ANNOUNCE: NUMERICAL OPTIMIZATION
WS 2015/16

Tu, Fr 14–16, Albertstr.

We 16: GKA 101

TUTOR: DIMITRIS

MATH: 9 ECTS	ENGINEERS: 6 ECTS
(6+3 ECTS)	

GIVEN

$$\boxed{\min_{x,u} \int_0^T L(x,u)\,dt \quad \text{s.t. } \dot{x} = f(x,u)}$$

NLP by Dir. Collocation

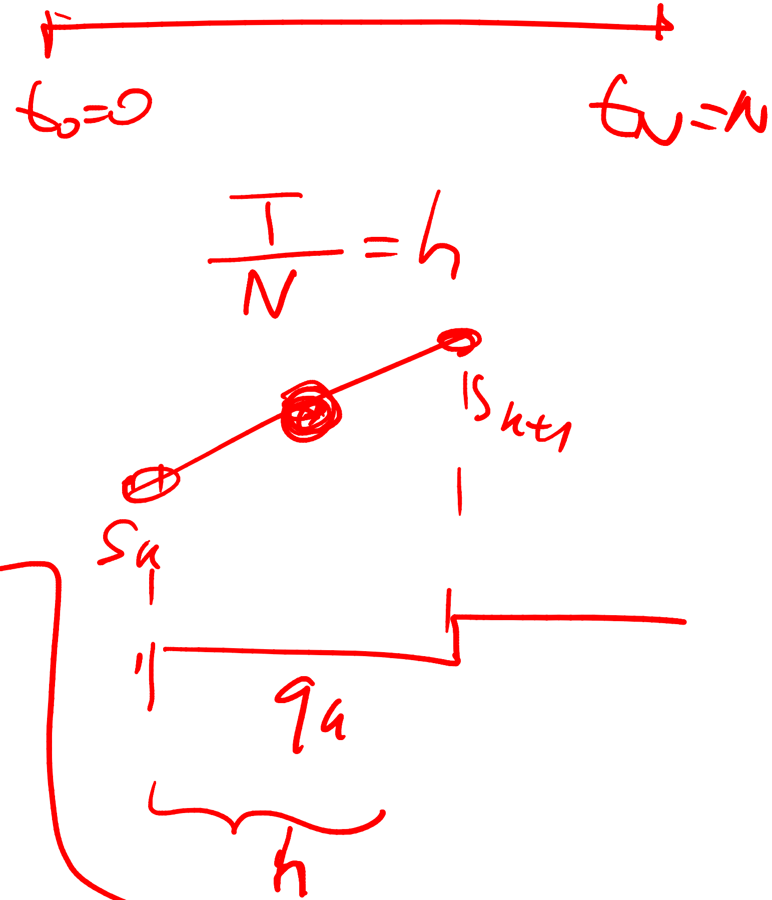
MIDPOINT-RULE

(GL2)

$$\min_{s,q} \sum_{k=0}^{N-1} L\left(\frac{s_k+s_{k+1}}{2}, q_k\right)\cdot h$$

$$\text{s.t.} \quad \frac{s_{k+1}-s_k}{h} = f\left(\frac{s_k+s_{k+1}}{2}, q_k\right), \quad k=0,\dots,N-1$$

$$\left[s_{k+1} = s_k + h\cdot f\left(\frac{s_k+s_{k+1}}{2}, q_k\right)\right]$$ (SKIP 9.4/9.5)



CH 10: NONLINEAR MODEL PREDICTIVE CONTROL (NMPC)

AIM: MAKE OPTIMAL CONTROL PRACTICALLY APPLICABLE FOR FEEDBACK CONTROL

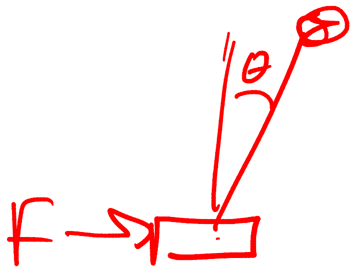
DREAM: $u^*_\infty(0) = \arg\min \int_0^\infty L(x,u)\,dt$

