Solution for Exercise 5: YALMIP for convex optimization

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TEMPO Summer School on Numerical Optimal Control and Embedded Optimization
University of Freiburg, July 27 - August 7, 2015
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- Part 2: Piecewise linear ground constraints
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Part 1: The unconstrained minimization problem

```
clc;
clear all;
close all;

N = 40;

y = sdpvar(N,1);
z = sdpvar(N,1);

mi = 4/N;
Di = 70;
g0 = 9.81;

Vchain = 0;
for i = 1:N
   Vchain = Vchain + g0*mi*z(i);
   if i < N
      Vchain = Vchain + 1/2*Di*((y(i)-y(i+1))^2 + (z(i)-z(i+1))^2);
   end
end

constr = [  y(1) == -2; ...
            z(1) == 1; ...
            y(N) == 2; ...
            z(N) == 1 ];

% Setting options and solving the problem with quadprog:
options = sdpsettings('solver', 'quadprog','verbose',2);
solvesdp(constr, Vchain, options);

Y = double(y); Z = double(z);

figure;
plot(Y,Z,'--or'); hold on;
plot(-2,1,'xg','MarkerSize',10);
plot(2,1,'xg','MarkerSize',10);
xlabel('y'); ylabel('z');
title('Optimal solution hanging chain (without extra constraints)')
```

```
MOSEK Version 7.1.0.32 (Build date: 2015-6-26 08:10:43)
Copyright (c) 1998-2015 MOSEK ApS, Denmark. WWW: http://mosek.com
Platform: Linux/64-X86

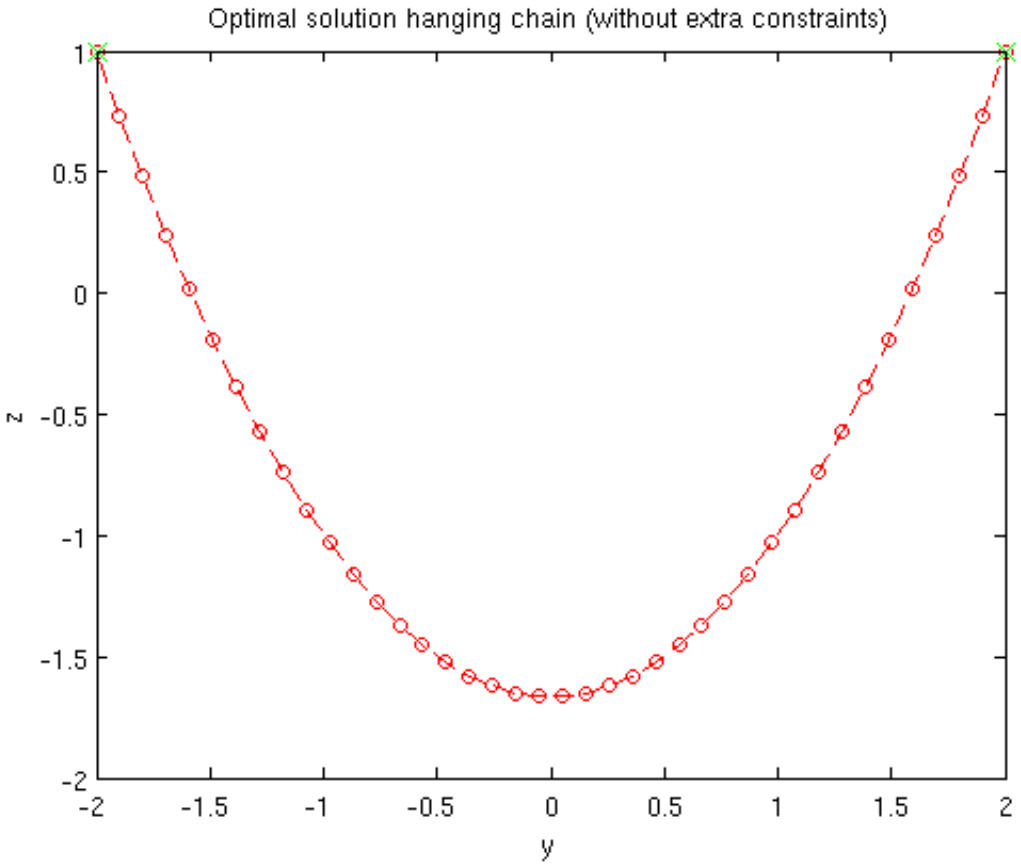
Computer
  Platform               : Linux/64-X86

Problem
  Name                   :
  Objective sense        : min
  Type                   : QO (quadratic optimization problem)
  Constraints            : 4
  Cones                  : 0
  Scalar variables       : 80
  Matrix variables       : 0
  Integer variables      : 0

Optimizer started.
Matrix reordering started.
Local matrix reordering started.
Local matrix reordering terminated.
Matrix reordering terminated.
Optimizer terminated. Time: 0.03

Interior-point solution summary
  Problem status  : PRIMAL_AND_DUAL_FEASIBLE
  Solution status : OPTIMAL
  Primal.  obj: 1.9641379073e+01    Viol.  con: 0e+00    var: 0e+00
  Dual.    obj: 1.9641379080e+01    Viol.  con: 0e+00    var: 2e-09
Optimizer summary
  Optimizer                 -                        time: 0.03
    Interior-point          - iterations : 6         time: 0.01
      Basis identification  -                        time: 0.00
        Primal              - iterations : 0         time: 0.00
        Dual                - iterations : 0         time: 0.00
        Clean primal        - iterations : 0         time: 0.00
        Clean dual          - iterations : 0         time: 0.00
        Clean primal-dual   - iterations : 0         time: 0.00
    Simplex                 -                        time: 0.00
      Primal simplex        - iterations : 0         time: 0.00
      Dual simplex          - iterations : 0         time: 0.00
      Primal-dual simplex   - iterations : 0         time: 0.00
    Mixed integer           - relaxations: 0         time: 0.00

Optimization terminated successfully.
```



