Exercises for Lecture Course on Modelling and System Identification (MSI)
Albert-Ludwigs-Universität Freiburg – Winter Term 2015

# Exercise 9: The Kalman Filter

**(to be returned on Feb 9, 2016, 8:15 in HS 26, or before in building 102, 1st floor, 'Anbau')**

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Your MATLAB solution has to run from a main script called `main.m`. When running this script all the necessary results and plots should be clearly visible. Please send it to `fabiangirrbach@gmail.com`. State also your name and the names of your team members in the e-mail.

## Exercise Task

In this exercise, you will implement your own Kalman Filter from scratch. The measuring setup will be as follows: consider $N+1$ connected compartments, with permeable walls, as in the figure below.

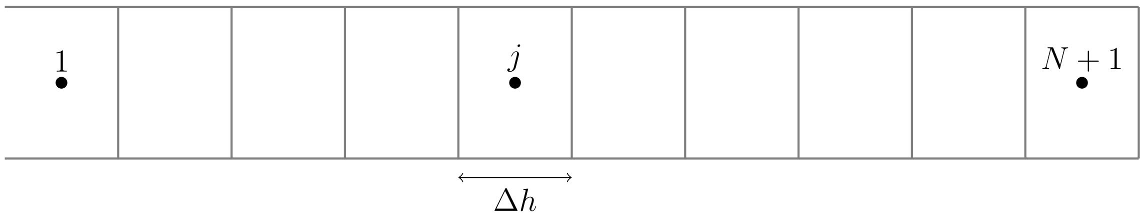


Figure 1: The measurement setup for this exercise. Compartment $j$ is connected to compartments $j-1$ and $j+1$.

Each compartment has a temperature $T_j[°\mathrm{C}],\ j = 1, \ldots, N+1$, where $T_{N+1} = 20\,°\mathrm{C}$. The continuous-time dynamics of the temperature in a compartment is given by

$$\dot{T}_j = \frac{k(T_{j+1} + T_{j-1} - 2T_j)}{(\Delta h)^2}, \quad j = 2, \ldots, N, \tag{1}$$

with $k = 0.025$ being some constant coefficient. The only temperature we measure is the one of compartment $N/2$:

$$y = T_{N/2} + v, \tag{2}$$

with $v \sim \mathcal{N}(0, 0.1)$ being the unknown measurement error. Each compartment is of width $\Delta h = 0.5\,\mathrm{m}$. The temperature in the first compartment varies between 12°C and 23°C and is modelled as a random walk defined by

$$T_{1,t+1} = T_{1,t} + w,\ w \sim \mathcal{N}(0, 2.5) \tag{3}$$

1. Which are the conditions on the model and noise for using the Kalman filter for recursive state estimation purposes? Are these conditions met for this particular problem setup?
   (1 point)

2. Derive on paper the complete vector and matrix expressions for this problem in a Kalman filter framework. In particular give the state vector $x$, the process model matrix $A$ and the measurement model matrix $C$ as a general expression. (2 points)

3. Choose an appropriate expression for the process noise matrix $W$ and the measurement noise matrix $V$. Take into account that the model is not perfect and that the measurement error is small. Justify in one sentence your answer. (2 points)

4. Implement the function for the prediction step using the following prototype:
`[x_pre, P_pre]=predict(x_k,P_k,A_k,W_k).` (1 point)

5. Implement the function for the innovation update step using the following prototype:
`[x_upd, P_upd]=update(y_k,x_k,P_k,C_k,V_k).` (1 point)

6. Download the dataset provided on the course website which contains the temperature measurements in room $N/2$. Use this dataset with a sampling time $\Delta t = 5\text{s}$ to develop a Kalman filter based estimation of the temperatures in the single compartments. Initialize $x_0$ and $P_0$ to some reasonable values corresponding to the knowledge of the setup and the average ambient temperature being $20°\text{C}$. Plot the temperature of compartment $1$ and $N/2$ over time using the `subplot` command. (3 points)

7. Plot how the uncertainty of each compartment's temperature estimate evolves with each filter iteration in a single plot (x-axis: compartments, y-axis uncertainty using `semilogy`). Is the result as expected? (1 point)

8. If the conditions in task 1 are violated, how can you still use the Kalman filter framework for recursive state estimation problems? What are the main extensions to the original Kalman filter? (1 point)

*Note: It is important that you address all questions in words, either in the code as comments or on a seperate paper.*

*This sheet gives in total 12 points*