

Tuesday June 5: Roskopf

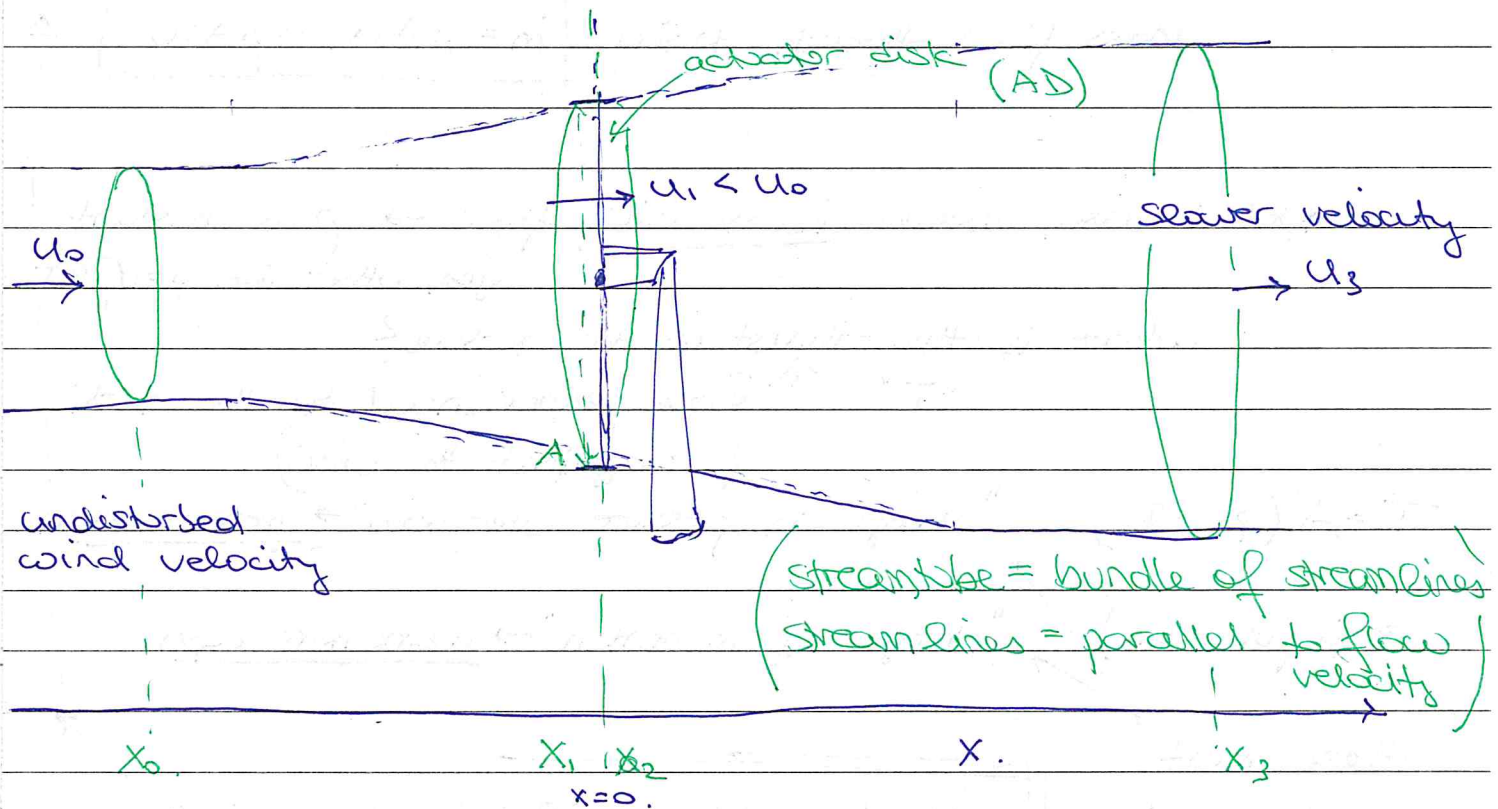
### 3. Aerodynamics of Wind Turbines

#### 3.1 A view on the wake.

#### 3.2 Actuator Disk Model & the Betz Limit

facts: - the wind is slower in and after the wind turbine.