Part 2: Piecewise linear ground constraints

```
constr2 = constr;
for i = 1:N
   constr2 = [constr2; z(i) >= 0.5];
   constr2 = [constr2; z(i)-0.1*y(i) >= 0.5];
end

% Setting options and solving the problem with quadprog:
options = sdpsettings('solver', 'quadprog','verbose',2);
solvesdp(constr2, Vchain, options);

Y2 = double(y); Z2 = double(z);

figure;
plot(Y2,Z2,'--or'); hold on;
plot(-2,1,'xg','MarkerSize',10);
plot(2,1,'xg','MarkerSize',10);
plot([-2 2],[0.5 0.5],':b');
plot([-2 2],[0.3 0.7],':b');
xlabel('y'); ylabel('z');
title('Optimal solution hanging chain (with linear ground constraints)')
```

```
MOSEK Version 7.1.0.32 (Build date: 2015-6-26 08:10:43)
Copyright (c) 1998-2015 MOSEK ApS, Denmark. WWW: http://mosek.com
Platform: Linux/64-X86

Computer
  Platform              : Linux/64-X86

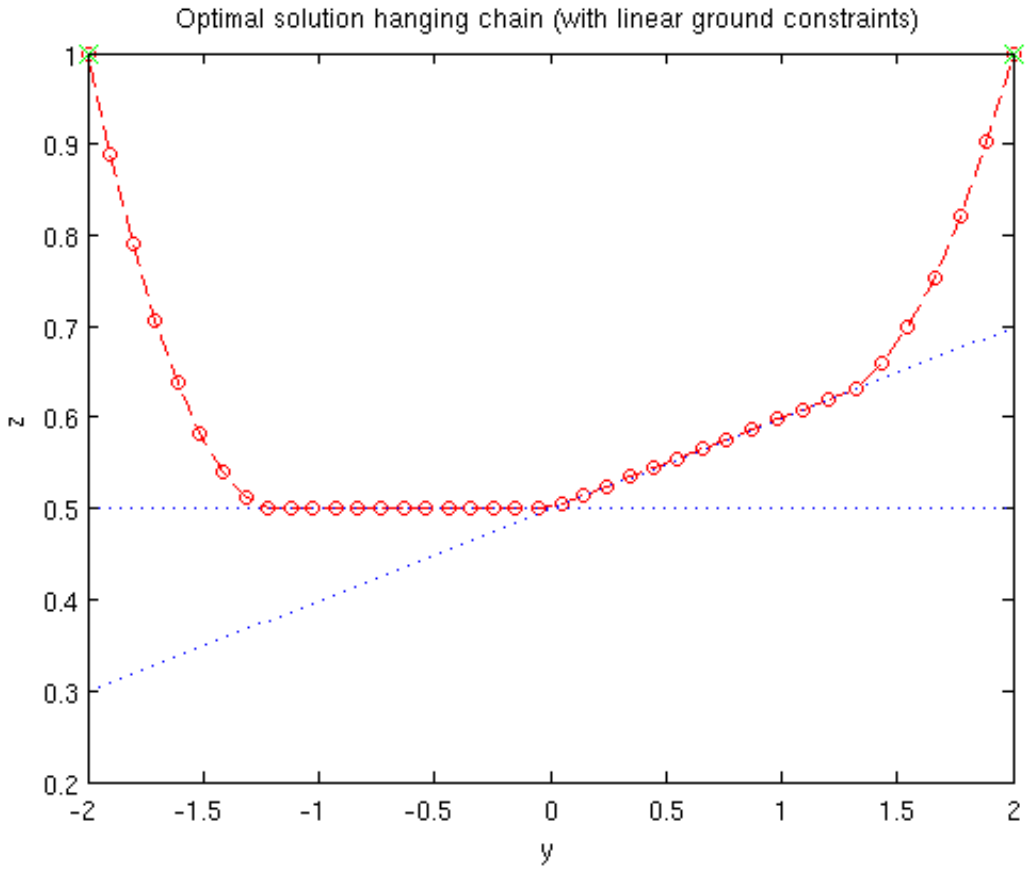
Problem
  Name                  :
```

```
  Objective sense        : min
  Type                   : QO (quadratic optimization problem)
  Constraints            : 84
  Cones                  : 0
  Scalar variables       : 80
  Matrix variables       : 0
  Integer variables      : 0

Optimizer started.
Matrix reordering started.
Local matrix reordering started.
Local matrix reordering terminated.
Matrix reordering terminated.
Optimizer terminated. Time: 0.03

Interior-point solution summary
  Problem status  : PRIMAL_AND_DUAL_FEASIBLE
  Solution status : OPTIMAL
  Primal.  obj: 4.0659942936e+01    Viol.  con: 6e-15    var: 0e+00
  Dual.    obj: 4.0659942935e+01    Viol.  con: 0e+00    var: 9e-13
Optimizer summary
  Optimizer                 -                        time: 0.03
    Interior-point          - iterations : 14        time: 0.01
      Basis identification  -                        time: 0.00
        Primal              - iterations : 0         time: 0.00
        Dual                - iterations : 0         time: 0.00
        Clean primal        - iterations : 0         time: 0.00
        Clean dual          - iterations : 0         time: 0.00
        Clean primal-dual   - iterations : 0         time: 0.00
    Simplex                 -                        time: 0.00
      Primal simplex        - iterations : 0         time: 0.00
      Dual simplex          - iterations : 0         time: 0.00
      Primal-dual simplex   - iterations : 0         time: 0.00
    Mixed integer           - relaxations: 0         time: 0.00

Optimization terminated successfully.
```



Part 3: Convex quadratic ground constraint

```
constr3 = constr;
for i = 1:N
   constr3 = [constr3; z(i) >= -0.2+0.1*y(i)^2];
end

% Setting options and solving the problem with SDPT3:
options = sdpsettings('solver', 'SDPT3','verbose',2);
solvesdp(constr3, Vchain, options);

Y3 = double(y); Z3 = double(z);

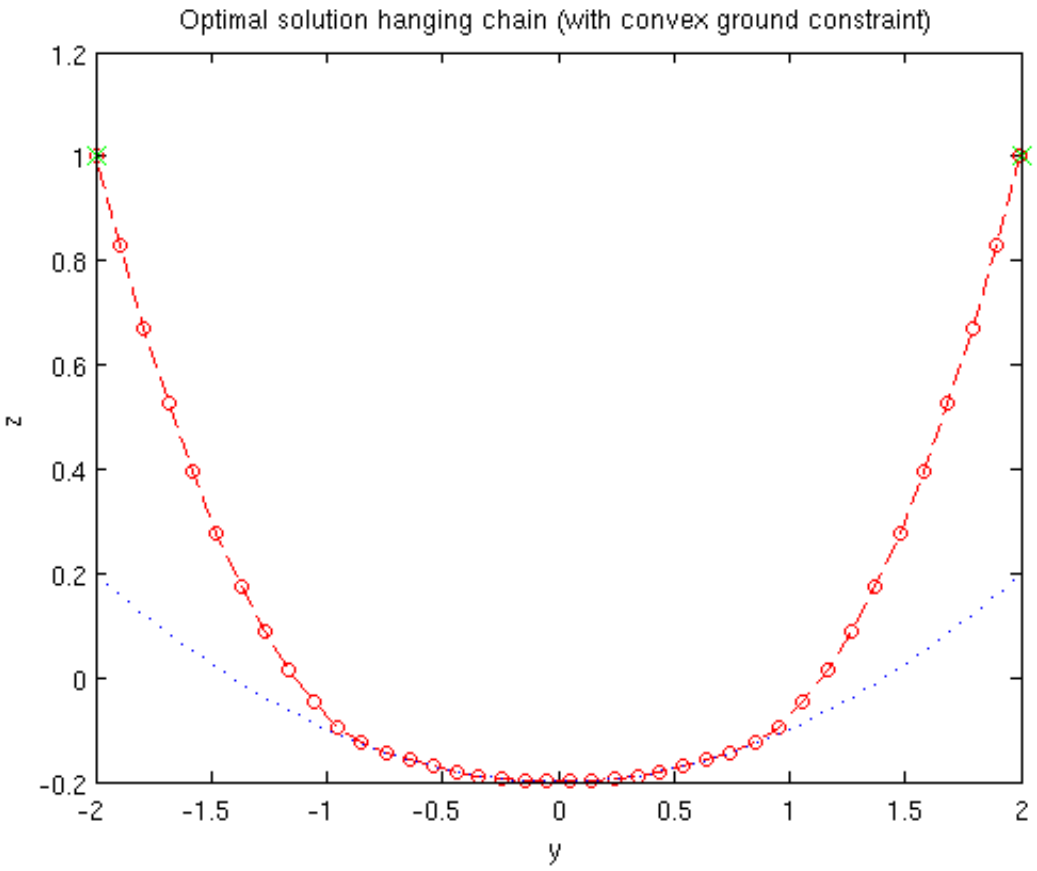
x = linspace(-2,2,100);
f = -0.2+0.1*x.^2;

figure;
plot(Y3,Z3,'--or'); hold on;
plot(-2,1,'xg','MarkerSize',10);
plot(2,1,'xg','MarkerSize',10);
plot(x,f,':b');
xlabel('y'); ylabel('z');
title('Optimal solution hanging chain (with convex ground constraint)')
```

```
 num. of constraints = 81
 dim. of socp   var  = 200,   num. of socp blk  = 41
 dim. of free   var  =  4
 *** convert ublk to linear blk
******************************************************************************
*****
   SDPT3: homogeneous self-dual path-following algorithms
```

```
*******************************************************************************************
*****
 version  predcorr  gam  expon
   HKM      1      0.000   1
it pstep dstep pinfeas dinfeas  gap     mean(obj)    cputime    kap   tau    theta
-------------------------------------------------------------------------------------------
-----
 0|0.000|0.000|2.5e+01|3.7e+01|1.4e+03| 2.531655e+01| 0:0:00|1.4e+03|1.0e+00|1.0e+00| c
hol 1  1
 1|0.657|0.657|1.1e+01|1.6e+01|7.1e+02|-5.311990e+00| 0:0:01|5.3e+02|9.8e-01|4.3e-01| c
hol 1  1
 2|0.775|0.775|3.1e+00|4.6e+00|2.1e+02|-7.462653e+00| 0:0:01|1.1e+02|1.0e+00|1.2e-01| c
hol 1  1
 3|0.790|0.790|8.3e-01|1.2e+00|5.5e+01|-1.970607e+01| 0:0:01|1.3e+01|1.1e+00|3.6e-02| c
hol 1  1
 4|0.881|0.881|1.7e-01|2.5e-01|1.3e+01|-2.542812e+01| 0:0:01|8.8e-01|1.2e+00|8.2e-03| c
hol 1  1
 5|0.904|0.904|2.4e-02|3.6e-02|1.7e+00|-2.778757e+01| 0:0:01|1.0e-01|1.3e+00|1.2e-03| c
hol 1  1
 6|0.848|0.848|5.3e-03|1.1e-02|4.0e-01|-2.817253e+01| 0:0:01|2.3e-02|1.3e+00|2.7e-04| c
hol 1  1
 7|0.838|0.838|1.3e-03|7.6e-03|1.0e-01|-2.827388e+01| 0:0:01|5.8e-03|1.3e+00|6.7e-05| c
hol 1  1
 8|0.818|0.818|3.7e-04|3.8e-03|3.2e-02|-2.837552e+01| 0:0:01|1.6e-03|1.3e+00|1.9e-05| c
hol 1  1
 9|0.918|0.918|6.1e-05|1.4e-03|5.6e-03|-2.843208e+01| 0:0:01|3.2e-04|1.3e+00|3.1e-06| c
hol 1  1
10|0.927|0.927|6.0e-06|5.2e-04|5.6e-04|-2.845123e+01| 0:0:01|5.6e-05|1.3e+00|3.1e-07| c
hol 1  1
11|0.953|0.953|4.8e-07|2.0e-04|4.3e-05|-2.845832e+01| 0:0:01|6.2e-06|1.3e+00|2.4e-08| c
hol 1  1
12|0.940|0.940|8.0e-08|8.1e-05|7.2e-06|-2.846088e+01| 0:0:01|6.5e-07|1.3e+00|4.0e-09| c
hol 1  1
13|0.883|0.883|1.8e-08|2.2e-05|1.6e-06|-2.846213e+01| 0:0:01|1.2e-07|1.3e+00|9.0e-10| c
hol 1  1
14|0.925|0.925|2.7e-09|4.4e-06|2.6e-07|-2.846252e+01| 0:0:01|1.9e-08|1.3e+00|1.4e-10| c
hol 1  1
15|0.613|0.613|2.1e-09|2.1e-06|1.3e-07|-2.846257e+01| 0:0:01|8.3e-09|1.3e+00|5.8e-11| c
hol 1  1
16|0.607|0.607|1.0e-09|8.1e-07|6.9e-08|-2.846260e+01| 0:0:01|3.8e-09|1.3e+00|2.7e-11| c
hol 1  1
17|0.602|0.602|4.6e-10|3.2e-07|3.6e-08|-2.846261e+01| 0:0:01|1.8e-09|1.3e+00|1.5e-11| c
hol 1  1
18|0.598|0.598|5.6e-10|1.3e-07|1.9e-08|-2.846261e+01| 0:0:01|8.5e-10|1.3e+00|1.2e-11| c
hol 1  1
19|0.596|0.596|1.7e-09|5.3e-08|9.8e-09|-2.846261e+01| 0:0:01|4.1e-10|1.3e+00|1.7e-11|
  Stop: max(relative gap,infeasibilities) < 1.00e-07
-------------------------------------------------------------------
 number of iterations   = 19
 primal objective value = -2.84626157e+01
 dual   objective value = -2.84626134e+01
 gap := trace(XZ)       = 9.81e-09
 relative gap           = 3.33e-10
 actual relative gap    = -3.91e-08
 rel. primal infeas     = 1.71e-09
 rel. dual   infeas     = 5.26e-08
 norm(X), norm(y), norm(Z) = 2.8e+01, 3.0e+01, 3.6e+01
 norm(A), norm(b), norm(C) = 1.8e+02, 1.0e+00, 6.5e+00
 Total CPU time (secs)  = 1.49
 CPU time per iteration = 0.08
 termination code       =  0
```

```
 DIMACS: 1.7e-09  0.0e+00  5.3e-08  0.0e+00  -3.9e-08  1.7e-10
------------------------------------------------------------------------
 Percentage of CPU time spent in various parts
------------------------------------------------------------------------
 preproc Xchol Zchol pred pred_steplen  corr corr_steplen  misc
   14.0   4.9   2.7   38.0  3.7  3.7     14.8  1.5  1.5     15.2
------------------------------------------------------------------------
```



