Lecture Energy Systems: Hardware and Control - Control Part
University of Freiburg – Winterterm 2017/2018

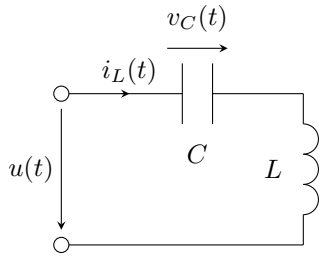
Exercise Sheet 2 with solutions

Prof. Dr. Moritz Diehl, Dr. Gianluca Frison and Benjamin Stickan

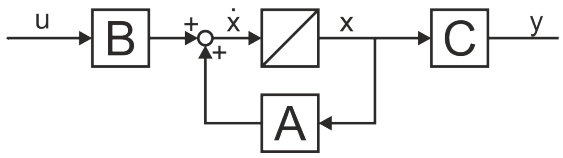
For questions on the exercise please contact Benjamin Stickan (benjamin.stickan@imtek.uni-freiburg.de)

Problem 4: LQR and prefilter

1. The aim of this task is to design an LQR controller for a given CL-oscillator circuit to **track** a voltage reference w at the capacitor. The state vector is defined as $\mathbf{x} := \begin{bmatrix} v_C & i_L \end{bmatrix}^\intercal$ and the system is assumed to be controllable.



(a) Draw the block-diagram of an $(\mathbf{A}, \mathbf{B}, \mathbf{C})$-system and use the provided template files to simulate the system without applying any control ($u = 0$). The initial state-vector is $\mathbf{x}_0 := \begin{bmatrix} 10 & 0 \end{bmatrix}^\intercal$ and the simulation time is 0.2 s. What can you say about the eigenvalues?



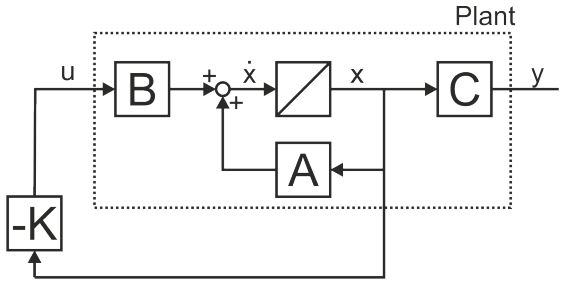
(b) Define weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ such that the perfomance-index reads as

$$J = \frac{1}{2}\int_0^\infty \left(\frac{v_C^2(t)}{[V]^2} + 0.1\frac{i_L^2(t)}{[I]^2} + \frac{u^2(t)}{[V]^2} \right) \mathrm{d}t \tag{1}$$

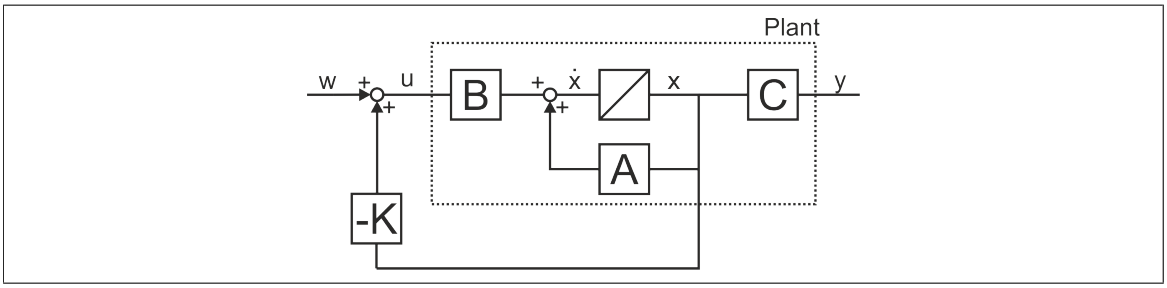
and calculate the feedback-gain $\mathbf{K}$ using the MATLAB function `lqr(A,B,Q,R,[])`.

$$\mathbf{Q} = \begin{bmatrix} \frac{1}{[V]^2} & 0 \\ 0 & \frac{0.1}{[I]^2} \end{bmatrix} \quad \mathbf{R} = \frac{1}{[V]^2}$$

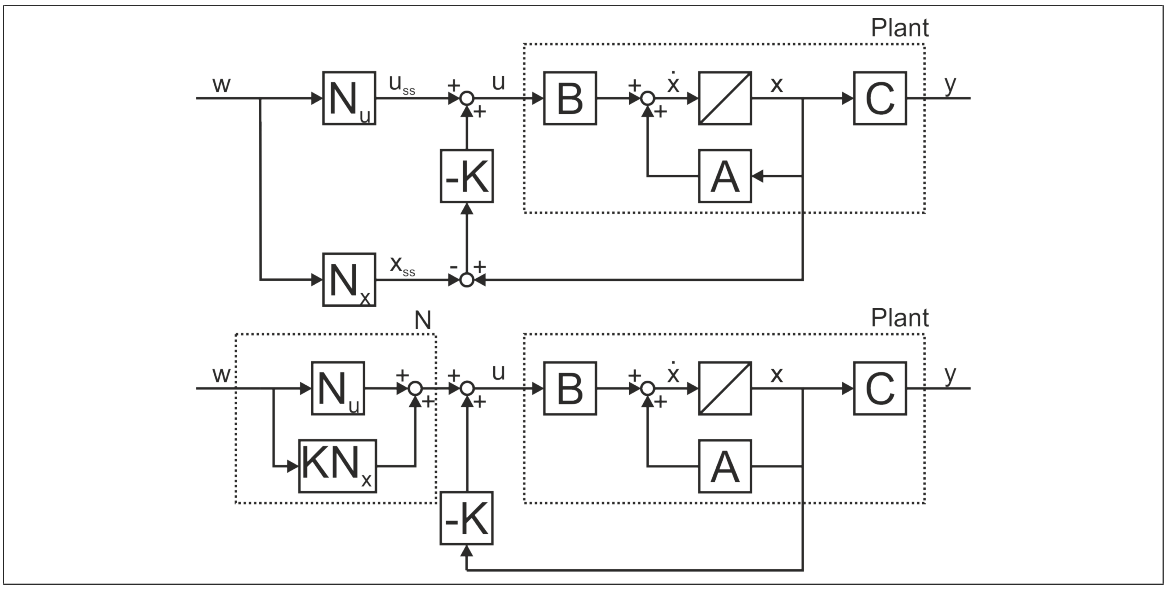
(c) Draw the closed-loop block diagram, write down matrix $\mathbf{A}_{\mathrm{cl}}$ and simulate the system in MATLAB.



(d) Draw the closed-loop block diagram including input w and simulate the system with input $w = 100$. What do you observe for the steady-state value of the capacitor voltage? How can you make the voltage v_C follow the input w?
(Remark: we denote the input w to distinguish from actual control u)



(e) Implement the prefilter $\mathbf{N}$ and simulate the system that can track a referenc w at the capacitor voltage v_C.



(f) What do you observe regarding the control u when you tune the controller to be more agressive ($\mathbf{R}$ small, e.g. $\frac{0.01}{[V]^2}$)?