s.t. $x(0) = x_0$

$\dot{x} = f(x,u)$

OPTIMAL FEEDBACK CONTROL BY HJB/DP

$u^*_\infty(x_0)$



$L = \theta^2 + F^2$

$f = \dots$

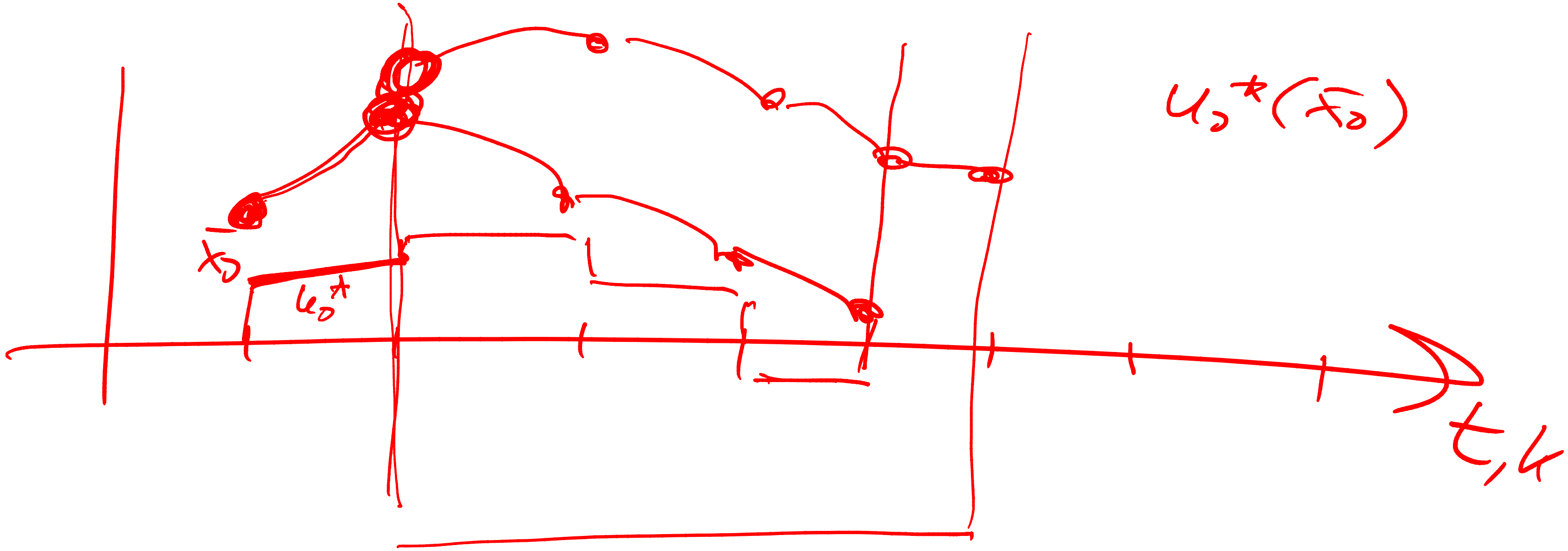
IN PRACTICE, CAN ONLY PREDICT ON <u>FINITE</u> HORIZON

