

# CH 12 MOVING HORIZON ESTIMATION (MHE)

AIM: GIVEN MEASUREMENTS  $y_0, \dots, y_{N-1}$   
ESTIMATE STATES  $x_0, \dots, x_N$

$$x_{k+1} = f_k(x_k, w_k) \quad k=0, \dots, N-1$$

$$y_k = g_k(x_k, w_k) + v_k$$

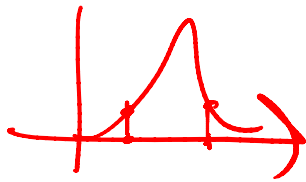
ASS:  $f_k, g_k$  KNOWN

$w_k$ : ~~NOISE~~ DISTURBANCES  
 $v_k$ : MEAS. NOISE

+ A PRIORI:  $v_k, w_k$  UNCERTAIN, BUT STOCHASTIC  
 $x_0$  KNOWN STOCHASTIC PROPERTIES KNOWN

$v_0, \dots, v_{N-1}, w_0, \dots, w_{N-1}, x_0$   
INDEPENDENT

$P(v_n) \equiv$  PDF OF RANDOM VAR  $v_n$   
TO TAKE VALUE  $v_n$



$p_{v_n}(z)$

$$\int P(v) dv = 1$$

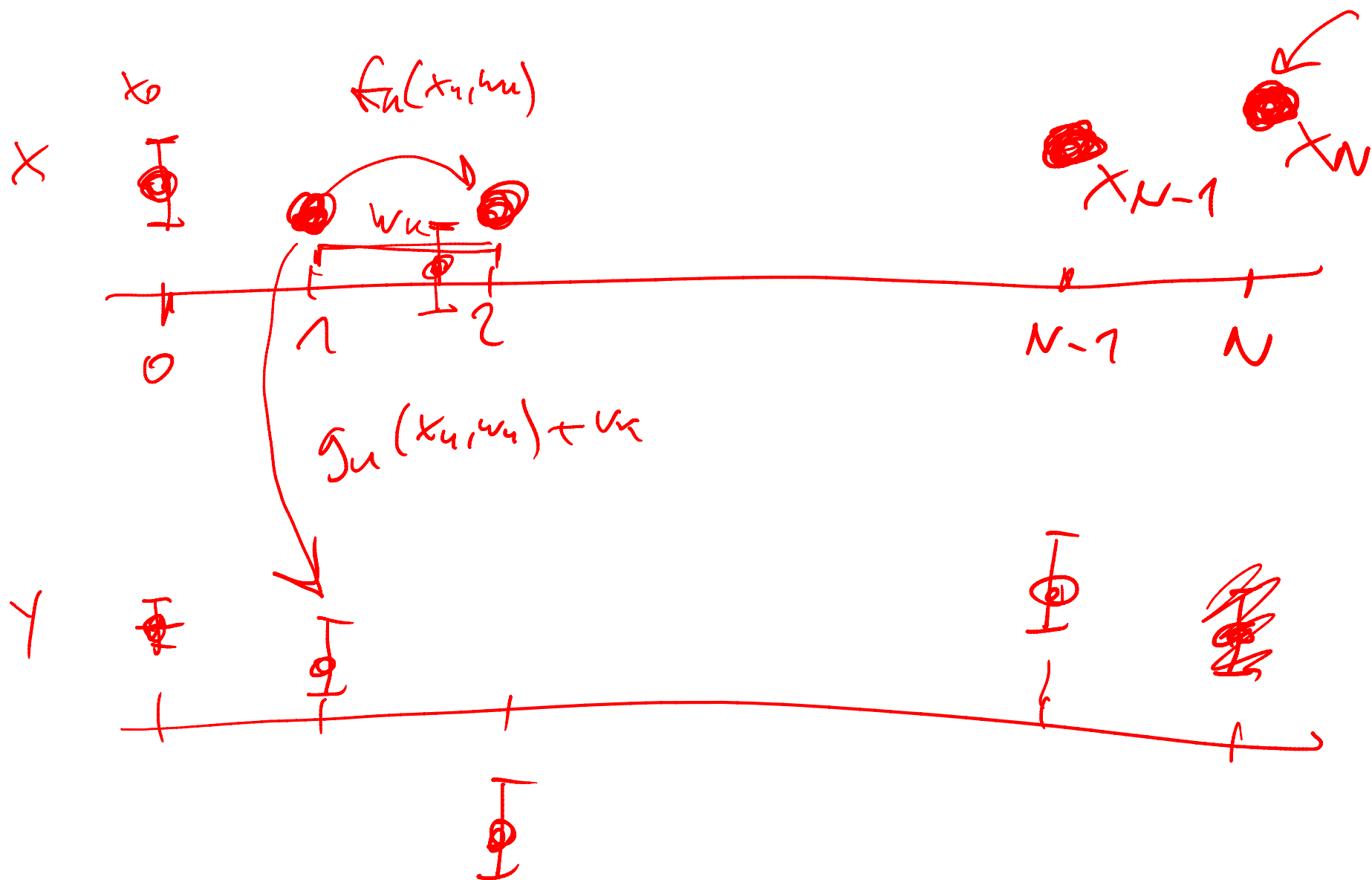
$p_{v_n}(\bar{v})$

$P(w_n)$

$P(\bar{v})$

$\beta_n(\bar{v}) =$

|                                                                                                                                                                                      |                         |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|
| $P(v_n) = \exp(-\Phi_n(v_n)) \cdot \text{CONST}$<br>$\uparrow$ "PENALTY"<br>$P(w_n) = \exp(-\beta_n(w_n)) \cdot \text{CONST}$<br>$P(x_0) = \exp(-\alpha_0(x_0)) \cdot \text{CONST.}$ | $\Phi_n = -\log P(v_n)$ |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------|



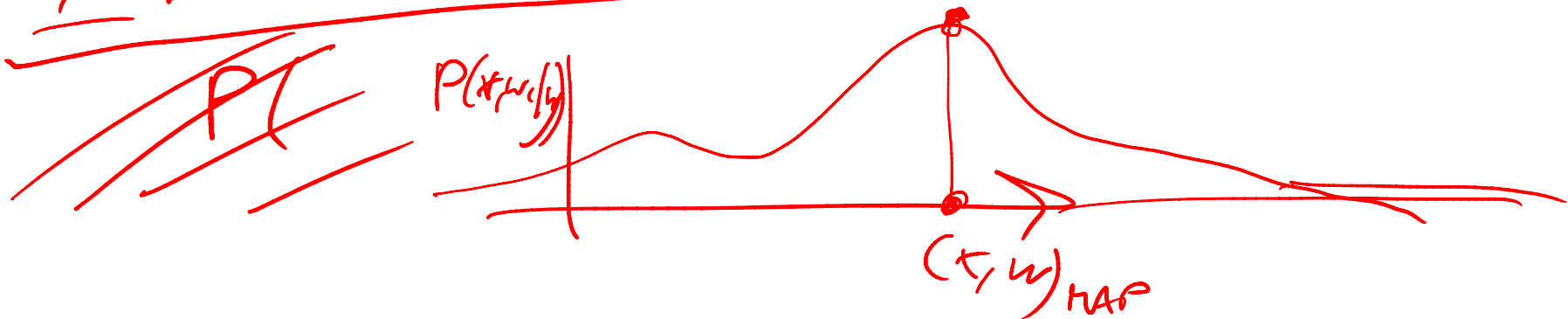
### 12.1.1 GENERALITY $\rightarrow$ PLEASE READ

## 12.2 TRAJECTORY ESTIMATION PROBLEM

BAYESIAN FRAMEWORK: GIVEN  $\gamma = (\gamma_0, \dots, \gamma_{n-1})$   
FIND  $(x_0, \dots, x_n), (w_0, \dots, w_{n-1})$

THAT MAXIMIZE A POSTERIORI PROB  
LIKELIHOOD  
(MAP)

$$\underset{x, w}{\text{argmax}} \quad \underline{\underline{P(x, w | \gamma) = \binom{x}{w} \pi_{AP}}}$$



