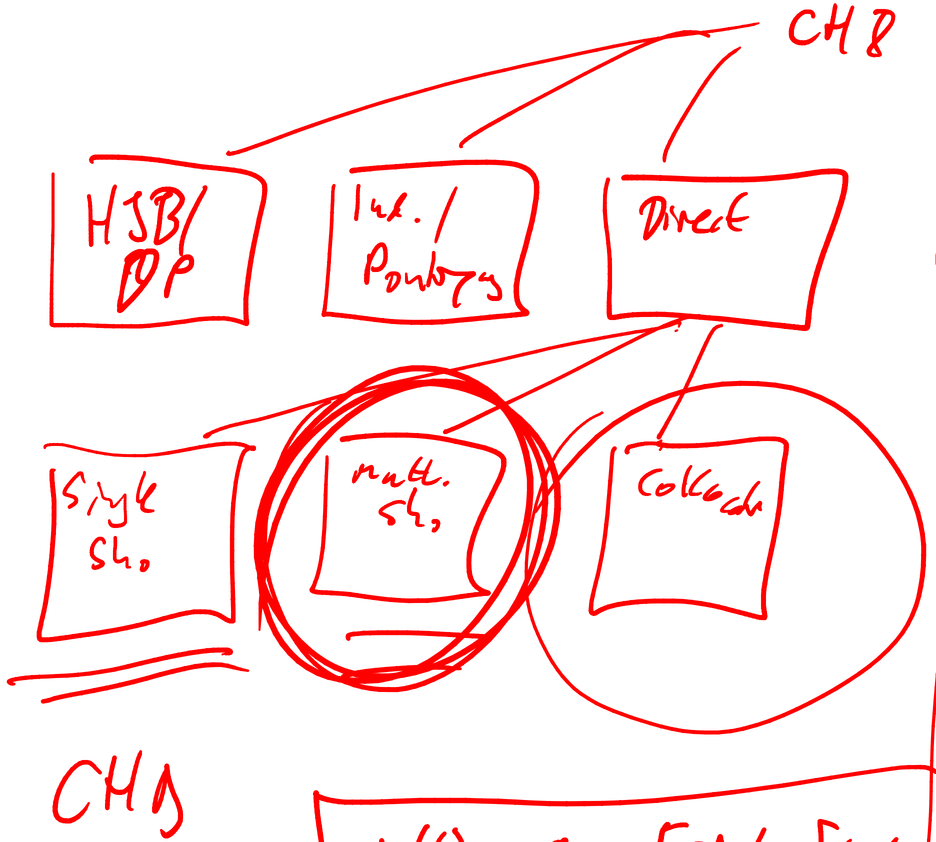
$\lambda_k = \nabla_x J_k(x_k)$

CH 7,



$u(t) = q_i \quad$ For $t \in [t_i, t_{i+1}]$

OCP

$$\min \int_0^T L \, dt + E(x(T))$$

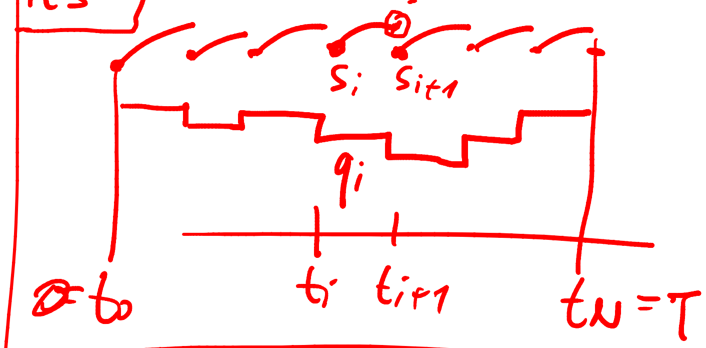
s.t. $\quad x(0) - \bar{x}_0 = 0$

$$\left.\begin{array}{l} \dot{x} - f(x,u) = 0 \\ h(x,u) \le 0 \end{array}\right\} t \in [0,T]$$

$r(x(T)) \le 0$

MS

(1.2)



$$\min_{s,q} \sum_{i=0}^{N-1} l_i(s_i, q_i) + E(s_N)$$

s.t. $\quad \bar{x}_0 - s_0 = 0$

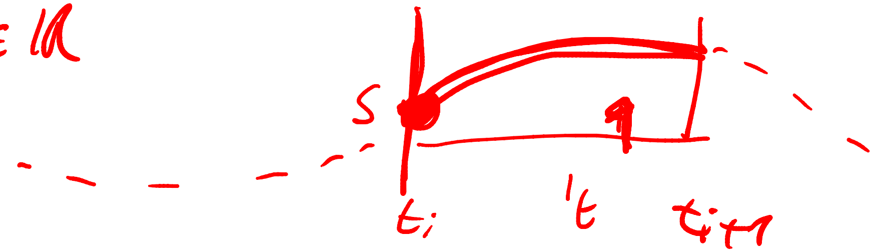
$$\left.\begin{array}{l} x_i(t_{i+1}; s_i, q_i) - s_{i+1} = 0 \\ h(s_i, q_i) \le 0 \end{array}\right\} i = 0, \dots, N-1$$