Part 4: Nonconvex quadratic ground constraint

```
constr4 = constr;
for i = 1:N
   constr4 = [constr4; z(i) >= -y(i)^2];
end

% Setting options and solving the problem with fmincon:
options = sdpsettings('solver', 'fmincon','verbose',2,'usex0',1);
solvesdp(constr4, Vchain, options);

Y4 = double(y); Z4 = double(z);

x = linspace(-1,1,100);
f = -x.^2;

figure;
plot(Y4,Z4,'--or'); hold on;
plot(-2,1,'xg','MarkerSize',10);
plot(2,1,'xg','MarkerSize',10);
plot(x,f,':b');
xlabel('y'); ylabel('z');
title('Optimal solution hanging chain (with nonconvex ground constraint)')
```

Your initial point x0 is not between bounds lb and ub; FMINCON
shifted x0 to strictly satisfy the bounds.

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	1	2.846262e+01	1.974e-01	5.596e-01	
1	13	2.848503e+01	1.886e-01	5.100e-01	2.826e-02
2	15	2.847083e+01	1.860e-01	8.567e-01	2.582e-02
3	20	2.873192e+01	1.506e-01	1.801e+00	1.121e-01
4	22	2.871047e+01	1.471e-01	1.454e+00	6.511e-02
5	29	2.884092e+01	1.292e-01	8.968e-01	5.617e-02
6	31	2.883420e+01	1.276e-01	1.087e+00	2.690e-02
7	37	2.904623e+01	1.096e-01	1.270e+00	5.451e-02
8	39	2.906354e+01	1.070e-01	1.483e+00	5.444e-02
9	46	2.926402e+01	8.862e-02	1.541e+00	5.520e-02
10	48	2.927148e+01	8.693e-02	1.724e+00	4.562e-02
11	54	2.984237e+01	4.978e-02	2.306e+00	1.075e-01
12	56	2.983971e+01	4.759e-02	1.685e+00	4.355e-02
13	62	3.033126e+01	7.842e-03	2.348e+00	1.124e-01
14	64	3.037950e+01	6.089e-03	2.231e+00	7.363e-02
15	70	3.162508e+01	0.000e+00	2.778e+00	2.183e-01
16	73	3.164343e+01	0.000e+00	2.560e+00	9.597e-02
17	80	3.163672e+01	0.000e+00	2.375e+00	1.089e-01
18	82	3.161328e+01	0.000e+00	2.421e+00	2.758e-02
19	86	3.164860e+01	0.000e+00	4.146e+00	2.010e-01
20	95	3.135568e+01	0.000e+00	3.079e+00	9.403e-02
21	99	3.132735e+01	0.000e+00	2.845e+00	3.272e-02
22	105	3.119474e+01	0.000e+00	1.223e+00	4.610e-02
23	115	3.116811e+01	0.000e+00	8.860e-01	2.579e-02
24	117	3.114459e+01	0.000e+00	8.989e-01	2.487e-02
25	123	3.113998e+01	0.000e+00	1.168e+00	4.314e-02
26	125	3.112080e+01	0.000e+00	1.197e+00	1.963e-02
27	131	3.108200e+01	0.000e+00	6.273e-01	3.816e-02
28	134	3.107686e+01	0.000e+00	1.130e+00	2.398e-02
29	140	3.106135e+01	0.000e+00	7.342e-01	4.143e-02
30	142	3.104287e+01	0.000e+00	7.148e-01	2.177e-02

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
31	149	3.101733e+01	0.000e+00	4.740e-01	2.675e-02
32	151	3.099921e+01	0.000e+00	5.872e-01	1.686e-02
33	158	3.099542e+01	0.000e+00	7.638e-01	1.872e-02
34	160	3.098247e+01	0.000e+00	6.585e-01	8.678e-03
35	163	3.092173e+01	0.000e+00	1.683e+00	9.056e-02
36	165	3.070935e+01	0.000e+00	1.645e+00	1.283e-01
37	175	3.063473e+01	0.000e+00	1.850e+00	5.823e-02
38	177	3.062228e+01	0.000e+00	2.085e+00	4.851e-02
39	185	3.053042e+01	0.000e+00	8.309e-01	3.098e-02
40	187	3.052751e+01	0.000e+00	1.519e+00	2.573e-02
41	193	3.050256e+01	0.000e+00	8.156e-01	4.853e-02
42	196	3.049881e+01	0.000e+00	1.250e+00	3.201e-02
43	203	3.048716e+01	0.000e+00	7.436e-01	2.719e-02
44	205	3.047790e+01	0.000e+00	9.685e-01	1.625e-02
45	209	3.045033e+01	0.000e+00	9.694e-01	4.096e-02
46	218	3.044651e+01	0.000e+00	9.628e-01	2.568e-02
47	220	3.043875e+01	0.000e+00	8.488e-01	1.296e-02
48	227	3.042028e+01	0.000e+00	9.846e-02	1.324e-02
49	240	3.041976e+01	0.000e+00	1.333e-01	3.382e-03
50	242	3.041826e+01	0.000e+00	1.368e-01	1.572e-03
51	245	3.038644e+01	0.000e+00	1.463e+00	2.438e-01

