3.3

- ROTOR DISC THEORY & WAKE ROTATION

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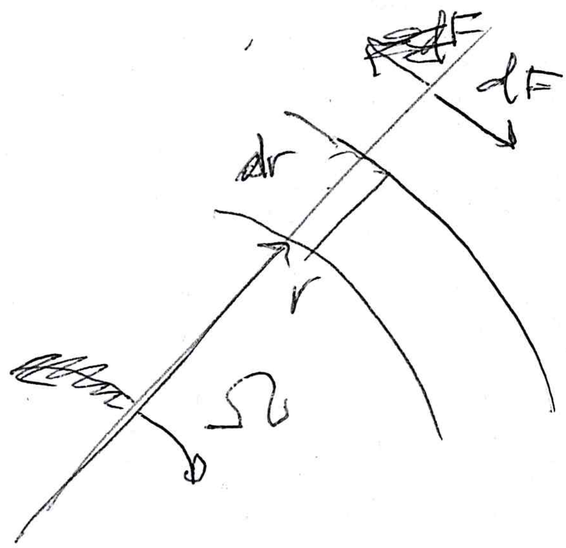
$a(r)$
INDUCTION AS FUNCTION OF r

$$C_p(a) = 4a(1-a)^2$$

REGARD ANNULUS OF WIDTH dr



ENERGY EXTRACTED IN ANNULUS OF AREA

$$dA = 2\pi r \cdot dr$$

$$dP = \frac{1}{2} \rho u_\infty^3 \cdot dA \cdot c_p(a) \qquad (1)$$

THIS ENERGY IS EXTRACTED VIA A ROTATING MOTION WITH TANGENTIAL FORCE dF LEADING TO ~~MOMENT~~ TORQUE $dF \cdot r$ AND WITH ROTATION VELOCITY Ω WE GET

$$dP = dF \cdot r \cdot \Omega \qquad (2)$$

THE FORCE RESULTS IN A <u>TANGENTIAL</u> CHANGE OF MOMENTUM OF THE AIRFLOW [WAKEROTATION !

LET'S CALL THE TANG. VELOCITY AT THE DISC V_1 AND THE ONE IN THE FAR WAKE V_3 (AS BEFORE). UPSTREAM THE TANG. VELOCITY WAS ZERO. THUS, THE TOTAL MOMENTUM CHANGE PER SECOND IS

$$dF = d\dot{m}\,(V_3 - 0) \qquad \text{WITH } d\dot{m} = 2\pi r \cdot dr \cdot \rho U_\infty (1-a) \quad (3)$$

EQUATING (1) & (3) USING (2) YIELDS

$$\frac{1}{2}\rho U_\infty^3 \cdot 2\pi r\, dr \cdot 4a(1-a)^2 = \rho U_\infty (1-a)\, 2\pi r\, dr \cdot V_3 \cdot r \cdot \Omega \cdot$$

WHICH GIVES

$$\frac{U_\infty^2\, 2a(1-a)}{r \cdot \Omega} = V_3$$

FOR CONSTANT $a(r)$, $V_3(r) \propto \frac{1}{r}$!

IT IS CUSTOMARY TO ASSUME THAT $V_1 = \frac{1}{2} V_3$ AND TO INTRODUCE THE TANGENTIAL INDUCTION FACTOR a' SUCH THAT

$$\boxed{V_1 = a' \cdot r \cdot \Omega}$$

OR, EQUIVALENTLY

$$a' = \frac{\frac{1}{2} V_3}{r \cdot \Omega}$$

SUCH THAT

$$a' = a(1-a) \frac{U_\infty^2}{r^2 \Omega^2} = \frac{a(1-a)}{\lambda_r^2}$$

WITH $\lambda_r := \dfrac{r \cdot \Omega}{U_\infty}$ $\left[= \mu \cdot \left(\dfrac{R\Omega}{U_\infty} \right) = \mu \cdot \lambda \right.$
$\left. \mu = \dfrac{r}{R} \right]$

WE CONCLUDE THAT THE WAKE ROTATES MORE IF THE TURBINE MOVES RELATIVELY SLOWER. HIGH λ LEAD TO LESS WAKE ROTATION.