# ~~MINI~~ COURSE ON PROBABILITY ~~MICRO~~ NANO

$P(A)$  PROB. OF EVENT A

$P(A \cap B)$  PROB TO GET BOTH A & B

$P(A, B)$

$P(A|B)$  PROB. OF EVENT A

GIVEN B

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

SAME FOR PDF



$$P(x, v | y) = \frac{P(y | x, w) P(x, w)}{P(y)} \leftarrow \text{constant}$$

(B)
(A)

$$(A) P(x, w) = \begin{cases} 0 & \text{IF NOT } x_{u+1} = f_u(x_u, w_u) \forall u=0, \dots, n-1 \\ P(x_0) \cdot P(w_0) \cdot P(w_1) \dots P(w_{n-1}) & \text{ELSE} \end{cases}$$

$$-\log P(x, w) = \begin{cases} \infty & \text{IF NOT } x_{u+1} = f_u \dots \\ \alpha_0(x_0) + \sum_{u=0}^{n-1} \beta_u(w_u) \end{cases}$$

---


$$(B) P(y | x, w) = \prod_{u=0}^{n-1} P(y_u | x_u, w_u) = \prod_{u=0}^{n-1} P(w_u)$$

$$-\log P(y | x, w) = \sum_{u=0}^{n-1} \Phi_u(y_u - \mathcal{J}_u(x_u, w_u))$$

$$\arg \max_{x, u} P(x, u | y) = \arg \max_{x, u} P(y | x, u) \cdot P(x, u)$$

$$= \arg \min_{x, u} -\log P(y | x, u) \\ -\log P(x, u)$$

$$= \arg \min_{x, u} \alpha_0(x_0) + \sum_{n=0}^{N-1} \underbrace{\left[ \beta_n(w_n) + \phi_n(x_n - g_n(x_n, w_n)) \right]}_{\substack{k=0, \dots, N-1 \\ \phi_k(x_k, w_k)}}$$

$$\text{s.t. } x_{n+1} = f_n(x_n, u_n)$$

$$k=0, \dots, N-1$$

AN OCP!

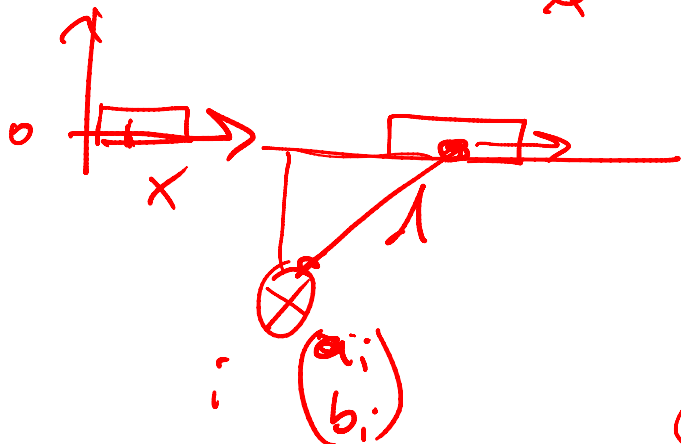
$$\phi_k(x_k, w_k)$$

$$\begin{aligned}
 & \min_{x_0, \dots, x_N, w_0, \dots, w_{N-1}} \quad \alpha_0(t_0) + \sum_{u=0}^{N-1} \varphi_u(x_u, w_u) \\
 & \text{s.t.} \quad x_{u+1} = f_u(x_u, w_u), \quad u=0, \dots, N-1
 \end{aligned}$$

PARTIALLY SEPARABLE LAGRANGIAN ETC.