(MOVING HORIZON), IN DISCRETE TIME

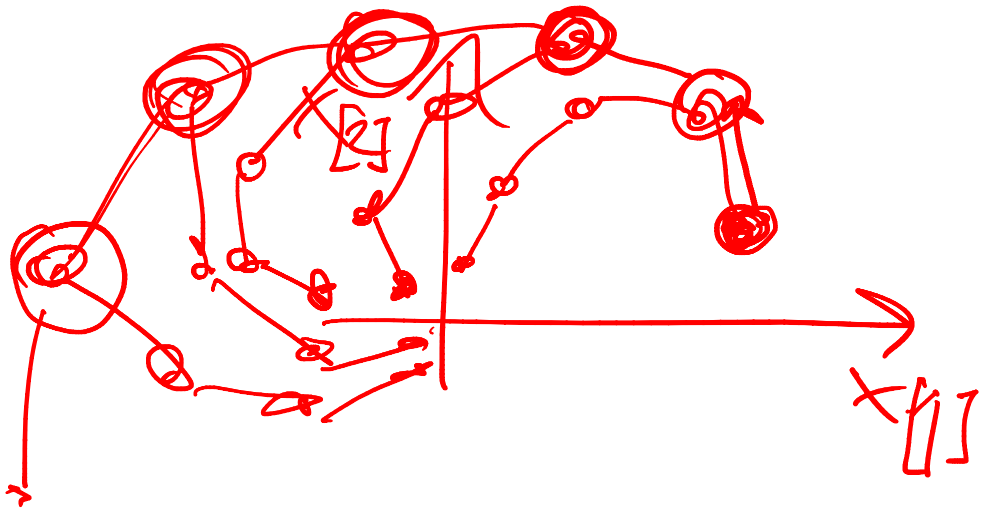
$\bar{x}_0 \rightarrow$

$$\boxed{\begin{aligned}&\min_{x,z,u} \sum_{k=0}^{N-1} L(x,z,u) + E(x_N)\\ &\text{s.t.} \quad x_0 - \bar{x}_0 = 0\\ &\qquad x_{k+1} - f(x_k, z_k, u_k) = 0\\ &\qquad g(x_k, z_k, u_k) = 0\\ &\qquad h(x_k, z_k, u_k) \leq 0\\ &\qquad r(x_N) \leq 0\end{aligned}} \rightarrow u_0^*$$

$\left.\begin{matrix} x_{k+1} - f(x_k, z_k, u_k) = 0 \\ g(x_k, z_k, u_k) = 0 \\ h(x_k, z_k, u_k) \leq 0 \end{matrix}\right\} k = 0, \ldots, N-1$



GET FEEDBACK CONTROL $u_0^*(x_0)$ BY REPEATEDLY SOLVING OPEN-LOOP OCPS.



PROBLEM: EVEN IN <u>NOMINAL</u> CASE, I.E. NO MODEL-PLANT MISMATCH, NO DISTURBANCE, STABILITY CANNOT ALWAYS BE GUARANTEED.