$r(s_N) \le 0$

Ⓧ FOR $x_i(t;s,q)$ & $l_i(s,q)$ ON $t\in[t_i,t_{i+1}]$

$\dot{x} = a\cdot x + b\cdot u \qquad x\in\mathbb{R} \quad , q,s\in\mathbb{R}$

$L = x^2 + \underline{u}^2$



$x_i(t;s,q) = ?$

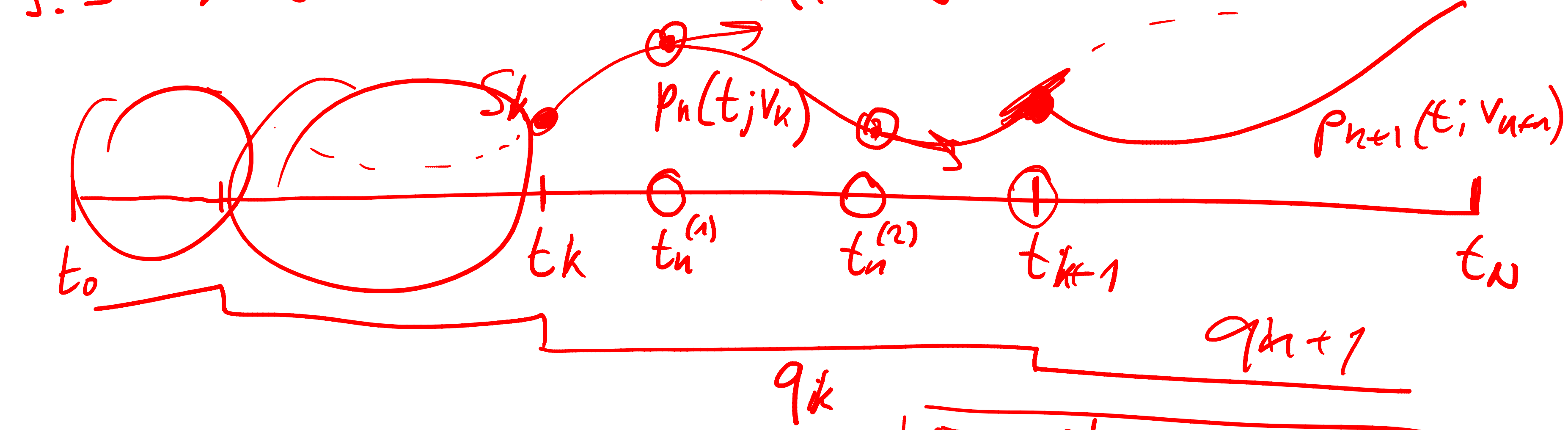
$l_i(s,q) = ? = \int_{t_i}^{t_{i+1}} \left(x_i(t;s,q)\right)^2 + (\underline{q})^2 \, dt$

$= \int (\dots)^2 dt + (t_{i+1}-t_i)\cdot q^2$

$\underline{x(t) = e^{a(t-t_i)}s + \int_{t_i}^{t} e^{a(t-\tau)} b\cdot q \cdot d\tau}$

$\underline{x_i(t; s,q)}$

9.3 DIRECT COLLOCATION



v_k POLYNOMIAL COEFFICIENTS

$$p_k(t_k; v_k) = s_k$$

$$[\,p_k(t_{k+1}; v_k) = s_{k+1}\,]$$

$$p_k'(t_k^{(i)}; v_k) = f(p_k(t_k^{(i)}; v_k), q_k) \quad i = 1, \dots, m$$

$$\boxed{c_k(s_k, v_k, q_k) = 0}$$

FOR M-th order polynomials

EXAMPLE

$x \in \mathbb{R}^{n_x}$, 2ND ORDER POLYN

$$p(t; v) = v_{(0)} + t \cdot v_{(1)} + t^2 \cdot v_{(2)}$$

$$= v'_{(0)} + (t - t_i)\, v'_{(1)} + (t - t_i)^2\, v'_{(2)}$$

$$= v'_{(0)} - t_i\, v'_{(1)} + \dots$$

$$+\, t_i^2\, v'_{(2)} + \dots$$

$$p'(t; v) = v_{(1)} + 2 \cdot t \cdot v_{(2)}$$

NLP 1

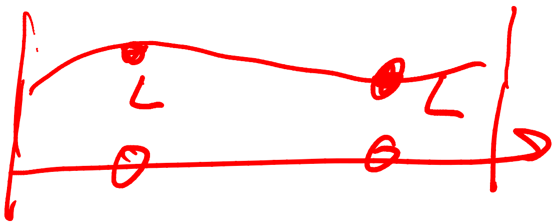
$$\min_{s,q,v} \sum_{k=0}^{N-1} L_k(s_k, v_k, q_k) + E(s_N)$$

