Wind Energy Systems
Albert-Ludwigs-Universität Freiburg – Summer Semester 2018

Exercise Sheet 4 SOLUTION: Mechanics for Wind Turbine (Continued)

Prof. Dr. Moritz Diehl und Rachel Leuthold

Deadline: midnight before July 4, 2018
`https://goo.gl/forms/1kW0G8jtlIeo7c062`

In this exercise sheet we'll continue our preliminary study of the role of mechanics in wind turbine tower design. Here, we'll look at vibrations, and will use the Rayleigh energy method, and the Campbell diagram.

preliminary tower design (continued) **[*6 pt*]**

1. We would like to make a preliminary design of a wind turbine tower. This tower should support an un-yawed and un-tilted three-bladed wind turbine ('Turbine B'), with the following dimensions:

Table 1: wind turbine dimensions and properties for Turbine B

property	symbol	value
tower height	L	84 m
nacelle + hub mass	m_{nac}	143 tonnes
rotor radius	R	66 m **
design tip speed ratio	λ_{rated}	5
cut-in wind speed	$u_{\text{cut-in}}$	3 m/s
rated wind speed	u_{rated}	12 m/s
cut-out wind speed	$u_{\text{cut-out}}$	25 m/s

** My appologies, that the exercise was previously published with the value of 12 m.

Some other information that you might find useful is as follows:

Table 2: other potentially useful information

property	symbol	value
density of A36 structural steel	ρ_{steel}	$7.8 \cdot 10^3$ kg/m^3
Young's modulus of A36 structural steel	E_{steel}	200 GPa
yield stress of A36 structural steel	U_{steel}	250 MPa
air density	ρ_{air}	1.225 kg/m^3
surface roughness length for low crops w. occasional obstacles	z_0	0.1 m
meterological mast height	z_{Ref}	10 m
approx. drag coefficient for cylinder	C_D	1
typical wind turbine structural safety factor	f_{safety}	1.35

(a) **tower natural frequency** **[*3 pt*]**

Let's use Rayleigh's energy method to estimate the natural frequency of the tower. In this method, we assume that the strain energy from bending perfectly trades off with the kinetic energy of the tower's displacement x. We will again approximate the tower as a cantilevered beam.

Let's assume that the tower's displacement is sinusoidal in time:

$$x(t) = x_0 \sin(\omega t)$$

and that the tower remains approximately straight during its displacement.

Futher, we know that the strain energy from bending can be found as:

$$V = \frac{1}{2} k x^2, \text{ where } k = 3 \frac{E_{\text{steel}} I_x}{L^3}.$$

i. What is $\dot{x}(t)$? *[0.25 pt]*

Let's differentiate:

$$\dot{x}(t) = \frac{\mathrm{d}x}{\mathrm{d}t} = x_0 \omega \cos(\omega t)$$

ii. What is the kinetic energy due to the nacelle displacement T_{nac}? *[0.25 pt]*

We can find the kinetic energy with the standard expression:

$$T_{\mathrm{nac}} = \frac{1}{2} m_{\mathrm{nac}} \dot{x}(t)^2 = \frac{1}{2} m_{\mathrm{nac}} x_0^2 \omega^2 \cos^2(\omega t)$$

iii. What is the kinetic energy due to the displacement of the tower T_{t}? (*Hint: the tower is not massless...*) (*Hint: also, you might assume that the deflection of the tower is roughly proportional to the distance to the fixed point.*) *[0.5 pt]*

Here, we have to integrate:

$$T_{\mathrm{t}} = \int_0^L \frac{1}{2} m' \left(\dot{x}(t) \frac{y}{L}\right)^2 \mathrm{d}y$$

Here, m' is the mass per unit length of the tower: $m' = \rho_{\mathrm{steel}}(\pi)(r^2 - (r-\tau)^2)$. Also, y is the length along the tower.

The term $\left(\dot{x}(t)\frac{y}{L}\right)$ in the integrand is the deflection that is proportional to the distance to the fixed point.