MANY STABILITY ENFORCING SCHEMES EXIST,

ONE IS CALLED: ZERO TERMINAL CONSTRAINT

$\boxed{x_N = 0}$

(ASSUMPTIONS

$$\min_{x,u} \sum L(x_n, u_n)$$

$$x_0 - \bar{x}_0 = 0$$

$$x_{n+1} - f(x_n, u_n) = 0$$

$$x_N = 0$$

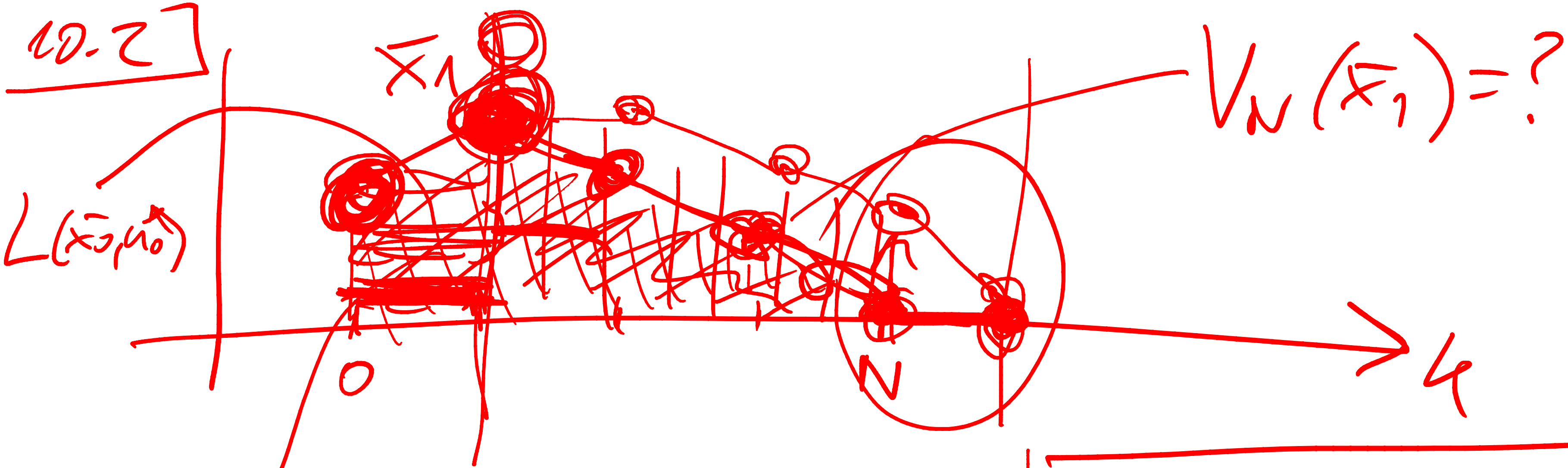
$$0 = f(0,0)$$

$$L(0,0) = 0$$

$$L(x,u) > 0$$

IF $x \neq 0$

10.2



$V_N(\bar{x}_0)$ = OPTIMAL COST TO GO TO END/ZERO IN N STEPS FROM $\bar{x}_0$

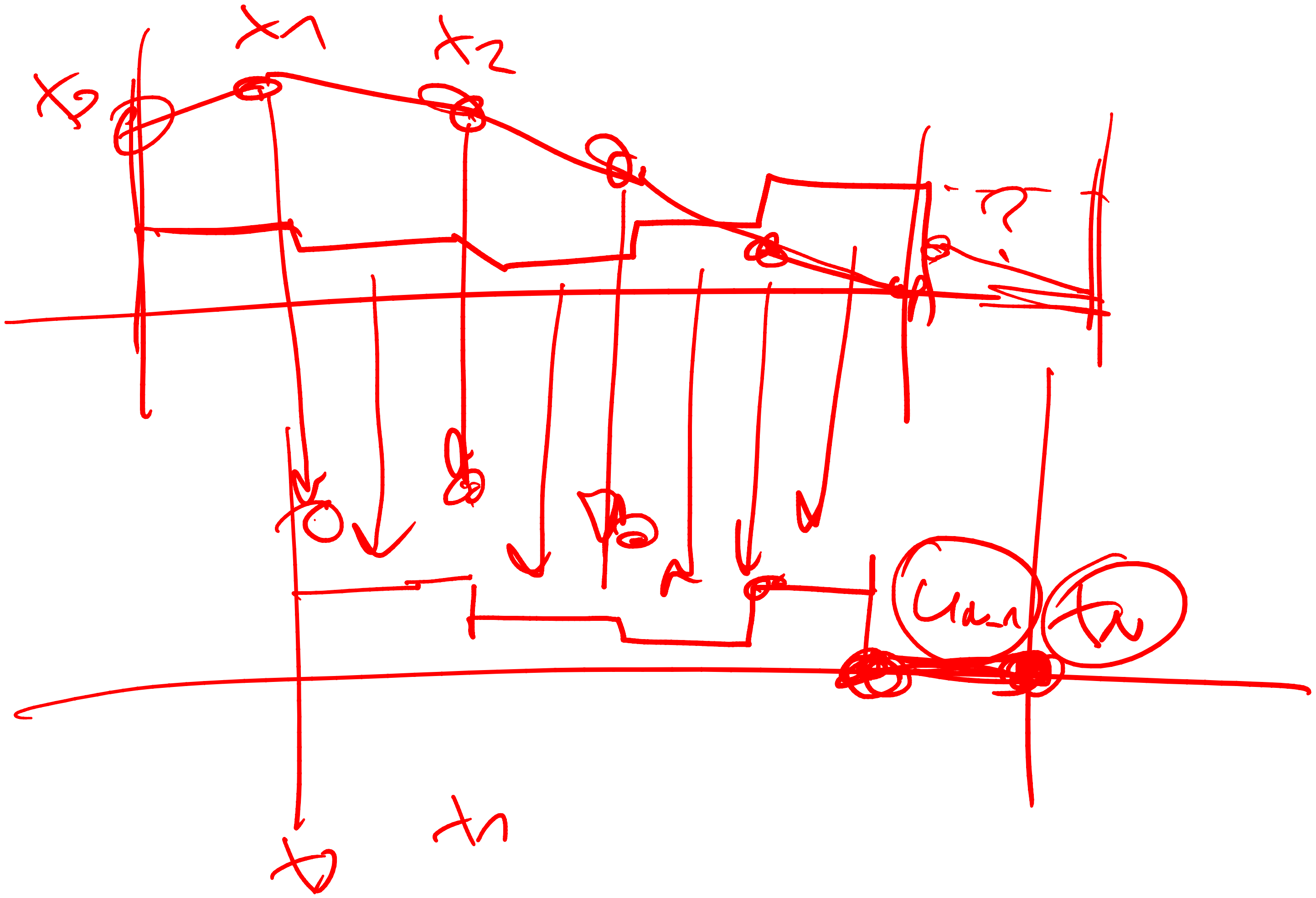
$$\boxed{V_N(\bar{x}_1) \leq V_N(\bar{x}_0) - L(\bar{x}_0, u_0^*)}$$

$V_N(0) = 0 \qquad V_N(x) > 0 \quad x \neq 0$

V_N IS LYAPUNOV FCT.