$$\text{s.t.} \quad s_0 - \bar{x}_0 = 0$$

$$\left.\begin{aligned} c_k(s_k, v_k, q_k) &= 0 \\ p_k(t_{k+1}; v_k) - s_{k+1} &= 0 \\ h(s_k, q_k) &\le 0 \end{aligned}\right\} \quad k = 0, \dots, N-1$$

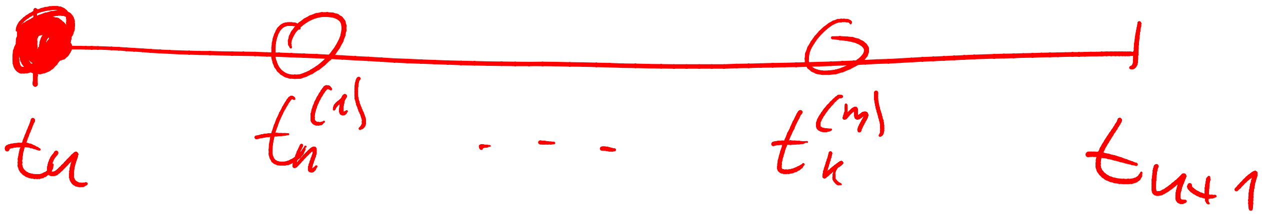
$$r(s_N) \le 0$$

$$\boxed{L_k(v_k, q_k) = \sum_{i=1}^{n} \underline{\alpha_i} \cdot L\left(p_k(t_k^{(i)}; v_k), q_k\right)}$$



$$\dot{g} = L(x,u) \qquad g(t_k) = 0$$

$$L_i = g(t_{k+1})$$



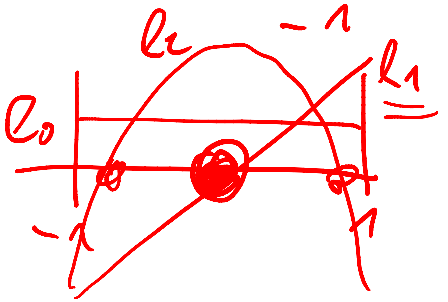
LOCATIONS OF COLLOCATION NODE POSITIONS $t^{(1)}, \dots, t^{(m)}$

A) GAUSS-LEGENDRE POINTS: INSIDE INTERVAL, LOCATED AT ROOTS OF LEGENDRE-POLYNOMIALS ℓ

G-L-POLY. DEF. ON $[-1,1]$

$$\int_{-1}^{1} \ell_i(t)\cdot \ell_j(t)\,dt = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{else} \end{cases}$$

$$\int_{-1}^{1} (\ell_i(t))^2\,dt = 1$$

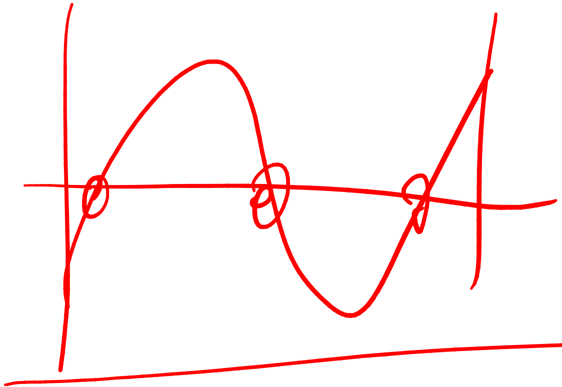


$\boxed{\text{USES GAUSS-QUADRATURE}}$

$$\ell_0 = \frac{1}{2}$$

$$\ell_1 = t \cdot c_1$$

$$\ell_2 = c_{20} + c_{22} t^2$$

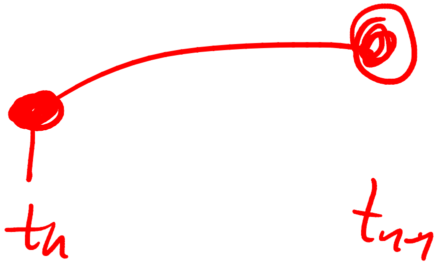


(SCALE TO $[t_n, t_{n+1}]$ FROM $[-1,1]$

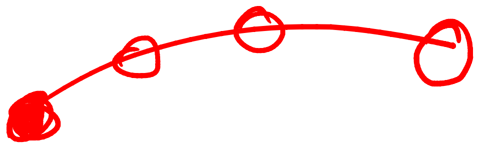
B) RADAU-COLLOCATION: $(m-1)$ POINTS INSIDE