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
52	256	3.036157e+01	0.000e+00	1.473e+00	2.387e-02
53	258	3.035232e+01	0.000e+00	1.248e+00	2.065e-02
54	265	3.034791e+01	0.000e+00	1.077e+00	2.498e-02
55	267	3.034234e+01	0.000e+00	9.946e-01	8.530e-03
56	273	3.033547e+01	0.000e+00	8.662e-01	2.334e-02
57	275	3.032948e+01	0.000e+00	8.443e-01	7.624e-03
58	282	3.031690e+01	0.000e+00	3.096e-01	1.260e-02
59	284	3.031290e+01	0.000e+00	3.019e-01	6.137e-03
60	291	3.031106e+01	0.000e+00	2.746e-01	8.593e-03

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
61	293	3.030727e+01	0.000e+00	2.598e-01	7.031e-03
62	299	3.030648e+01	0.000e+00	2.446e-01	1.727e-02
63	301	3.030268e+01	0.000e+00	2.642e-01	4.577e-03
64	308	3.029978e+01	0.000e+00	2.132e-01	1.183e-02
65	310	3.029664e+01	0.000e+00	2.361e-01	4.558e-03
66	313	3.028648e+01	0.000e+00	1.510e+00	8.699e-02
67	317	3.026457e+01	0.000e+00	1.195e+00	4.626e-02
68	327	3.024673e+01	0.000e+00	9.784e-01	1.819e-02
69	329	3.023627e+01	0.000e+00	7.158e-01	1.277e-02
70	335	3.022670e+01	0.000e+00	7.972e-01	2.801e-02
71	337	3.022235e+01	0.000e+00	9.330e-01	2.922e-02
72	344	3.020908e+01	0.000e+00	8.250e-01	2.717e-02
73	347	3.020260e+01	0.000e+00	7.214e-01	1.256e-02
74	354	3.019095e+01	0.000e+00	3.457e-01	1.534e-02
75	356	3.018741e+01	0.000e+00	3.217e-01	7.052e-03
76	359	3.017375e+01	0.000e+00	1.024e+00	6.937e-02
77	369	3.016444e+01	0.000e+00	1.093e+00	2.562e-02
78	371	3.015867e+01	0.000e+00	9.851e-01	7.036e-03
79	377	3.015282e+01	0.000e+00	7.734e-01	2.365e-02
80	379	3.014744e+01	0.000e+00	6.939e-01	1.077e-02
81	387	3.014140e+01	0.000e+00	3.008e-01	1.130e-02
82	389	3.013900e+01	0.000e+00	2.671e-01	5.101e-03
83	393	3.012818e+01	0.000e+00	4.980e-01	2.820e-02
84	397	3.011866e+01	0.000e+00	5.653e-01	2.694e-02
85	401	3.010747e+01	0.000e+00	5.152e-01	3.399e-02
86	405	3.010045e+01	0.000e+00	6.265e-01	6.893e-02
87	414	3.009827e+01	0.000e+00	7.133e-01	3.125e-02
88	416	3.009475e+01	0.000e+00	6.604e-01	7.615e-03
89	420	3.009184e+01	0.000e+00	7.366e-01	6.436e-02
90	424	3.008236e+01	0.000e+00	5.703e-01	6.119e-02

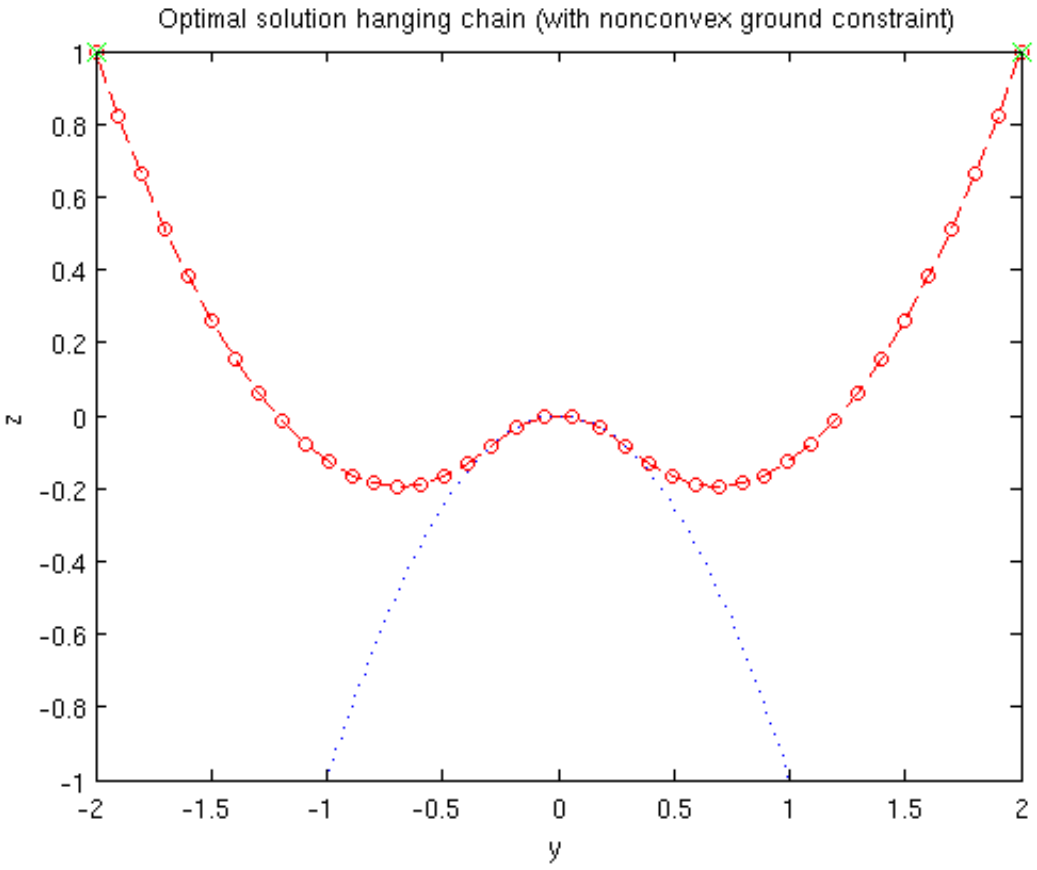
Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
91	433	3.008010e+01	0.000e+00	5.847e-01	2.682e-02
92	435	3.007750e+01	0.000e+00	5.403e-01	8.181e-03
93	440	3.007242e+01	0.000e+00	3.931e-01	4.702e-02
94	442	3.006852e+01	0.000e+00	4.531e-01	2.816e-02
95	449	3.006583e+01	0.000e+00	3.562e-01	2.469e-02
96	451	3.006339e+01	0.000e+00	3.504e-01	8.038e-03
97	455	3.005767e+01	0.000e+00	3.169e-01	1.764e-02
98	459	3.005249e+01	0.000e+00	3.784e-01	1.393e-02
99	462	3.004213e+01	0.000e+00	4.367e-01	3.367e-02
100	472	3.003901e+01	0.000e+00	1.968e-01	2.263e-02
101	474	3.003713e+01	0.000e+00	2.450e-01	1.181e-02
102	478	3.003317e+01	0.000e+00	2.575e-01	2.115e-02
103	482	3.003214e+01	0.000e+00	3.851e-01	1.931e-02
104	485	3.002663e+01	0.000e+00	3.587e-01	2.464e-02
105	488	3.002319e+01	0.000e+00	3.636e-01	3.220e-02
106	497	3.002112e+01	0.000e+00	2.021e-01	2.796e-02

```
 107     499    3.002009e+01    0.000e+00    1.711e-01    4.516e-03
 108     503    3.001834e+01    0.000e+00    2.404e-01    2.007e-02
 109     506    3.001494e+01    0.000e+00    3.494e-01    2.429e-02
 110     510    3.001275e+01    0.000e+00    1.936e-01    1.278e-02
 111     512    3.000829e+01    0.000e+00    1.602e-01    2.799e-02
 112     515    3.000711e+01    0.000e+00    9.344e-02    1.420e-02
 113     517    3.000533e+01    0.000e+00    7.567e-02    1.173e-02
 114     520    3.000508e+01    0.000e+00    6.198e-02    1.704e-02
 115     521    3.000343e+01    0.000e+00    5.795e-02    9.005e-03
 116     525    3.000381e+01    0.000e+00    3.177e-02    5.657e-03
 117     527    2.999308e+01    0.000e+00    2.195e-01    5.430e-02
 118     528    2.998117e+01    0.000e+00    1.535e-01    3.611e-02
 119     529    2.998151e+01    0.000e+00    1.204e-01    4.471e-02
 120     530    2.998051e+01    0.000e+00    3.155e-02    2.070e-02

                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
 121     531    2.998046e+01    0.000e+00    2.465e-02    6.553e-03
 122     532    2.998046e+01    0.000e+00    5.654e-03    4.358e-03
 123     533    2.997632e+01    0.000e+00    2.802e-02    9.373e-03
 124     534    2.997640e+01    0.000e+00    6.233e-03    5.764e-04
 125     535    2.997640e+01    0.000e+00    1.724e-03    5.719e-04
 126     536    2.997562e+01    0.000e+00    3.457e-03    2.004e-03
 127     537    2.997562e+01    0.000e+00    1.356e-03    2.970e-04
 128     538    2.997562e+01    0.000e+00    9.688e-04    2.167e-04
 129     539    2.997562e+01    0.000e+00    2.260e-03    1.268e-03
 130     540    2.997563e+01    0.000e+00    6.079e-03    3.258e-03
 131     541    2.997563e+01    0.000e+00    7.534e-03    3.495e-03
 132     542    2.997562e+01    0.000e+00    4.548e-03    1.266e-03
 133     543    2.997562e+01    0.000e+00    1.185e-03    9.824e-05
 134     544    2.997562e+01    0.000e+00    1.749e-04    9.625e-05
 135     545    2.997547e+01    0.000e+00    6.196e-04    3.642e-04
 136     546    2.997547e+01    0.000e+00    1.844e-05    3.329e-05
 137     547    2.997543e+01    0.000e+00    1.325e-04    9.204e-05
 138     548    2.997543e+01    0.000e+00    4.002e-05    1.162e-05
 139     549    2.997543e+01    0.000e+00    2.063e-06    1.735e-06

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the selected value of the function tolerance,
and constraints are satisfied to within the selected value of the constraint tolerance.
```



