Modeling and System Identification – Microexam 3

Prof. Dr. Moritz Diehl, IMTEK, Universität Freiburg

February 6, 2017, 8:15-9:15, Freiburg

Surname: Name: Matriculation number:

Study: Studiengang: Bachelor ☐ Master ☐

Please fill in your name above and tick exactly **ONE** box for the right answer of each question below.

1. Given measurement sequences $u(k)$ and $y(k)$, we identify a model by solving the following optimization problem: $\underset{\theta\in\mathbb{R}^2}{\operatorname{argmin}} \frac{1}{\sigma_y^2} ||y(k) - \theta_1 y(k-1) - \theta_2 \widetilde{u}(k-1)||_2^2 + \frac{1}{\sigma_u^2} (u(k) - \widetilde{u}(k))^2$. What model assumptions have we made?

(a) ☐ iid. Gaussian equation errors & input noise	(b) ☐ iid. non-Gaussian noise on inputs and outputs
(c) ☐ iid. non-Gaussian equation errors & output noise	(d) ☒ iid. Gaussian noise on inputs and outputs

1

2. Consider a FIR model with output errors. Which statement is NOT true:

(a) ☐ The Model estimation problem is convex.	(b) ☒ $\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \sum_{k=K+1}^{N} (y_k - h(p, u_k, ...y_{k-1}, ...))^2$
(c) ☐ A good fit may require a large FIR model order K.	(d) ☐ $\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \sum_{k=K+1}^{N} (y_k - (u_k, u_{k-1}, ..., u_{k-K})\,\theta)^2$

1

3. Consider an equation error model. Which statement is NOT true:

(a) ☐ A LIP model has a convex estimation problem.	(b) ☐ The problem might not be analytically solvable.
(c) ☒ Noise is iid. and enters the model additively.	(d) ☐ Estimation is by minimizing the prediction errors.

1

4. Consider the discrete LTI system $y_{k+1} = \theta_1 y_k + \theta_2 u_k + \theta_3 u_{k-1} + \epsilon_k$ with scalar input u, output$y$ and noise ϵ. What of the following abbreviations best describes this system?

(a) ☐ AR	(b) ☒ ARX	(c) ☐ ARMA	(d) ☐ FIR

1

5. Consider the discrete LTI system $y_{k+1} = \theta_1 y_k + \theta_2 \epsilon_{k+1} + \theta_3 \epsilon_k$ with scalar input u and noise ϵ. What of the following abbreviations best describes this system?

(a) ☐ ARX	(b) ☒ ARMA	(c) ☐ AR	(d) ☐ FIR

1

6. Consider the one-step ahead prediction model $\boxed{y_k = \theta_1 y_{k-1} + \theta_2 y_{k-2}^2 + \epsilon_k}$ with unknown parameter vector $\theta = (\theta_1, \theta_2)^T$ and iid. Gaussian noise ϵ_k $\mathcal{N}(0, \sigma_\epsilon^2)$. Given a sequence of N scalar input and output measurements $u_1, \ldots, u_N$ and $y_1, \ldots, y_N$, we want to compute the linear least squares (LLS) estimate $\hat{\theta}$ by minimizing a function $f(\theta) = ||y - \Phi\theta||_2^2$. How do we need to choose the matrix Φ and vector y?

(a) ☐ $y = \begin{bmatrix} y_3 \\ \ldots \\ y_N \end{bmatrix}$, $\Phi = \begin{bmatrix} y_2 & y_1 \\ \ldots & \ldots \\ y_{N-1} & y_{N-2} \end{bmatrix}$	(b) ☐ $y = \begin{bmatrix} y_1 \\ \ldots \\ y_{N-2} \end{bmatrix}$, $\Phi = \begin{bmatrix} y_2^2 & y_3 \\ \ldots & \ldots \\ y_{N-1}^2 & y_N \end{bmatrix}$	(c) ☐ $y = \begin{bmatrix} y_3 \\ \ldots \\ y_N \end{bmatrix}$, $\Phi = \begin{bmatrix} y_2^2 & y_1 \\ \ldots & \ldots \\ y_{N-1}^2 & y_{N-2} \end{bmatrix}$	(d) ☒ $y = \begin{bmatrix} y_3 \\ \ldots \\ y_N \end{bmatrix}$, $\Phi = \begin{bmatrix} y_2 & y_1^2 \\ \ldots & \ldots \\ y_{N-1} & y_{N-2}^2 \end{bmatrix}$

1

Points on page (max. 6)

7. For a system that is known to be unstable, which type of model is most appropriate?

(a) [ ] Equation-Error	(b) [ ] Output-Error	(c) [x] Input-Output-Error	(d) [ ] LIP, additive noise

1

8. For which type of system can a global minimum to the estimation problem be guaranteed?

(a) [ ] Equation-Error	(b) [x] LIP, additive noise	(c) [ ] Input-Output-Error	(d) [ ] Output-Error

1

9. Which expression relates the Kalman Filter P to the Recursive Least Squares Q, for a known deterministic linear system defined as $x_i = A_{i-1}x_{i-1}$:

(a) [x] $P[k\|m] = A_{k-1}...A_0 Q_m^{-1} A_0^\top ...A_{k-1}^\top$	(b) [ ] $P[m\|k] = A_{k-1}...A_0 Q_m^{-1} A_0^\top ...A_{k-1}^\top$
(c) [ ] $P[k\|k] = P[k\|k-1]^{-1} + Q_{k-1}^T Q_{k-1}$	(d) [ ] $P[m\|k] = A_k...A_1 Q_m^{-1} A_1^\top ...A_k^\top$

1

10. Which statement concerning the set-up of the Kalman Filter is NOT, typically, true:

(a) [ ] Measurement noise effects decrease progressively.	(b) [x] The larger P, the smaller the "innovation".
(c) [ ] The more trustworthy the model, the smaller P.	(d) [ ] Process noise effects increase progressively.

1

11. What does the expression $Q_k^{-1}\phi_k(y_k - \phi_k^\top \hat{\theta}_{k-1})$ mean in the context of Recursive Least Squares?
The "innovation update", aka, the part you add to the old "best-guess" to get the new "best-guess"

1

12. Consider a linear system defined as $x_i = A_{i-1}x_{i-1} + b_i + w_i$ with a linear measurement equation $y_i = C_i x_i + v_i$. If $x_i \in \mathbb{R}^n$ and $y_i \in \mathbb{R}^m$, what is NOT true about the covariance of the process noise W_i and measurement noise V_i of the Kalman Filter?

(a) [x] W_i and V_i are positive semi-definite.	(b) [ ] W_i has n non-zero singular values.
(c) [ ] W_i and V_i are diagonal matrices.	(d) [ ] $W_i \in \mathbb{R}^{n\times n}$, and $V_i \in \mathbb{R}^{m\times m}$.

1

13. Given the continuous time transfer function G(s), what quantity does the magnitude plot of the Bode diagram show on its y-axis?

(a) [ ] $\|\|G(j\omega)\|\|$	(b) [ ] $G(j\omega)^2$	(c) [ ] $\log\|G(j\omega)\|$	(d) [x] $\|G(j\omega)\|$

1

14. Given the continuous time transfer function G(s), what quantity does the phase plot of the Bode diagram show in a single logarithmic scale?

(a) [ ] $cos(G(j\omega))$	(b) [ ] $\log(\arg G(j\omega))$	(c) [x] $\arg G(j\omega)$	(d) [ ] $\arg \|G(j\omega)\|$

1

15. What is the output of an LTI system that is excited with a sinusoidal input $u(t) = U_0 \cdot \sin(\omega \cdot t)$? $y(t) = \ldots$

(a) [ ] $\frac{U_0}{\|G(j\omega)\|} \cdot \sin(\omega \cdot t + \alpha$	(b) [ ] $\|G(j\omega)\|U_0 \cdot \sin(\omega \cdot t) + \alpha$
(c) [x] $\|G(j\omega)\|U_0 \cdot \sin(\alpha \cdot t + \omega)$	(d) [ ] $\|G(j\omega)\|U_0 \cdot \sin(\omega \cdot t + \alpha)$

1

16. A system is excited with the same sine wave as in the previous question. The magnitude Y_0 and the phase shift α of the output of the system are recorded. How can you compute the transfer function $G(j\omega)$ for the specific frequency ω? $G(j\omega) = \ldots$

(a) [ ] $\frac{Y_0}{U_0}e^{j\omega}$	(b) [x] $\frac{Y_0}{U_0}e^{j\alpha}$	(c) [ ] $\frac{U_0}{Y_0}e^{j\omega}$	(d) [ ] $\frac{U_0}{Y_0}e^{j\alpha}$

1

17. Regard a periodic signal with base frequency f_{base} that is sampled every $\Delta t = 1s$ (with f_{base} a multiple of $1/\Delta t$). How many

Points on page (max. 10)

different frequencies are contained in the discretized signal?

(a) ☐ $(2\Delta t f_{base})/1$	(b) ☐ $2\Delta t/f_{base}$	(c) ☒ $1/(2\Delta t f_{base})$	(d) ☐ $f_{base}/2\Delta t$

1	

18. At which frequency f [Hz] is the resonance peak of the following transfer function: $G(s) = \frac{a^2}{s^2+2as+a^2}$, with $a \in \mathbb{R}$? ...

(a) ☐ $f = \frac{a}{j2\pi}$	(b) ☐ $f = \frac{j2\pi}{a}$	(c) ☒ $f = \frac{ja}{2\pi}$	(d) ☐ $f = \frac{-ja}{2\pi}$

1	

19. Which slope has the Bode amplitude diagram of a system described by the following transfer function: $G(s) = \frac{1}{s^2+2s+1}$ for high frequencies f [Hz]?

