3.4 BLADE ELEMENT / MOMENTUM (BEM) THEORY

RECALL:

ACTUATOR DISC: AXIAL MOMENTUM ONLY

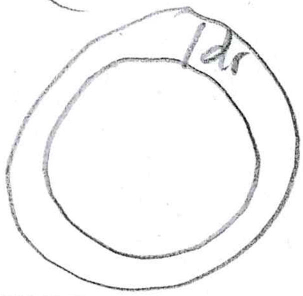
$C_T(a) = 4a(1-a)$, $C_P(a) = 4a(1-a)^2$

BETZ LIMIT

ROTOR DISC: ALSO TANGENTIAL MOMENTUM

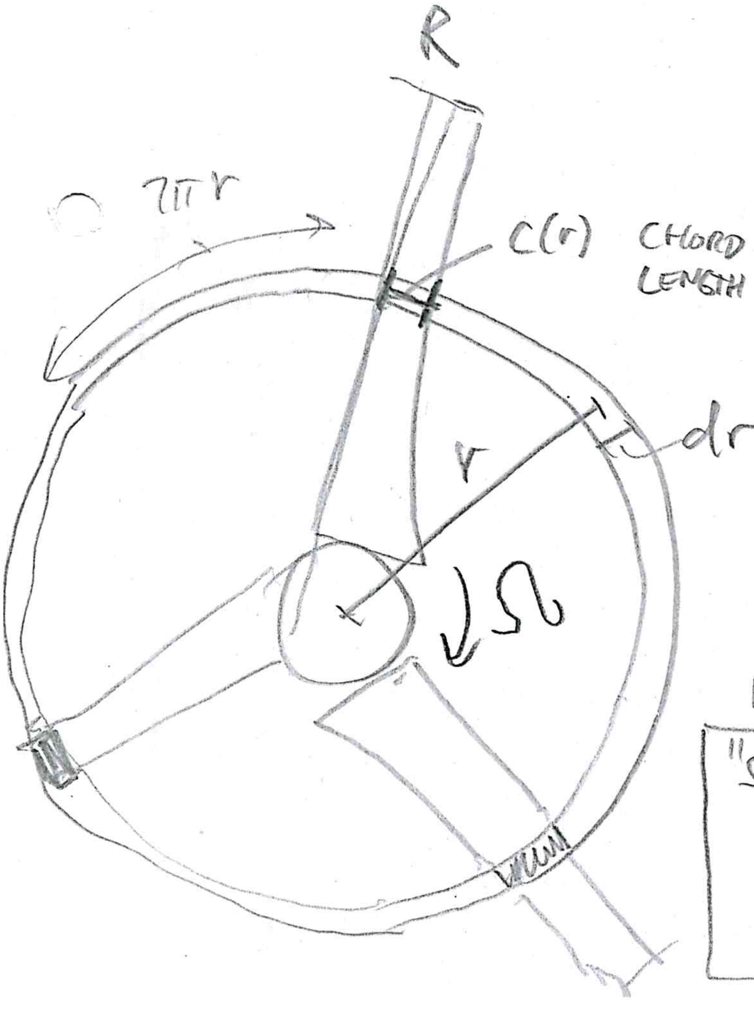
$$a'(r) = \frac{a(1-a)}{\lambda_r^2} \qquad \left(V_3 = 2a' r \cdot \Omega\right)$$

REGARDS ANNULI



NOW: BLADE ELEMENT / MOMENTUM:

- REGARD ANNULI COMPLETELY INDEPENDENT FROM EACH OTHER (LIKE ROTOR DISC)
- ASSUME AERODYNAMIC LIFT & DRAG ACCORDING TO 2D-AIRFOIL PROPERTIES C_L & C_D



NUMBER OF BLADES: B (e.g. $B=3$)

$$\boxed{\text{"SOLIDITY" AT RADIUS } r: \quad \sigma_r := \frac{c(r) \cdot B}{2\pi \cdot r}}$$

