KATHOLIEKE UNIVERSITEIT LEUVEN

Faculteit Ingenieurswetenschappen
Departement Werktuigkunde

H04S3A: Master’s thesis

Sensitivity analysis of optimal divertor configurations

in nuclear fusion reactors

De Schutter Jochem

Outline

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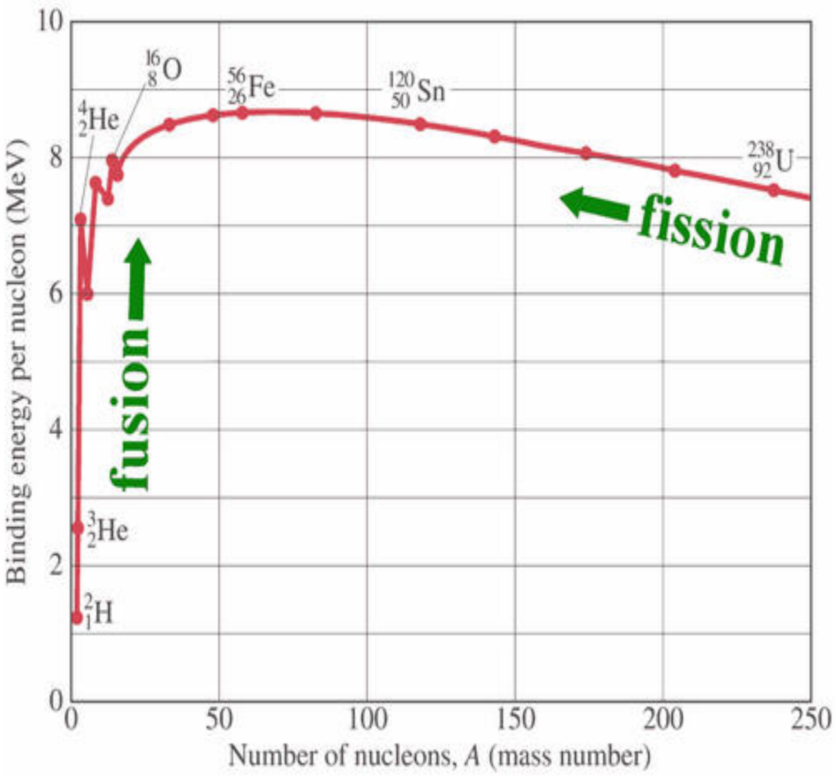
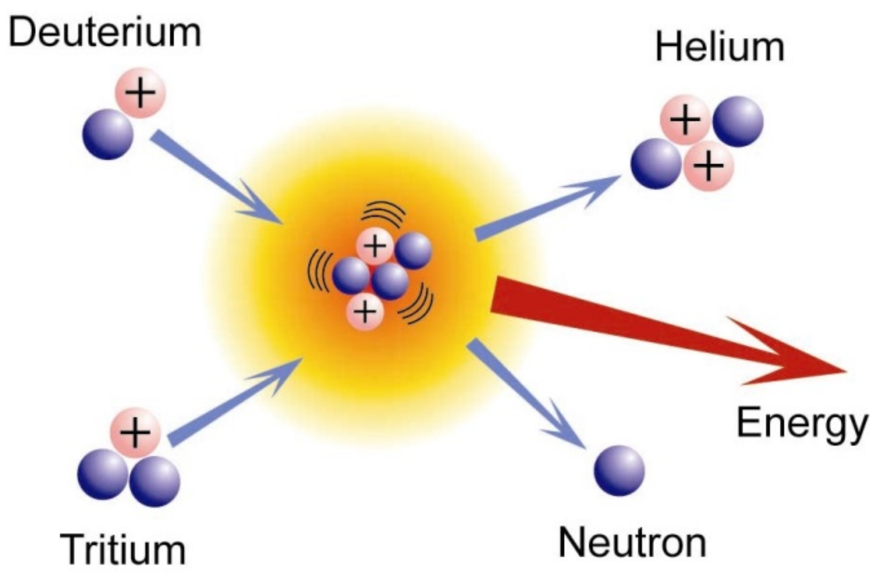
- **Introduction**
- Objectives
- Methodology
- Forward sensitivity analysis
- Parameter estimation
- General conclusions & future work