(a) ☐ -10 dB/decade	(b) ☒ -20 dB/decade	(c) ☐ 20 dB/decade	(d) ☐ 0 dB/decade

1	

20. A continuous function in the time domain $u_c(t)$ is sampled with a frequency f_s, resulting in a set of discrete values $u = [u(0), \ldots, u(N-1)]^\top$. Applying the DFT to u yields $U = [8, 2, 0, 8, 0, 8, 0, 2]^\top$. Reconstruct the function $u_c(t)$ from U.
DFT: $U(m) = \sum_{k=0}^{N-1} u(k) e^{i\frac{-2\pi mk}{N}}$, for $m = 0, \ldots, N-1$
IDFT: $u(n) = \frac{1}{N}\sum_{k=0}^{N-1} U(k) e^{i\frac{2\pi kn}{N}}$, for $n = 0, \ldots, N-1$

(a) ☒ $u_c(t) = 1 + \frac{1}{2}\cos(\frac{\pi f_s t}{4}) + 2\cos(\frac{3\pi f_s t}{4})$	(b) ☐ $u_c(t) = 1 + \frac{1}{2}\cos(\frac{\pi f_s t}{8}) + 2\cos(\frac{\pi f_s t}{4})$
(c) ☐ $u_c(t) = 2 + \cos(\frac{\pi f_s t}{4}) + 2\cos(\pi f_s t)$	(d) ☐ $u_c(t) = 2 + \cos(\frac{\pi f_s t}{4}) + 2\cos(\frac{3\pi f_s t}{4})$

1	

21. You want to do multisine excitation to identify the transfer function of an unknown system. How should the multisine be designed, to prevent Aliasing errors?
The multisine should not contain any frequency higher than the Nyquist frequency $f_{Nyquist} = \frac{1}{\Delta t}$.

1	

22. You want to do multisine excitation to identify the transfer function of an unknown system. How should the multisine be designed, to prevent Leakage errors?
The multisine should only contain frequencies which are multiples of the base frequency $f_{base} = 1/T$, with the window length $T = N \cdot \Delta t$.

1	

23. What is the frequency resolution of the DFT of a given periodic signal with period T, sampling frequency f_s, and the number of samples per period N?

(a) ☒ $\omega = \frac{2\pi f_s}{N}$	(b) ☐ $\omega = \frac{2\pi}{N \cdot T}$	(c) ☐ $\omega = \frac{2\pi f_s}{T}$	(d) ☐ $\omega = 2\pi f_s$

1	

24. You identify an LTI system with periodic multisine excitations, where each window has length T and the total duration of your experiment is MT with a large integer M. Which procedure should you follow to identify the transfer function $\hat{G}(j\omega_k)$ at a given frequency $\omega_k = \frac{2\pi k}{T}$?

(a) ☒ compute the DFTs of each window, then average the DFTs and then build the quotient of the average	(b) ☐ build the quotients of the M windows, average the quotients and apply the DFT on the quotients
(c) ☐ compute the DFTs of each window, build the DFT quotients and then average the quotients	(d) ☐ average the M windows, build the quotient of the average and apply DFT

1	

25. Which expression describes the Kalman filter Innovation Step of the state?

Points on page (max. 8)	

(a) ☐ $\hat{x}_{[k\|k-1]} = \hat{x}_{[k\|k-1]} + P_{[k\|k]} \cdot C_k^\top V^{-1}(y_k - C_k \hat{x}_{[k\|k-1]})$	(b) ☐ $\hat{x}_{[k\|k]} = \hat{x}_{[k-1\|k]} + P_{[k-1\|k]} \cdot C_{k-1}^\top V^{-1}(y_{k-1} - C_{k-1}\hat{x}_{[k-1\|k})$
(c) ☒ $\hat{x}_{[k\|k]} = \hat{x}_{[k\|k-1]} + P_{[k\|k]} \cdot C_k^\top V^{-1}(y_k - C_k \hat{x}_{[k\|k-1]})$	(d) ☐ $\hat{x}_{[k\|k]} = \hat{x}_{[k\|k-1]} + P_{[k\|k]} \cdot C_k^\top V(y_k - C_k \hat{x}_{[k\|k-1]})$

1	

26. What signal-to-noise-ratio (SNR) at a certain frequency f_0 of a measured signal do you want to achieve, in order to be able to estimate the amplitude of this frequency component with an accuracy of 10%?

(a) ☒ 20dB	(b) ☐ 40dB	(c) ☐ 1dB	(d) ☐ 10dB

1	

27. Which are the apropriate matrices of the state space model of a system described by the following ODE: $\ddot{a} = c_1\dot{a} + c_2 a + \dot{s}$ $\ddot{s} = c_3\dot{a} + c_4\dot{s} + s$ and the output $y = c_5 s + c_6 \dot{a}$, with $c_1 \ldots c_6 \in \mathbb{R}$.

(a) ☒ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} 0 & c_6 & c_5 & 0 \end{bmatrix}$	(b) ☐ $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \\ 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$
(c) ☐ $A = \begin{bmatrix} c_2 & c_1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} 0 & c_6 & c_5 & 0 \end{bmatrix}$	(d) ☐ $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ c_2 & c_1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & c_3 & 1 & c_4 \end{bmatrix}, C = \begin{bmatrix} c_5 & 0 & 0 & c_6 \end{bmatrix}$

1	

Points on page (max. 3)	