OVERALL SOLIDITY:

$$\sigma = \frac{\int_0^R c(r)\,dr \cdot B}{\pi R^2}$$

← TOTAL BLADE AREA

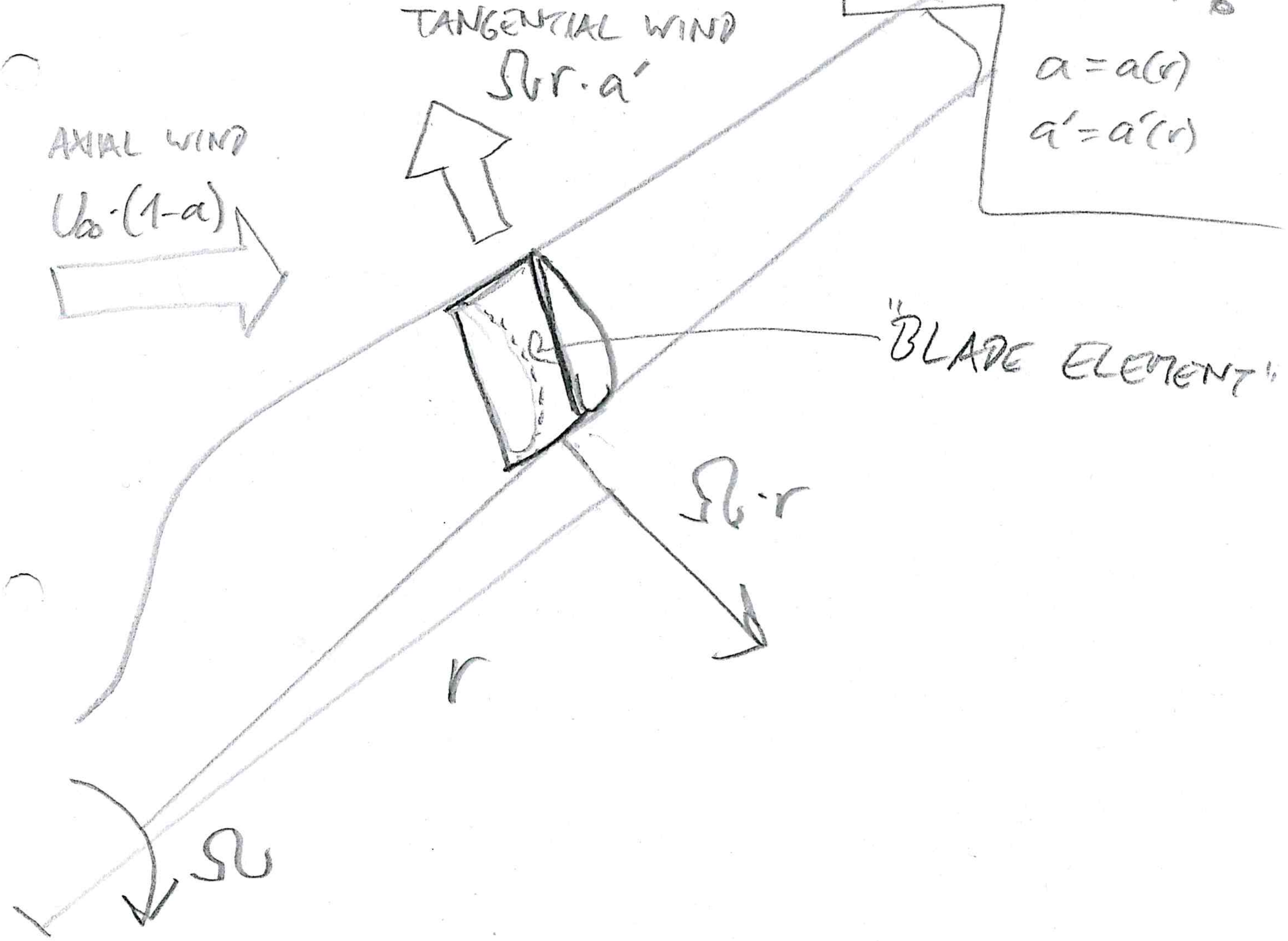
← DISC AREA

GEOMETRY AND SPEEDS

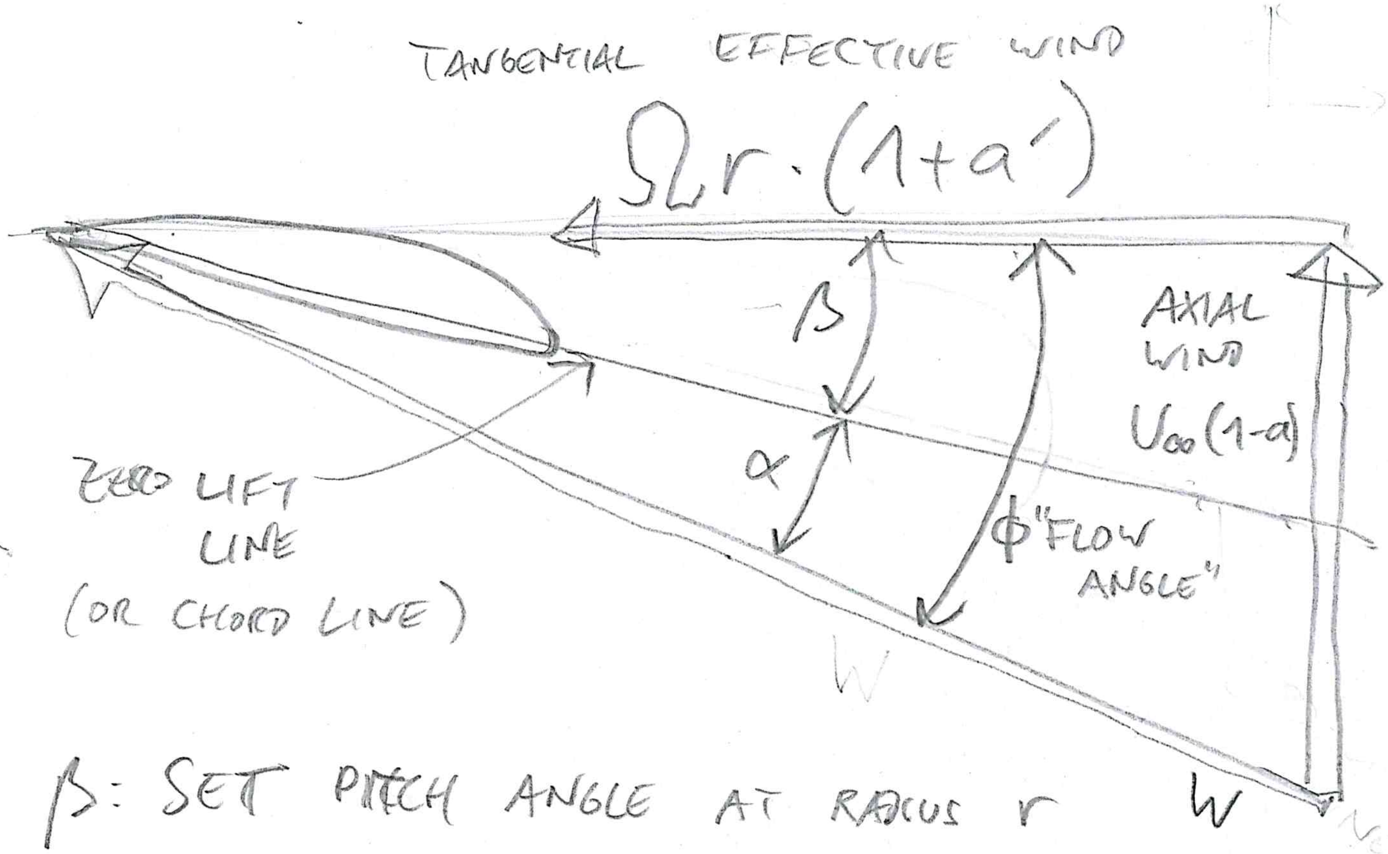
a & a' CAN DEPEND ON r:

$a = a(r)$

$a' = a'(r)$



BLADE ELEMENT FROM TOP (FOR ONE r)



β: SET PITCH ANGLE AT RADIUS r

$\alpha = \phi - \beta \doteq$ ANGLE OF ATTACK

$C_L(\alpha)$: 2D-LIFT COEFFICIENT

$C_D(\alpha)$: 2D-DRAG "

EFFECTIVE WIND :