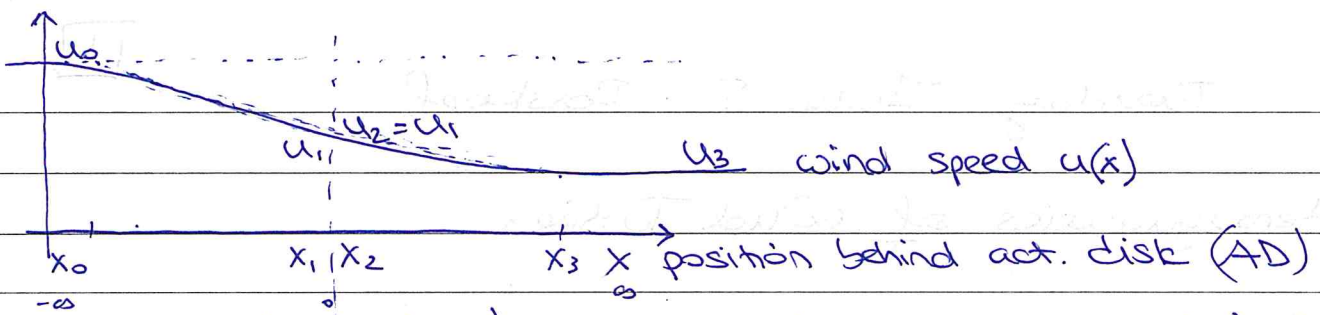
Side view on a wind turbine.



$$P_{\text{wind}} = \frac{1}{2} \rho u_0^3 A$$

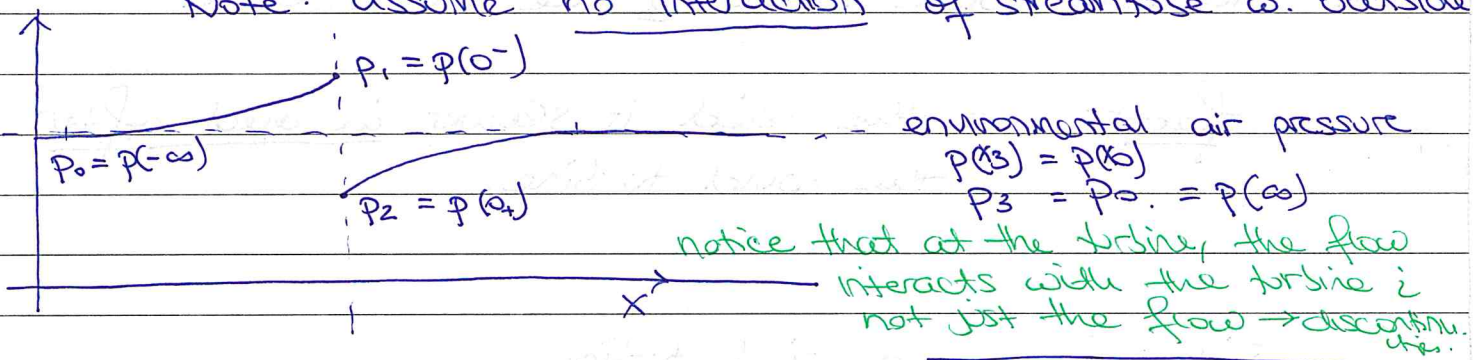
"a first guess" - not achievable

↑ power in the ~~are~~ air that would flow through AD if AD weren't actually there...



$\rightarrow u(-\infty) = u_0, \quad u(0) = u_1 = u_2, \quad u(\infty) = u_3$

Note: assume no interaction of streamtube w. outside



mass flow through turbine:  $\boxed{\dot{m} = \rho A u_1 = \rho A u_2}$

Note: also, assume incompressible flow  $\rightarrow \rho$  is constant.  
yes, otherwise  $p = \sqrt{\rho R T}$ ...

what is the thrust on the turbine?

"T"  $\uparrow$  force experienced by the turbine  
(... against the wind)

$T = A (p_2 - p_1)$  "change in pressure over area"  $\rightarrow$  pressure eqn.

$= \dot{m} (u_0 - u_3)$  "change of momentum"  $\rightarrow$  momentum eqn.

these must be equivalent.

think of: balloon. if the balloon is inflated, the pressure is the same within the balloon except at the skin where the flow interacts w. the solid.

what is the <sup>power</sup> ~~energy~~ extracted by the turbine?  
"P" "P"

$$\frac{P}{\hbar} =$$

$$= m \left( \frac{1}{2} u_0^2 - \frac{1}{2} u_3^2 \right) \quad \text{by change of kinetic energy}$$

the question we want to ask...

- what's  $u_1$  &  $u_3$ ?

- what's  $p_1$  &  $p_2$ ?

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