NO TERMINAL COST, BUT "ARRIVAL COST"  $\alpha_0(t_0)$

Q: 1D-ROBOT (TRAIN): MEASUREMENTS AT  $t_n$ :  
DISTANCE TO  $n$  LANDMARKS,



$x_n$  POSITION AT  $t_n$

$s_n$  SPEED COMMAND BETWEEN

$$\begin{aligned}
 & x_{n+1} = x_n + (s_n + w_n)(t_{n+1} - t_n) \\
 & y_n^{[i]} = \sqrt{(x_n - a_i)^2 + b_i^2} + v_n
 \end{aligned}$$

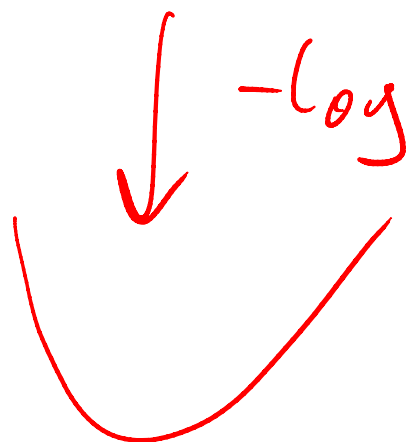
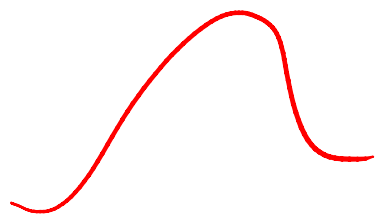
$[t_n, t_{n+1}]$   
 $\phi = ?$   
 $\beta = ?$



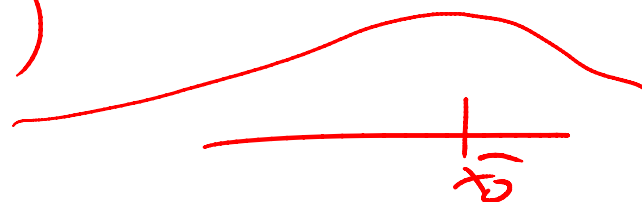
$$p(v_u) \sim N(0, \sigma^2)$$

$$P(v_u) = \text{const} \cdot e^{-\frac{(v_u)^2}{2\sigma^2}}$$

$$\begin{aligned} \Phi_u(v_u) &= -\log P(v_u) \\ &= \frac{v_u^2}{2\sigma^2} + \text{const.} \end{aligned}$$



- 1  $w_u \sim N(0, \sigma_w^2)$
- 2  $v_u \sim N(0, \sigma_v^2)$
- 3  $x_o \sim N(\bar{x}_o, \sigma_{x_o}^2)$

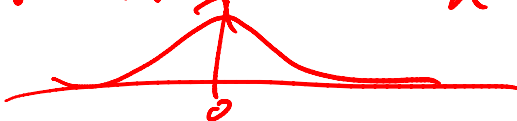


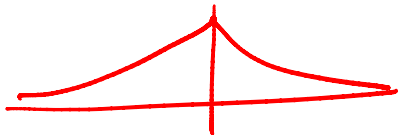
- 1  $\beta_u(w_u) = \frac{w_u^2}{2\sigma_w^2}$
- 2  $\Phi_u(v_u) = \frac{v_u^2}{2\sigma_v^2}$
- 3  $\alpha_o(x_o) = \frac{(x_o - \bar{x}_o)^2}{2\sigma_{x_o}^2}$

$$\min_{x, v} \frac{(x_0 - \bar{x}_0)^2}{2\sigma_x^2} + \sum_{n=0}^{N-1} \left[ \frac{v_n^2}{2\sigma_w^2} + \sum_{i=1}^M \frac{\left( y_n^{(i)} - \sqrt{(x_n - a_i)^2 + b_i^2} \right)^2}{2\sigma_v^2} \right]$$

$$\text{s.t.} \quad x_{n+1} = x_n + (S_n + v_n)(t_{n+1} - t_n) \quad n=0, \dots, N-1$$

EXAMPLES FOR STAGE COSTS CH12.2.1

a) GAUSSIANS :  $v_n \sim \mathcal{N}(0, \Sigma_v) \rightarrow \Phi_n(v_n) = v_n^T \Sigma_v^{-1} v_n$   
 WITH  $\Sigma_v \succ 0$

b) LAPLACE-DISTRIB.  
  $p(v_n) = e^{-\frac{|v_n|}{\sigma}} \rightarrow \Phi_n(v_n) = \frac{|v_n|}{\sigma}$

c) HUBER-PENALTY (GAUSSIAN WITH FAT TAILS)  
  $\rightarrow$  LATENT