Multiple local minima

```
figure;
plot(Y4,Z4,'--or'); hold on;
plot(-2,1,'xg','MarkerSize',10);
plot(2,1,'xg','MarkerSize',10);
plot(x,f,':b');
xlabel('y'); ylabel('z');
title('Multiple local minima (with nonconvex ground constraint)')

% Assign all variables on the left side:
assign(y,-2*ones(N,1))
assign(z,1*ones(N,1))

solvesdp(constr4, Vchain, options);

Y5 = double(y); Z5 = double(z);
plot(Y5,Z5,'--og');

% Assign all variables on the right side:
assign(y,2*ones(N,1))
assign(z,1*ones(N,1))

solvesdp(constr4, Vchain, options);

Y6 = double(y); Z6 = double(z);
plot(Y6,Z6,'--ob');
```

```
Your initial point x0 is not between bounds lb and ub; FMINCON
shifted x0 to strictly satisfy the bounds.
```

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	1	5.992400e+02	0.000e+00	1.732e+02	
1	7	3.570590e+02	0.000e+00	5.837e+01	1.307e+00
2	11	3.416393e+02	0.000e+00	5.696e+01	8.191e-01
3	16	2.876097e+02	0.000e+00	9.680e+01	1.465e+00
4	19	2.830372e+02	0.000e+00	1.160e+02	1.470e+00
5	24	2.602494e+02	0.000e+00	1.068e+02	2.288e+00
6	28	2.480282e+02	0.000e+00	1.054e+02	5.363e-01
7	31	2.424785e+02	0.000e+00	9.636e+01	6.788e-01
8	36	1.787374e+02	0.000e+00	6.453e+01	1.358e+00
9	43	1.535877e+02	0.000e+00	2.200e+01	6.554e-01
10	47	1.510769e+02	0.000e+00	2.724e+01	3.238e-01
11	51	1.456782e+02	0.000e+00	3.152e+01	5.656e-01
12	54	1.357795e+02	0.000e+00	3.734e+01	1.580e+00
13	58	1.295419e+02	0.000e+00	2.590e+01	1.696e+00
14	62	1.268406e+02	0.000e+00	3.268e+01	1.948e+00
15	66	1.232635e+02	0.000e+00	3.464e+01	2.168e+00
16	73	1.227025e+02	0.000e+00	4.309e+01	1.026e+00
17	76	1.172479e+02	0.000e+00	3.768e+01	5.198e-01
18	79	1.157415e+02	0.000e+00	5.100e+01	1.247e+00
19	83	1.114291e+02	0.000e+00	4.673e+01	5.603e-01
20	87	1.069852e+02	0.000e+00	4.544e+01	5.534e-01
21	90	1.060312e+02	0.000e+00	5.276e+01	1.245e+00
22	93	1.055610e+02	0.000e+00	4.190e+01	9.752e-01
23	97	9.982963e+01	0.000e+00	2.720e+01	7.852e-01
24	103	9.583909e+01	0.000e+00	4.119e+01	7.809e-01
25	107	9.413436e+01	0.000e+00	4.373e+01	1.845e-01
26	112	7.655108e+01	0.000e+00	8.826e+00	4.044e-01
27	115	7.584120e+01	0.000e+00	1.493e+01	3.790e-01
28	119	7.505355e+01	0.000e+00	1.531e+01	5.549e-01
29	125	6.935884e+01	0.000e+00	1.331e+01	5.322e-01
30	129	6.929227e+01	0.000e+00	1.612e+01	2.826e-01

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
31	134	6.793865e+01	0.000e+00	1.763e+01	5.202e-01
32	137	6.712445e+01	0.000e+00	1.163e+01	2.812e-01
33	143	6.218699e+01	0.000e+00	6.883e+00	2.861e-01
34	147	6.181484e+01	0.000e+00	8.044e+00	1.365e-01
35	152	6.109999e+01	0.000e+00	8.556e+00	2.353e-01
36	156	6.092821e+01	0.000e+00	7.445e+00	1.236e-01
37	159	6.032696e+01	0.000e+00	1.281e+01	6.222e-01
38	163	5.927973e+01	0.000e+00	1.386e+01	5.607e-01
39	170	5.676259e+01	0.000e+00	1.215e+01	3.061e-01
40	174	5.663415e+01	0.000e+00	1.018e+01	1.472e-01
41	179	5.423792e+01	0.000e+00	6.884e+00	2.600e-01
42	183	5.391898e+01	0.000e+00	7.738e+00	1.500e-01
43	189	5.292788e+01	0.000e+00	4.417e+00	1.428e-01
44	192	5.263421e+01	0.000e+00	4.226e+00	1.270e-01
45	196	5.234106e+01	0.000e+00	7.321e+00	4.080e-01
46	200	5.143265e+01	0.000e+00	7.326e+00	5.776e-01
47	207	5.115931e+01	0.000e+00	1.247e+01	3.070e-01
48	209	5.090286e+01	0.000e+00	1.173e+01	2.883e-01
49	215	4.987688e+01	0.000e+00	1.088e+01	2.839e-01
50	218	4.928822e+01	0.000e+00	8.384e+00	1.390e-01
51	223	4.904719e+01	0.000e+00	9.167e+00	2.544e-01
52	226	4.858503e+01	0.000e+00	6.539e+00	1.376e-01
53	231	4.778878e+01	0.000e+00	6.020e+00	2.770e-01
54	234	4.765645e+01	0.000e+00	4.859e+00	2.876e-01
55	240	4.747875e+01	0.000e+00	8.513e+00	2.995e-01

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  56     243    4.707834e+01    0.000e+00    5.279e+00    1.411e-01
  57     248    4.659595e+01    0.000e+00    7.135e+00    2.946e-01
  58     251    4.625419e+01    0.000e+00    5.657e+00    1.342e-01
  59     257    4.506670e+01    0.000e+00    2.620e+00    1.531e-01
  60     260    4.502176e+01    0.000e+00    3.721e+00    1.069e-01