Then, the kinetic energy gives:

$$T_{\mathrm{t}} = \left(\frac{m' \omega^2 y^3 x_0^2 \cos^2(\omega t)}{6L^2}\right)\Bigg]_0^L = \frac{m' L \omega^2 x_0^2 \cos^2(\omega t)}{6L^2}$$

iv. What is the total kinetic energy T of the swaying cantilevered beam? *[0.25 pt]*

Then, the total kinetic energy T is the sum of the tower and nacelle kinetic energies:

$$T = T_{\mathrm{nac}} + T_{\mathrm{t}} = \frac{1}{6}\left(3m_{\mathrm{nac}} + Lm'\right) \omega^2 x_0^2 \cos^2(\omega t)$$

v. What equation can you formulate, that would implicitly define the vibration frequency ω? *[0.5 pt]*

If all of the strain energy from bending gets transformed into kinetic energy, then the strain energy when the tower is at greatest deflection $(\sin(\omega t) = 1)$ must equal the kinetic energy when the tower deflection is smallest $(\cos(\omega t) = 1)$.

That is:

$$V(t = \pi/(2\omega)) = T(t = 0)$$

Alternatively:

$$\begin{aligned} \frac{3}{2}\frac{E_{\mathrm{steel}} I_x}{L^3} x_0^2 &= \frac{1}{6}\left(3m_{\mathrm{nac}} + Lm'\right) \omega^2 x_0^2 \\ \omega^2 &= 9\frac{E_{\mathrm{steel}} I_x}{(3m_{\mathrm{nac}} + Lm')L^3} \end{aligned}$$

vi. Please find ω. *[0.25 pt]*

Starting from above, we get:

$$\omega^2 = \frac{3E_{\text{steel}}I_x}{(m_{\text{nac}} + m_{\text{t}}/3)\,L^3}$$

Where, $m_{\text{t}} = L\rho_{\text{steel}}\pi(r^2 - (r-\tau)^2)$, based on previous definitions...

Then:

$$\omega = \left(\frac{3E_{\text{steel}}I_x}{(m_{\text{nac}} + m_{\text{t}}/3)\,L^3}\right)^{\frac{1}{2}}$$

Notice that negative frequencies are not particularly meaningful, such that only the positive root of ω should be given.

vii. What is the natural frequency f_{nat} of the cantilevered tower? [*0.25 pt*]

We can convert from ω to f_{nat} as:

$$f_{\text{nat}} = \omega/(2\pi)$$

viii. What is the natural frequency of each of the three potential tower designs (defined by r and τ) that you determined in Exericse Sheet 3, Problem 2b)? (*Hint: If you do not have this solution, you can use the following combinations of* (r,τ)*:* $(1.5m, 0.05m)$*,* $(3.0m, 0.015m)$*,* $(5.5m, 0.005m)$*.*)

A. $r = 5.5$ m [*0.25 pt*]

Plugging in the given values, gives: $f_{\text{nat}} \approx 0.6$ Hz.

B. $r = 3.0$ m [*0.25 pt*]

Plugging in the given values, gives: $f_{\text{nat}} \approx 0.4$ Hz.

C. $r = 1.5$ m [*0.25 pt*]

Plugging in the given values, gives: $f_{\text{nat}} \approx 0.2$ Hz.

(b) **Campbell diagram** **[*3 pt*]**

i. With what frequency (1P, 2P, 3P, ...) would you expect the tower to experience the following effects? What is this frequency (in Hertz), as a function of the wind turbine's rotor speed (in RPM)?

A. 'rotor-rotation' effects, such as having unequally dirty blades? [*0.25 pt*]

The load imbalance will occur at the same frequency as the entire rotor rotates. So, this occurs with a frequency of 1P (once periodic). The 1P frequency is equivalent to converting the RPM into Hertz:

$$f_{1\text{P}} = \text{RPM}\frac{1\,\text{min}}{60\text{s}}$$

B. 'blade-passing' effects, such as tower shadow? [*0.25 pt*]

Since $B = 3$ blades pass behind the shadow per rotation, the frequency of such effects is 3P.

$$f_{1\text{P}} = 3f_{1\text{P}} = 3\text{RPM}\frac{1\,\text{min}}{60\text{s}}$$

ii. Make a plot of frequency [Hz] vs rotor speed [RPM] that we will call the Campbell plot. Add the 'rotor-rotation' and 'blade-passing' frequencies into the Campbell plot. Include a 15 percent safety margin to either side of each curve. [*0.25 pt*]