$$W = \sqrt{(\Omega r (1+a'))^2 + (U_\infty(1-a))^2}$$

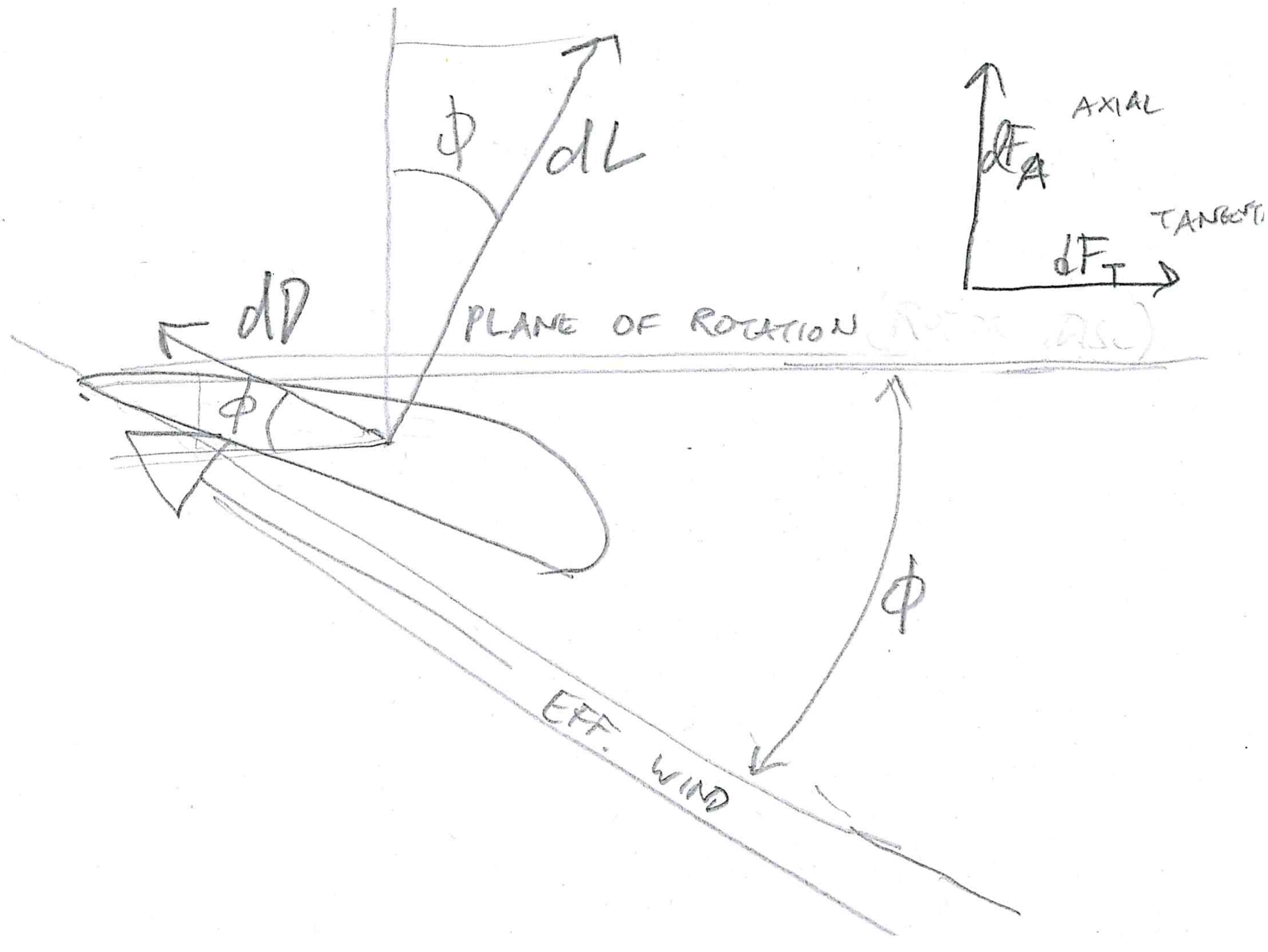
$$\sin\phi = \frac{V_\infty(1-a)}{W}$$

$$\cos\phi = \frac{\Omega r(1+a')}{W}$$

AREA OF BLADE ELEMENT : $dA_B = c \cdot dr$

LIFT " " " : $dL = \frac{1}{2}\rho W^2 dA_B \cdot c_L$

DRAG " " " $dD = \frac{1}{2}\rho W^2 dA_B \cdot c_D$



AXIAL THRUST FORCE OF COMPLETE ANNULUS :

$$dF_A = (dL \cdot \cos\phi + dD \cdot \sin\phi) \cdot B$$

$$= \frac{1}{2}\rho W^2 B \cdot c\,(c_L \cdot \cos\phi + c_D \cdot \sin\phi) \quad (1)$$

TANGENTIAL FORCE :

$$dF_T = -(dD \cdot \cos\phi - dL \cdot \sin\phi) \cdot B$$

$$= \frac{1}{2}\rho W^2 Bc\,(c_L \cdot \sin\phi - c_D \cos\phi) \quad (2)$$

AXIAL THRUST & TORQUE CAUSE THE INDUCTION (a & a') OF THE FLOW BY MOMENTUM BALANCE (AS BEFORE)

AXIAL:

$$dF_A = d\dot{m} \cdot (2 \cdot a \cdot V_\infty) \quad \text{WITH} \quad d\dot{m} = \rho \cdot 2\pi r \cdot dr \; V_\infty (1-a)$$

$$= \frac{1}{2}\rho\, 2\pi r\, dr\, V_\infty^2 \underbrace{4a(1-a)}_{= C_T(a)} \quad (3)$$

TANGENTIAL FORCE:

$$dF_T = d\dot{m}\,(2a' r \Omega)$$

$$= \frac{1}{2}\rho\, 2\pi r \cdot dr \cdot U_\infty\, 4a'(1-a)\, r \cdot \Omega$$