                                            First-order      Norm of
Iter F-count            f(x)  Feasibility   optimality         step
  61     263    4.457553e+01    0.000e+00    9.183e+00    5.097e-01
  62     266    4.428952e+01    0.000e+00    8.362e+00    6.204e-01
  63     270    4.349275e+01    0.000e+00    8.823e+00    4.479e-01
  64     277    4.235135e+01    0.000e+00    8.408e+00    2.465e-01
  65     280    4.199832e+01    0.000e+00    5.596e+00    2.108e-01
  66     286    4.086693e+01    0.000e+00    3.052e+00    2.061e-01
  67     290    4.078513e+01    0.000e+00    4.166e+00    1.070e-01
  68     295    4.055282e+01    0.000e+00    5.405e+00    2.007e-01
  69     298    4.037388e+01    0.000e+00    3.801e+00    1.031e-01
  70     303    3.999981e+01    0.000e+00    5.223e+00    1.960e-01
  71     307    3.990536e+01    0.000e+00    3.407e+00    1.090e-01
  72     312    3.960312e+01    0.000e+00    4.090e+00    1.807e-01
  73     315    3.946213e+01    0.000e+00    2.866e+00    1.094e-01
  74     319    3.921995e+01    0.000e+00    4.341e+00    2.329e-01
  75     323    3.888107e+01    0.000e+00    5.026e+00    1.794e-01
  76     327    3.849301e+01    0.000e+00    3.838e+00    1.912e-01
  77     334    3.825834e+01    0.000e+00    3.649e+00    1.388e-01
  78     336    3.806569e+01    0.000e+00    3.885e+00    1.179e-01
  79     342    3.792462e+01    0.000e+00    3.573e+00    1.268e-01
  80     344    3.775017e+01    0.000e+00    3.241e+00    9.713e-02
  81     348    3.746061e+01    0.000e+00    3.398e+00    1.566e-01
  82     352    3.699268e+01    0.000e+00    2.198e+00    1.688e-01
  83     360    3.677504e+01    0.000e+00    1.954e+00    7.951e-02
  84     362    3.657820e+01    0.000e+00    1.924e+00    1.024e-01
  85     366    3.621981e+01    0.000e+00    2.907e+00    2.602e-01
  86     370    3.601167e+01    0.000e+00    3.406e+00    1.808e-01
  87     377    3.589809e+01    0.000e+00    2.202e+00    1.149e-01
  88     379    3.573465e+01    0.000e+00    2.043e+00    6.896e-02
  89     382    3.548395e+01    0.000e+00    3.383e+00    3.299e-01
  90     386    3.516687e+01    0.000e+00    3.618e+00    2.670e-01

