(1) \Leftrightarrow \frac{x^2}{2} + 6c_3 x + 2c_2 \Big|_{x=L} = 0$$

$$(2) \Leftrightarrow x + 6c_3 \Big|_{x=L} = 0 \Leftrightarrow c_3 = -\frac{L}{6}$$

$$(1) \Leftrightarrow \frac{L^2}{2} - L^2 + 2c_2 = 0 \Leftrightarrow c_2 = \frac{1}{4}L^2$$

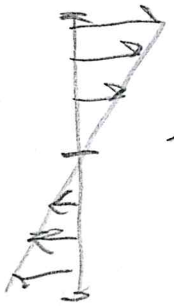
$$w(x) = \frac{q}{EI}\left(\frac{x^4}{24} - \frac{L}{6}x^3 + \frac{1}{4}L^2x^2\right) = \frac{q\,x^2}{EI \cdot 24}\left(x^2 - 4Lx + 6L^2\right)$$

$Q(x) =$

4.2.1

MAXIMUM STRESS AT BOUNDARY: $\sigma = E \cdot \varepsilon$

$$\varepsilon = z \cdot \frac{d^2w}{dx^2} = z \cdot \frac{M(x)}{EI}$$



d = Half Beam THICKNESS

$$\sigma_{max} = E \cdot \frac{d \cdot M(x)}{EI} = \frac{d}{I} \cdot M(x)$$