$\rightarrow$ GUARANTEES STAB.

10.3 ONLINE INITIALIZATION VIA SHIFT



10.4 REAL-TIME OPTIMIZATION STRATEGIES

AIM: FOR GIVEN $\bar{x}_0$, COMPUTE OPTIMAL SOLUTION <u>FAST</u>

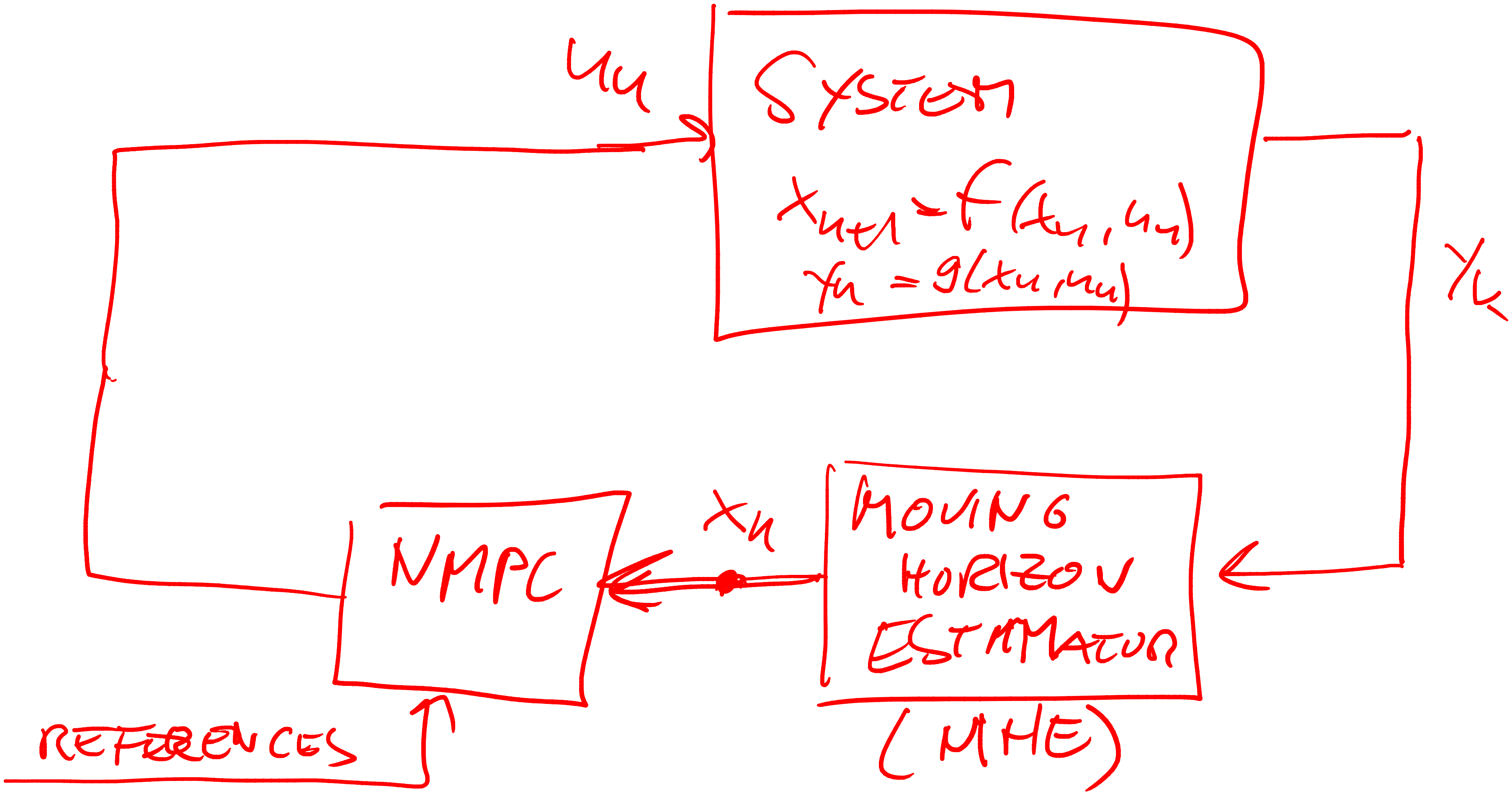
- OFFLINE PRECOMPUTATIONS / CODE GENERATION:
- DELAY COMPENSATION BY PREDICTION
- DIVISION INTO FEEDBACK / PREPARATION PHASE
- ITERATION WHILE THE PROBLEM CHANGES