See complete Campbell plot below...

iii. What is the design rotor speed (in RPM) of the wind turbine? [*0.25 pt*]

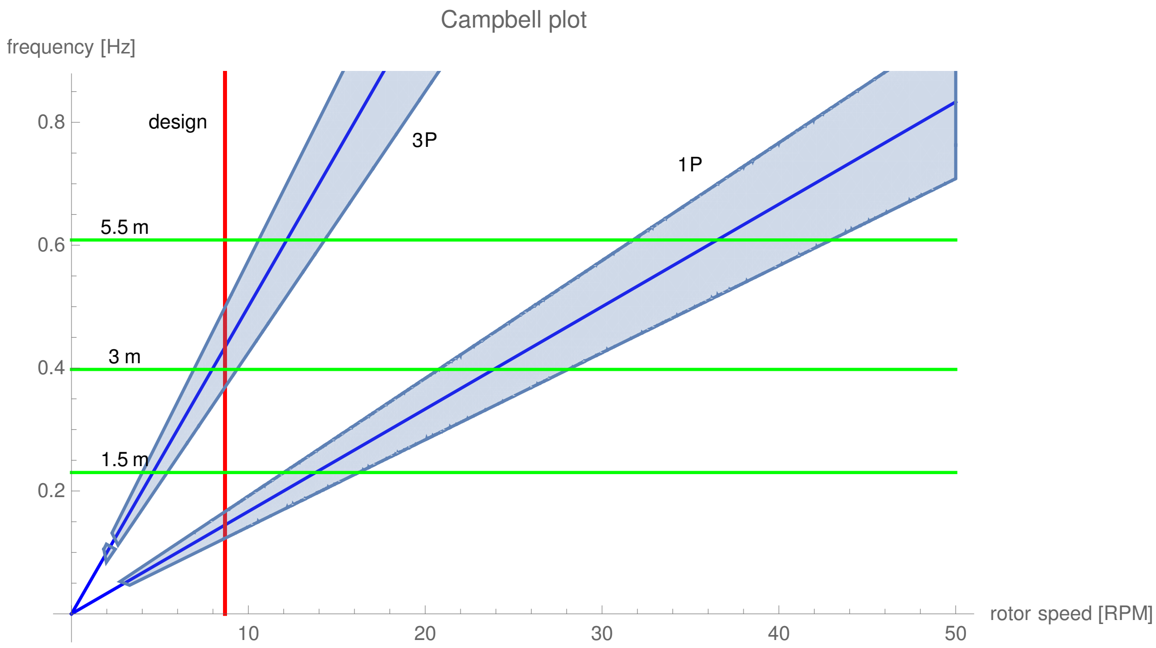
We can find the rotor speed from the design angular velocity Ω_{rated} of the rotor.

$$\text{RPM}_{\text{rated}} = \frac{60\text{s}}{1\,\text{min}} \frac{\Omega_{\text{rated}}}{2\pi}$$

iv. Please show the design rotor speed in the Campbell plot. [*0.25 pt*]

See complete Campbell plot below...

v. Please add the tower natural frequencies corresponding to your three possible tower designs (one for each of the outer diameters 5.5m, 3.0m and 1.5m) into the Campbell plot. [*0.25 pt*]



s

vi. Which of the three investigated tower designs (outer diameters 5.5m, 3.0m, 1.5m) can be classified as the following? Please explain briefly.

A. soft-soft [*0.25 pt*]

None of the investigated diameters gives a tower natural frequency less than the 1P frequency.

B. soft-stiff [*0.25 pt*]

The 1.5m diameter tower has a natural frequency between the 1P and 3P frequency, and is consequently soft-stiff.

C. stiff-stiff [*0.25 pt*]

The 5.5m diameter tower has a natural frequency above the 3P frequency, and is consequently stiff-stiff.

vii. Suggest some considerations you might have when chosing between your three proposed tower designs? [*1 pt*]

The 3.0m diameter tower is eliminated due to the risk of resonance. Then, the 5.5m diameter tower requires less steel overall (see the steel mass plot), but the 1.5m diameter tower would probably be easier to transport (think winding hairpin turns, in the mountains). The trade-off for selection would probably be determined by how easy it is to reach the site location, and how much steel (and machining) costs.