                                            First-order      Norm of
Iter F-count            f(x)  Feasibility   optimality         step
  91     390    3.504611e+01    0.000e+00    3.890e+00    1.708e-01
  92     394    3.493803e+01    0.000e+00    3.451e+00    2.057e-01
  93     400    3.477605e+01    0.000e+00    2.935e+00    2.624e-01
  94     403    3.466111e+01    0.000e+00    2.350e+00    6.772e-02
  95     407    3.445910e+01    0.000e+00    2.684e+00    1.495e-01
  96     411    3.415648e+01    0.000e+00    2.364e+00    2.307e-01
  97     418    3.412546e+01    0.000e+00    2.635e+00    1.372e-01
  98     420    3.395354e+01    0.000e+00    2.121e+00    8.896e-02
  99     423    3.357472e+01    0.000e+00    1.465e+00    2.546e-01
 100     425    3.329040e+01    0.000e+00    3.663e+00    4.693e-01
 101     427    3.302702e+01    0.000e+00    5.795e+00    6.003e-01
 102     430    3.267394e+01    0.000e+00    4.360e+00    3.395e-01
 103     433    3.233872e+01    0.000e+00    2.977e+00    6.532e-01
 104     436    3.217372e+01    0.000e+00    3.922e+00    3.766e-01
 105     438    3.157656e+01    0.000e+00    2.329e+00    3.270e-01
 106     441    3.138337e+01    0.000e+00    3.198e+00    2.532e-01
 107     443    3.118656e+01    0.000e+00    2.251e+00    3.018e-01
 108     445    3.095425e+01    0.000e+00    2.342e+00    3.191e-01
 109     448    3.090907e+01    0.000e+00    2.365e+00    2.406e-01
 110     450    3.075491e+01    0.000e+00    1.812e+00    2.142e-01
```

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
111	452	3.060087e+01	0.000e+00	2.244e+00	3.258e-01
112	454	3.048675e+01	0.000e+00	1.418e+00	1.941e-01
113	456	3.042922e+01	0.000e+00	1.218e+00	2.379e-01
114	457	3.041549e+01	0.000e+00	1.222e+00	4.404e-01
115	458	3.030491e+01	0.000e+00	2.073e+00	3.596e-01
116	461	3.031906e+01	0.000e+00	2.866e+00	2.643e-01
117	462	3.029043e+01	0.000e+00	5.353e-01	2.685e-01
118	463	3.018501e+01	0.000e+00	6.007e-01	1.958e-01
119	464	3.022223e+01	0.000e+00	5.903e-01	2.165e-01
120	465	3.020781e+01	0.000e+00	3.879e-01	1.923e-01

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
121	466	3.019548e+01	0.000e+00	4.243e-01	9.596e-02
122	467	3.018131e+01	0.000e+00	3.554e-01	9.640e-02
123	468	3.017693e+01	0.000e+00	4.054e-01	1.437e-01
124	469	3.016629e+01	0.000e+00	4.109e-01	1.024e-01
125	470	3.016250e+01	0.000e+00	4.484e-01	8.478e-02
126	471	3.015772e+01	0.000e+00	4.151e-01	5.528e-02
127	472	3.015683e+01	0.000e+00	4.366e-01	5.677e-02
128	473	3.015416e+01	0.000e+00	4.234e-01	4.292e-02
129	474	3.015281e+01	0.000e+00	4.248e-01	3.825e-02
130	475	3.015143e+01	0.000e+00	4.283e-01	2.915e-02
131	476	3.015050e+01	0.000e+00	4.331e-01	2.210e-02
132	477	3.014984e+01	0.000e+00	4.377e-01	1.617e-02
133	478	3.014940e+01	0.000e+00	4.418e-01	1.170e-02
134	479	3.014910e+01	0.000e+00	4.450e-01	8.363e-03
135	480	3.014890e+01	0.000e+00	4.475e-01	5.937e-03
136	481	3.014876e+01	0.000e+00	4.493e-01	4.194e-03
137	482	3.014866e+01	0.000e+00	4.506e-01	2.952e-03
138	483	3.014859e+01	0.000e+00	4.515e-01	2.074e-03
139	484	3.014855e+01	0.000e+00	4.522e-01	1.454e-03
140	485	3.014852e+01	0.000e+00	4.527e-01	1.018e-03
141	486	3.014849e+01	0.000e+00	4.530e-01	7.123e-04
142	487	3.014848e+01	0.000e+00	4.532e-01	4.983e-04
143	488	3.014847e+01	0.000e+00	4.534e-01	3.483e-04
144	489	3.014846e+01	0.000e+00	4.535e-01	2.434e-04
145	490	3.014846e+01	0.000e+00	4.536e-01	1.701e-04
146	491	3.014845e+01	0.000e+00	4.537e-01	1.188e-04
147	492	3.014845e+01	0.000e+00	4.537e-01	8.301e-05
148	493	3.014845e+01	0.000e+00	4.537e-01	5.800e-05
149	494	3.014845e+01	0.000e+00	4.537e-01	4.049e-05
150	495	3.014845e+01	0.000e+00	4.538e-01	2.829e-05

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
151	496	3.014845e+01	0.000e+00	4.538e-01	1.975e-05
152	497	3.014844e+01	0.000e+00	4.538e-01	1.382e-05
153	498	3.014844e+01	0.000e+00	4.538e-01	9.639e-06
154	499	3.014844e+01	0.000e+00	4.538e-01	6.735e-06
155	500	3.014844e+01	0.000e+00	4.538e-01	4.717e-06
156	501	3.014844e+01	0.000e+00	4.538e-01	3.264e-06
157	502	3.014844e+01	0.000e+00	4.538e-01	2.299e-06
158	503	3.014844e+01	0.000e+00	4.538e-01	1.605e-06
159	504	3.014844e+01	0.000e+00	4.538e-01	1.123e-06
160	505	3.014844e+01	0.000e+00	4.538e-01	7.856e-07
161	506	3.014844e+01	0.000e+00	4.538e-01	5.546e-07
162	507	3.014844e+01	0.000e+00	4.538e-01	3.809e-07
163	508	3.014844e+01	0.000e+00	4.538e-01	2.683e-07
164	509	3.014844e+01	0.000e+00	4.538e-01	1.874e-07
165	510	3.014844e+01	0.000e+00	4.538e-01	1.312e-07

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the default value of the step size tolerance and constraints are
satisfied to within the selected value of the constraint tolerance.

Your initial point x0 is not between bounds lb and ub; FMINCON
shifted x0 to strictly satisfy the bounds.

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	1	5.992400e+02	0.000e+00	1.732e+02	
1	7	3.570590e+02	0.000e+00	5.837e+01	1.307e+00
2	11	3.416393e+02	0.000e+00	5.696e+01	8.191e-01
3	16	2.876097e+02	0.000e+00	9.680e+01	1.465e+00
4	19	2.830372e+02	0.000e+00	1.160e+02	1.470e+00
5	24	2.602494e+02	0.000e+00	1.068e+02	2.288e+00
6	28	2.480282e+02	0.000e+00	1.054e+02	5.363e-01
7	31	2.424785e+02	0.000e+00	9.636e+01	6.788e-01
8	36	1.787374e+02	0.000e+00	6.453e+01	1.358e+00
9	43	1.535877e+02	0.000e+00	2.200e+01	6.554e-01
10	47	1.510769e+02	0.000e+00	2.724e+01	3.238e-01
11	51	1.456782e+02	0.000e+00	3.152e+01	5.656e-01
12	54	1.357795e+02	0.000e+00	3.734e+01	1.580e+00
13	58	1.295419e+02	0.000e+00	2.590e+01	1.696e+00
14	62	1.268406e+02	0.000e+00	3.268e+01	1.948e+00
15	66	1.232635e+02	0.000e+00	3.464e+01	2.168e+00
16	73	1.227025e+02	0.000e+00	4.309e+01	1.026e+00
17	76	1.172479e+02	0.000e+00	3.768e+01	5.198e-01
18	79	1.157415e+02	0.000e+00	5.100e+01	1.247e+00
19	83	1.114291e+02	0.000e+00	4.673e+01	5.603e-01
20	87	1.069852e+02	0.000e+00	4.544e+01	5.534e-01
21	90	1.060312e+02	0.000e+00	5.276e+01	1.245e+00
22	93	1.055610e+02	0.000e+00	4.190e+01	9.752e-01
23	97	9.982963e+01	0.000e+00	2.720e+01	7.852e-01
24	103	9.583909e+01	0.000e+00	4.119e+01	7.809e-01
25	107	9.413436e+01	0.000e+00	4.373e+01	1.845e-01
26	112	7.655108e+01	0.000e+00	8.826e+00	4.044e-01
27	115	7.584120e+01	0.000e+00	1.493e+01	3.790e-01
28	119	7.505355e+01	0.000e+00	1.531e+01	5.549e-01
29	125	6.935884e+01	0.000e+00	1.331e+01	5.322e-01
30	129	6.929227e+01	0.000e+00	1.612e+01	2.826e-01

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
31	134	6.793865e+01	0.000e+00	1.763e+01	5.202e-01
32	137	6.712445e+01	0.000e+00	1.163e+01	2.812e-01
33	143	6.218699e+01	0.000e+00	6.883e+00	2.861e-01
34	147	6.181484e+01	0.000e+00	8.044e+00	1.365e-01
35	152	6.109999e+01	0.000e+00	8.556e+00	2.353e-01
36	156	6.092821e+01	0.000e+00	7.445e+00	1.236e-01
37	159	6.032696e+01	0.000e+00	1.281e+01	6.222e-01
38	163	5.927973e+01	0.000e+00	1.386e+01	5.607e-01
39	170	5.676259e+01	0.000e+00	1.215e+01	3.061e-01
40	174	5.663415e+01	0.000e+00	1.018e+01	1.472e-01
41	179	5.423792e+01	0.000e+00	6.884e+00	2.600e-01
42	183	5.391898e+01	0.000e+00	7.738e+00	1.500e-01
43	189	5.292788e+01	0.000e+00	4.417e+00	1.428e-01

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
44	192	5.263421e+01	0.000e+00	4.226e+00	1.270e-01
45	196	5.234106e+01	0.000e+00	7.321e+00	4.080e-01
46	200	5.143265e+01	0.000e+00	7.326e+00	5.776e-01
47	207	5.115931e+01	0.000e+00	1.247e+01	3.070e-01
48	209	5.090286e+01	0.000e+00	1.173e+01	2.883e-01
49	215	4.987688e+01	0.000e+00	1.088e+01	2.839e-01
50	218	4.928822e+01	0.000e+00	8.384e+00	1.390e-01
51	223	4.904719e+01	0.000e+00	9.167e+00	2.544e-01
52	226	4.858503e+01	0.000e+00	6.539e+00	1.376e-01
53	231	4.778878e+01	0.000e+00	6.020e+00	2.770e-01