1 POINT AT END,

$m=1$: $t_n^{(1)} = t_{n+1}$



$m=3$

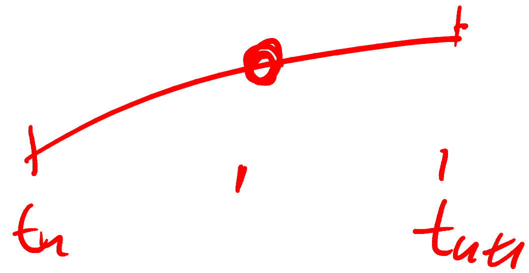


HOW TO SCALE:

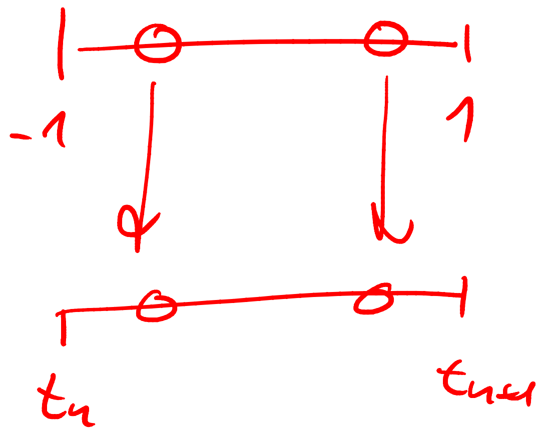
EX: GAUSS-LEGENDRE OF ORDER 2 (GL2)

WITH $m=1$.

$$t_n^{(1)} = \frac{t_{n+1} + t_n}{2}$$



EX: GL4 WITH $m=2$



EX. (FUNNY SCHEME)

$$\boxed{\begin{aligned} p_n(t_n; v_n) &= s_n \\ p_n'(t_n; v_n) &= f(s_n, q_n) \end{aligned}}$$

$$\boxed{p_n(t; v_n) = v_n^{(1)} + v_n^{(2)} (t - t_n)}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} v_n^{(1)} &= s_n \\ v_n^{(2)} &= f(s_n, q_n) \end{aligned}}$$

$$s_{n+1} - p(t_{n+1}; \overset{v_n}{\cancel{s_n, q_n}}) = 0$$

$$\Updownarrow$$

$$\boxed{s_{n+1} - s_n - (t_{n+1} - t_n) \cdot f(s_n, q_n) = 0}$$

$\longrightarrow$ EULER (EXPLICIT)

EX 2: RADAU WITH $m=1$:

(1) $p_n(t_n; v_n) = s_n$

(2) $\boxed{p_n'(t_{n+1}, v_n) = f(p_n(t_{n+1}; v_n), q_n)}$

(3) $p_n(t_{n+1}; v_n) = s_{n+1}$

$$p_n(t; v_n) = v_n^{(1)} + v_n^{(2)}(t - t_n)$$

$$v_n^{(1)} = s_n$$

$$v_n^{(1)} + v_n^{(2)}(t_{n+1} - t_n) = s_{n+1}$$

$$v_n^{(2)} = \frac{s_{n+1} - s_n}{t_{n+1} - t_n}$$

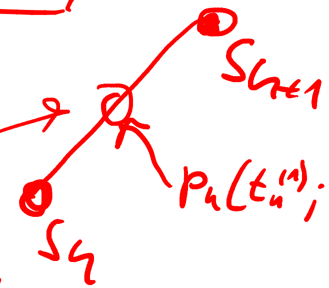
$$\boxed{\frac{s_{n+1} - s_n}{t_{n+1} - t_n} = f(s_{n+1}, q_n)}$$

IMPLICIT EULER

TASK GL2 $t_n^{(1)} = \frac{t_{n+1} + t_n}{2}$

ELIMINATE V & WRITE EQ.

$$\frac{s_{k+1} - s_k}{t_{n+1} - t_n} = f\left(\frac{s_n + s_{n+1}}{2}, q_n\right)$$



$p_n(t_n^{(1)}; v_n) = \frac{s_n + s_{n+1}}{2}$

$p' \ldots = \frac{s_{n+1} - s_n}{t_{n+1} - t_n}$