$\boxed{\text{ACADO CODE GENERATION}}$

SQP + GN + • + • + • $\boxed{1\,\text{MHz}}$

2 STATE, 1 CONTROL, 5 TIMESTEPS, $1\,\mu s$ CPU

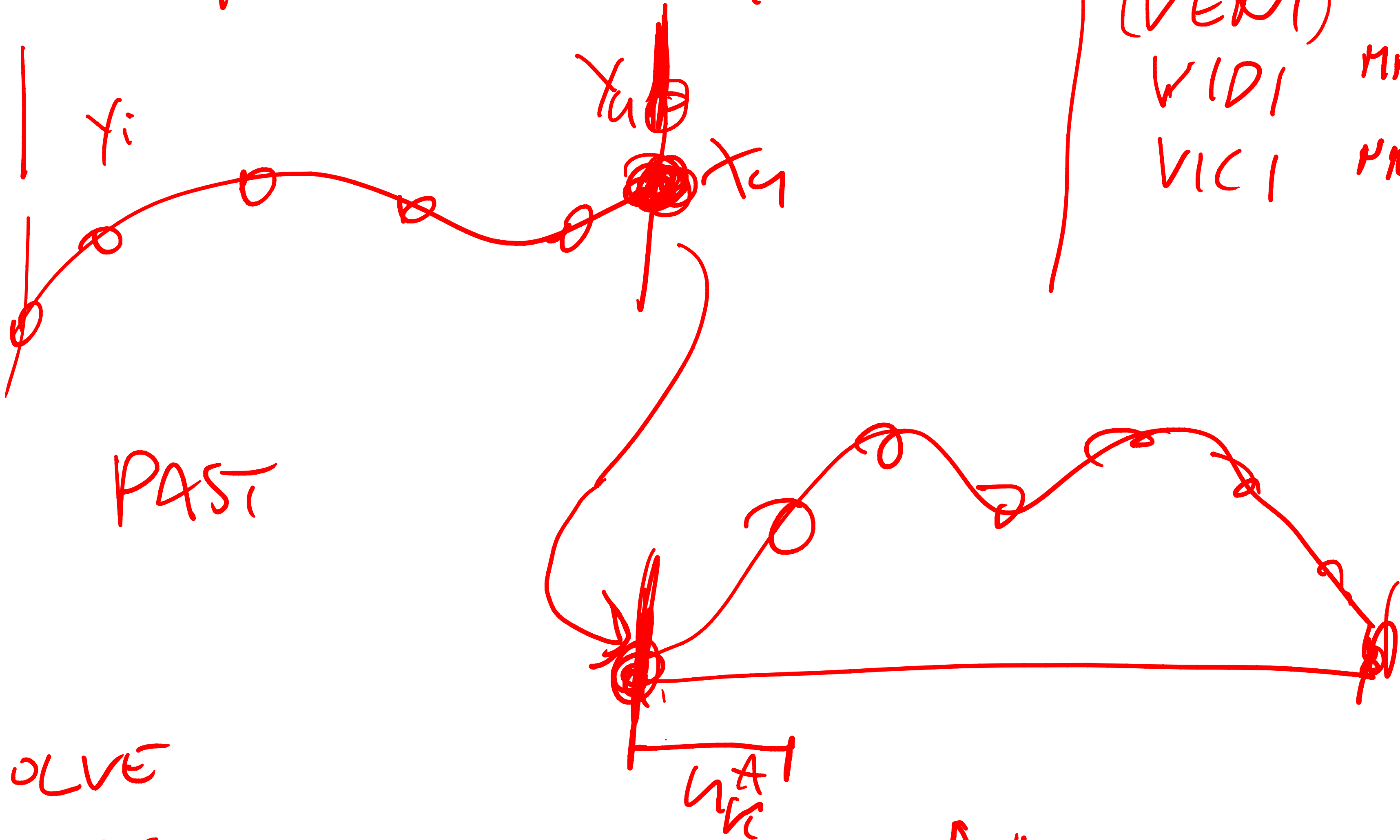
[SKIP CH. 11, PREPARE CH 12.]



MHE

t_n = NOW

(VENI)
VIDI MHE
VICI NMPC



SOLVE
2 OCPs
PER TIMESTEP

NMPC
FUTURE