54	234	4.765645e+01	0.000e+00	4.859e+00	2.876e-01
55	240	4.747875e+01	0.000e+00	8.513e+00	2.995e-01
56	243	4.707834e+01	0.000e+00	5.279e+00	1.411e-01
57	248	4.659595e+01	0.000e+00	7.135e+00	2.946e-01
58	251	4.625419e+01	0.000e+00	5.657e+00	1.342e-01
59	257	4.506670e+01	0.000e+00	2.620e+00	1.531e-01
60	260	4.502176e+01	0.000e+00	3.721e+00	1.069e-01

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
61	263	4.457553e+01	0.000e+00	9.183e+00	5.097e-01
62	266	4.428952e+01	0.000e+00	8.362e+00	6.204e-01
63	270	4.349275e+01	0.000e+00	8.823e+00	4.479e-01
64	277	4.235135e+01	0.000e+00	8.408e+00	2.465e-01
65	280	4.199832e+01	0.000e+00	5.596e+00	2.108e-01
66	286	4.086693e+01	0.000e+00	3.052e+00	2.061e-01
67	290	4.078513e+01	0.000e+00	4.166e+00	1.070e-01
68	295	4.055282e+01	0.000e+00	5.405e+00	2.007e-01
69	298	4.037388e+01	0.000e+00	3.801e+00	1.031e-01
70	303	3.999981e+01	0.000e+00	5.223e+00	1.960e-01
71	307	3.990536e+01	0.000e+00	3.407e+00	1.090e-01
72	312	3.960312e+01	0.000e+00	4.090e+00	1.807e-01
73	315	3.946213e+01	0.000e+00	2.866e+00	1.094e-01
74	319	3.921995e+01	0.000e+00	4.341e+00	2.329e-01
75	323	3.888107e+01	0.000e+00	5.026e+00	1.794e-01
76	327	3.849301e+01	0.000e+00	3.838e+00	1.912e-01
77	334	3.825834e+01	0.000e+00	3.649e+00	1.388e-01
78	336	3.806569e+01	0.000e+00	3.885e+00	1.179e-01
79	342	3.792462e+01	0.000e+00	3.573e+00	1.268e-01
80	344	3.775017e+01	0.000e+00	3.241e+00	9.713e-02
81	348	3.746061e+01	0.000e+00	3.398e+00	1.566e-01
82	352	3.699268e+01	0.000e+00	2.198e+00	1.688e-01
83	360	3.677504e+01	0.000e+00	1.954e+00	7.951e-02
84	362	3.657820e+01	0.000e+00	1.924e+00	1.024e-01
85	366	3.621981e+01	0.000e+00	2.907e+00	2.602e-01
86	370	3.601167e+01	0.000e+00	3.406e+00	1.808e-01
87	377	3.589809e+01	0.000e+00	2.202e+00	1.149e-01
88	379	3.573465e+01	0.000e+00	2.043e+00	6.896e-02
89	382	3.548395e+01	0.000e+00	3.383e+00	3.299e-01
90	386	3.516687e+01	0.000e+00	3.618e+00	2.670e-01

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
91	390	3.504611e+01	0.000e+00	3.890e+00	1.708e-01
92	394	3.493803e+01	0.000e+00	3.451e+00	2.057e-01
93	400	3.477605e+01	0.000e+00	2.935e+00	2.624e-01
94	403	3.466111e+01	0.000e+00	2.350e+00	6.772e-02
95	407	3.445910e+01	0.000e+00	2.684e+00	1.495e-01
96	411	3.415648e+01	0.000e+00	2.364e+00	2.307e-01
97	418	3.412546e+01	0.000e+00	2.635e+00	1.372e-01
98	420	3.395354e+01	0.000e+00	2.121e+00	8.896e-02

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 99     423    3.357472e+01    0.000e+00    1.465e+00    2.546e-01
100     425    3.329040e+01    0.000e+00    3.663e+00    4.693e-01
101     427    3.302702e+01    0.000e+00    5.795e+00    6.003e-01
102     430    3.267394e+01    0.000e+00    4.360e+00    3.395e-01
103     433    3.233871e+01    0.000e+00    2.977e+00    6.532e-01
104     436    3.217371e+01    0.000e+00    3.922e+00    3.766e-01
105     438    3.157655e+01    0.000e+00    2.329e+00    3.270e-01
106     441    3.138337e+01    0.000e+00    3.198e+00    2.532e-01
107     443    3.118656e+01    0.000e+00    2.251e+00    3.018e-01
108     445    3.095425e+01    0.000e+00    2.342e+00    3.191e-01
109     448    3.090907e+01    0.000e+00    2.365e+00    2.406e-01
110     450    3.075491e+01    0.000e+00    1.812e+00    2.142e-01
111     452    3.060087e+01    0.000e+00    2.244e+00    3.258e-01
112     454    3.048675e+01    0.000e+00    1.418e+00    1.941e-01
113     456    3.042922e+01    0.000e+00    1.218e+00    2.379e-01
114     457    3.041549e+01    0.000e+00    1.222e+00    4.404e-01
115     458    3.030491e+01    0.000e+00    2.073e+00    3.596e-01
116     461    3.031906e+01    0.000e+00    2.866e+00    2.643e-01
117     462    3.029043e+01    0.000e+00    5.353e-01    2.686e-01
118     463    3.018501e+01    0.000e+00    6.007e-01    1.958e-01
119     464    3.022223e+01    0.000e+00    5.902e-01    2.165e-01
120     465    3.020781e+01    0.000e+00    3.879e-01    1.923e-01

                                          First-order      Norm of
Iter F-count            f(x)  Feasibility   optimality         step
121     466    3.019548e+01    0.000e+00    4.243e-01    9.596e-02
122     467    3.018131e+01    0.000e+00    3.554e-01    9.640e-02
123     468    3.017693e+01    0.000e+00    4.054e-01    1.437e-01
124     469    3.016629e+01    0.000e+00    4.109e-01    1.024e-01
125     470    3.016250e+01    0.000e+00    4.484e-01    8.478e-02
126     471    3.015772e+01    0.000e+00    4.151e-01    5.528e-02
127     472    3.015683e+01    0.000e+00    4.366e-01    5.677e-02
128     473    3.015416e+01    0.000e+00    4.234e-01    4.292e-02
129     474    3.015281e+01    0.000e+00    4.248e-01    3.825e-02
130     475    3.015143e+01    0.000e+00    4.283e-01    2.915e-02
131     476    3.015050e+01    0.000e+00    4.331e-01    2.210e-02
132     477    3.014984e+01    0.000e+00    4.377e-01    1.617e-02
133     478    3.014940e+01    0.000e+00    4.418e-01    1.170e-02
134     479    3.014910e+01    0.000e+00    4.450e-01    8.363e-03
135     480    3.014890e+01    0.000e+00    4.475e-01    5.937e-03
136     481    3.014876e+01    0.000e+00    4.493e-01    4.194e-03
137     482    3.014866e+01    0.000e+00    4.506e-01    2.952e-03
138     483    3.014859e+01    0.000e+00    4.515e-01    2.074e-03
139     484    3.014855e+01    0.000e+00    4.522e-01    1.454e-03
140     485    3.014852e+01    0.000e+00    4.527e-01    1.018e-03
141     486    3.014849e+01    0.000e+00    4.530e-01    7.123e-04
142     487    3.014848e+01    0.000e+00    4.532e-01    4.983e-04
143     488    3.014847e+01    0.000e+00    4.534e-01    3.483e-04
144     489    3.014846e+01    0.000e+00    4.535e-01    2.434e-04
145     490    3.014846e+01    0.000e+00    4.536e-01    1.701e-04
146     491    3.014845e+01    0.000e+00    4.537e-01    1.188e-04
147     492    3.014845e+01    0.000e+00    4.537e-01    8.303e-05
148     493    3.014845e+01    0.000e+00    4.537e-01    5.801e-05
149     494    3.014845e+01    0.000e+00    4.537e-01    4.062e-05
150     495    3.014845e+01    0.000e+00    4.538e-01    2.830e-05

                                          First-order      Norm of
Iter F-count            f(x)  Feasibility   optimality         step
151     496    3.014845e+01    0.000e+00    4.538e-01    1.977e-05
152     497    3.014844e+01    0.000e+00    4.538e-01    1.376e-05
153     498    3.014844e+01    0.000e+00    4.538e-01    9.636e-06
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   154      499    3.014844e+01    0.000e+00    4.538e-01    6.721e-06
   155      500    3.014844e+01    0.000e+00    4.538e-01    4.689e-06
   156      501    3.014844e+01    0.000e+00    4.538e-01    3.275e-06
   157      502    3.014844e+01    0.000e+00    4.538e-01    2.285e-06
   158      503    3.014844e+01    0.000e+00    4.538e-01    1.597e-06
   159      504    3.014844e+01    0.000e+00    4.538e-01    1.115e-06
   160      505    3.014844e+01    0.000e+00    4.538e-01    8.073e-07
   161      506    3.014844e+01    0.000e+00    4.538e-01    5.517e-07
   162      507    3.014844e+01    0.000e+00    4.538e-01    3.852e-07
   163      508    3.014844e+01    0.000e+00    4.538e-01    2.673e-07
   164      509    3.014844e+01    0.000e+00    4.538e-01    1.861e-07
   165      510    3.014844e+01    0.000e+00    4.538e-01    1.294e-07

Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the default value of the step size tolerance and constraints are
satisfied to within the selected value of the constraint tolerance.
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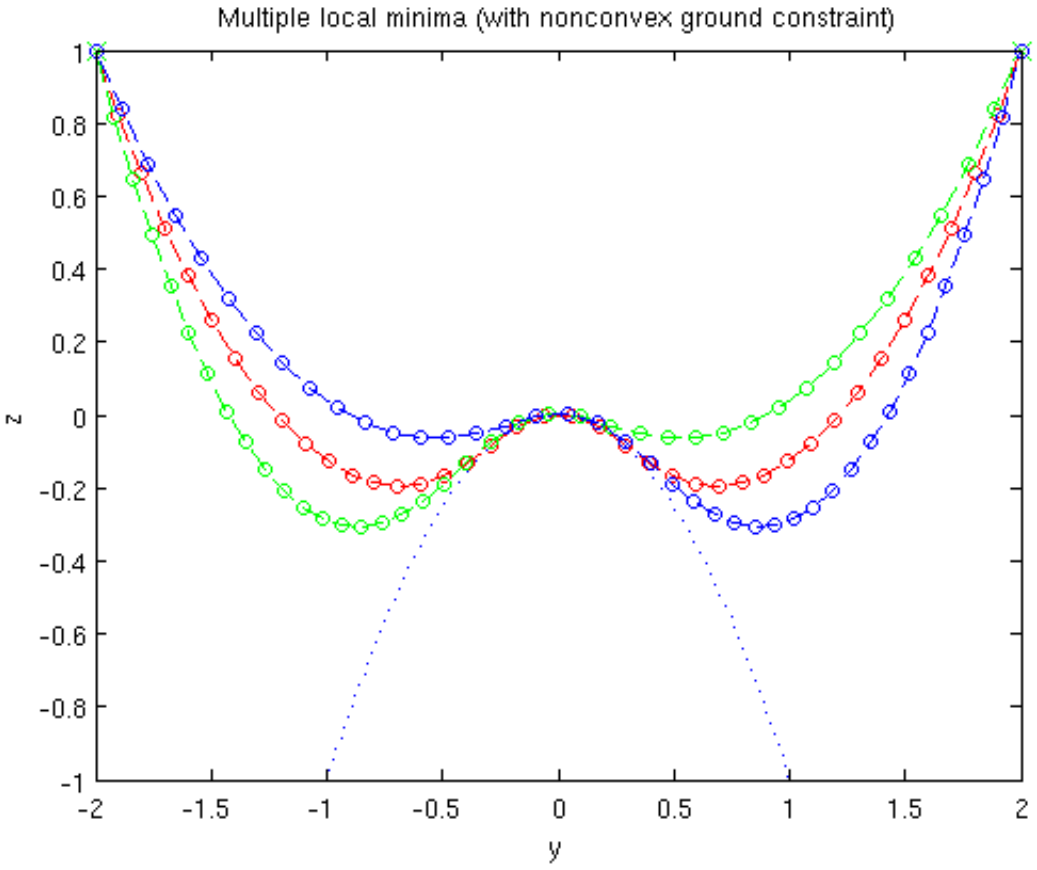

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