

Exercise 10: Inequality Constrained Optimization

(to be completed during exercise session on Jan 20, 2016 or sent by email to dimitris.kouzoupis@imtek.uni-freiburg.de before Jan 22, 2016)

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Aim of this last (and optional) exercise sheet is to teach you the concepts of inequality constrained optimization using CasADi, a powerful optimization tool that might be useful for your projects.

Exercise Tasks

1. **Hanging chain, the last episode:** Recall the optimization problem of the hanging chain with non-convex inequality constraints and without considering a rest length:

$$\underset{y,z}{\text{minimize}} \quad \frac{1}{2} \sum_{i=1}^{N-1} D \left((y_i - y_{i+1})^2 + (z_i - z_{i+1})^2 \right) + g_0 \sum_{i=1}^N m z_i \quad (1a)$$

$$\text{subject to} \quad (y_1, z_1) = (-2, 1) \quad (1b)$$

$$(y_N, z_N) = (2, 1) \quad (1c)$$

$$z_i \geq -y_i^2, \quad \text{for } i = 1, \dots, N. \quad (1d)$$

- (a) Download the latest CasADi binaries (currently v3.1.1) for your operating system from:

<http://install.casadi.org>

Unzip the folder and add it to your MATLAB path.

(0 bonus points)

- (b) Solve the optimization problem (1) with CasADi and the NLP solver IPOPT (included already in the CasADi binaries). For that purpose, complete the template file `main.m` to build the objective and constraints as CasADi *expressions*. Familiarize with the output of the NLP solver, did the algorithm converge? How many iterations were required?

(2 bonus points)

- (c) Identify the active set at the solution. Does LICQ hold? Are there any weakly active constraints? *Hint: For the latter you will need to check both constraint values and multipliers.*

(2 bonus points)

- (d) Evaluate the Hessian of the Lagrangian at the solution. You may first build a CasADi expression for the Lagrangian that you then differentiate twice with respect to the primal variables. *Hint: The linear constraints can be neglected as they do not contribute to the Hessian matrix.*

(2 bonus points)

- (e) Form the Jacobian of the active constraints at the solution and calculate its null-space using a QR decomposition. Use the result to calculate the reduced Hessian. Is the reduced Hessian positive definite? What does this mean?

(2 bonus points)

This sheet gives in total 8 bonus points.