Introduction: Nuclear Fusion

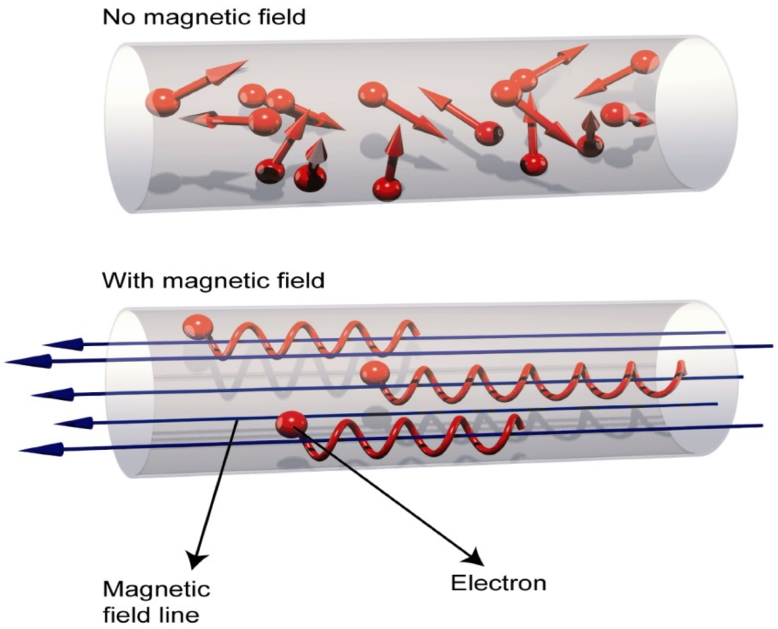


www.kentchemistry.com

www.universetoday.com

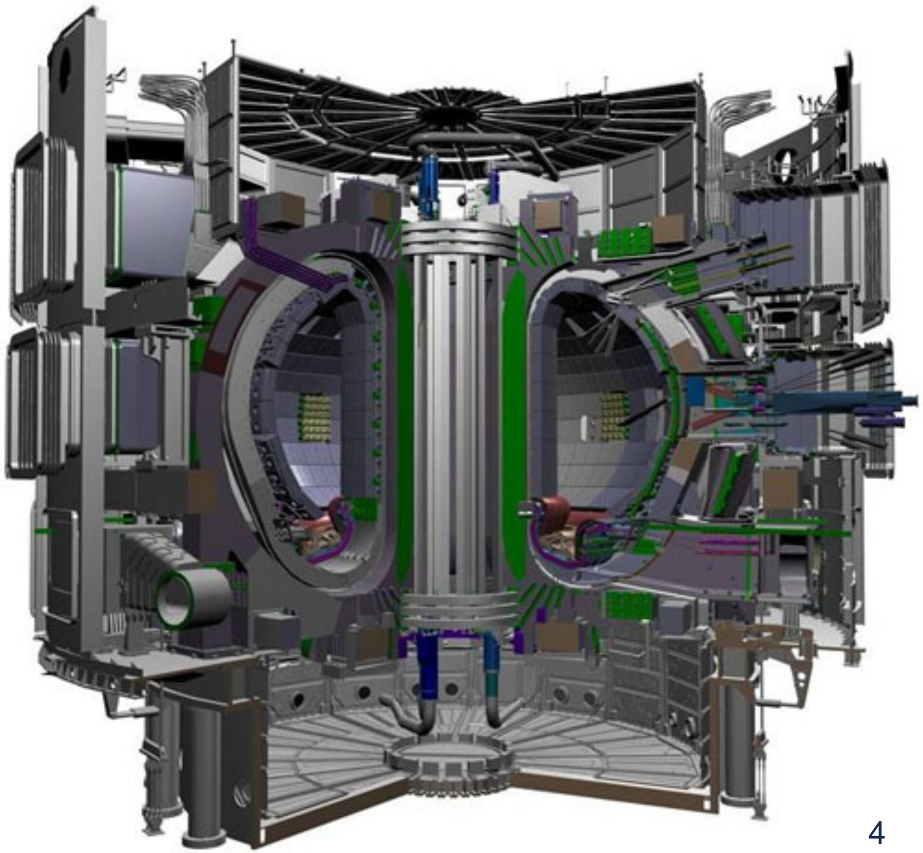


Magnetic confinement

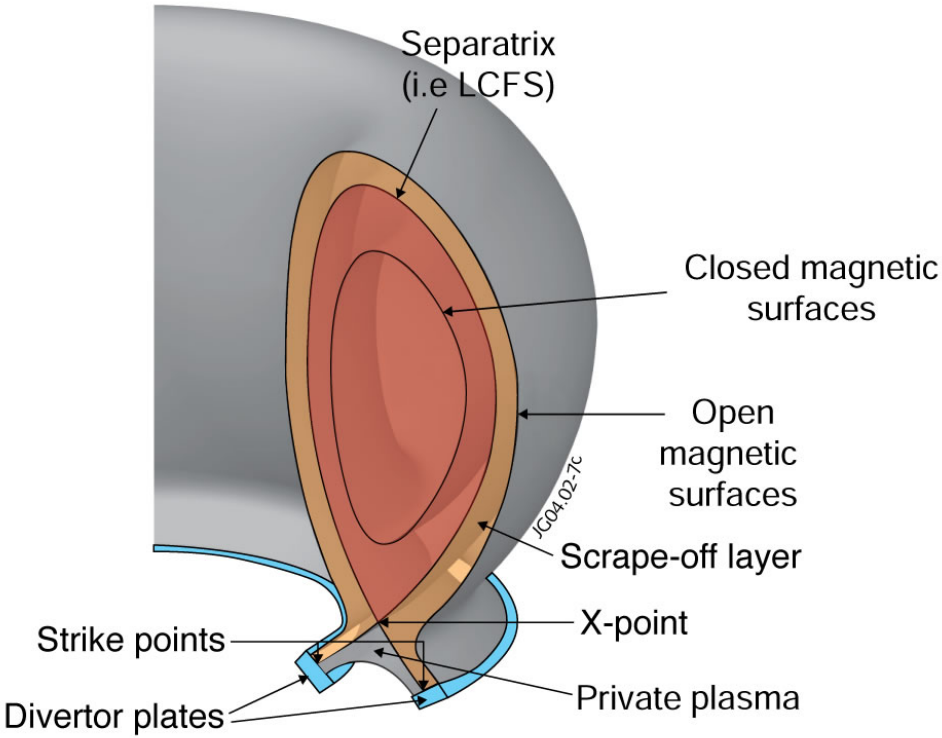


www.iter.rma.ac.be

www.scitechdaily.com

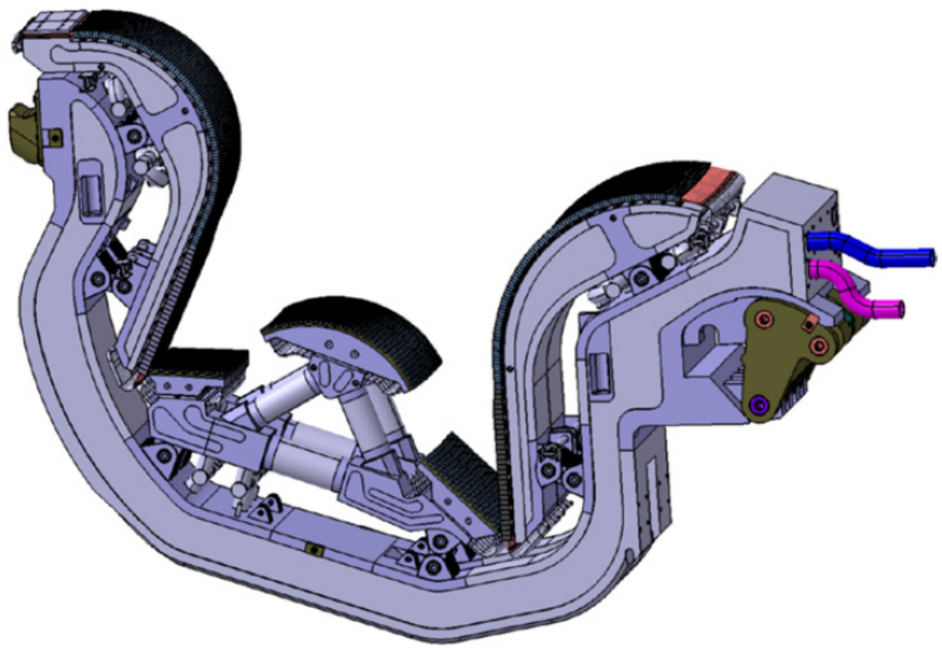


Divertor as exhaust system



www.efda.org/fusion/focus-on/limiters-and-divertors/

Finalizing the ITER divertor design: The key role of SOLPS modeling – Kukushkin,A.S.



Optimal design: uncertainties

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- Transport perpendicular to magnetic field
  - mass, momentum and energy transport
  - turbulent processes not well understood
  - modelled as diffusion processes
  - → uncertain anomalous transport coefficients!

- Neutral model

- Boundary conditions

- Large operational window
  - different power or density levels in life time
  - safe operation guaranteed?

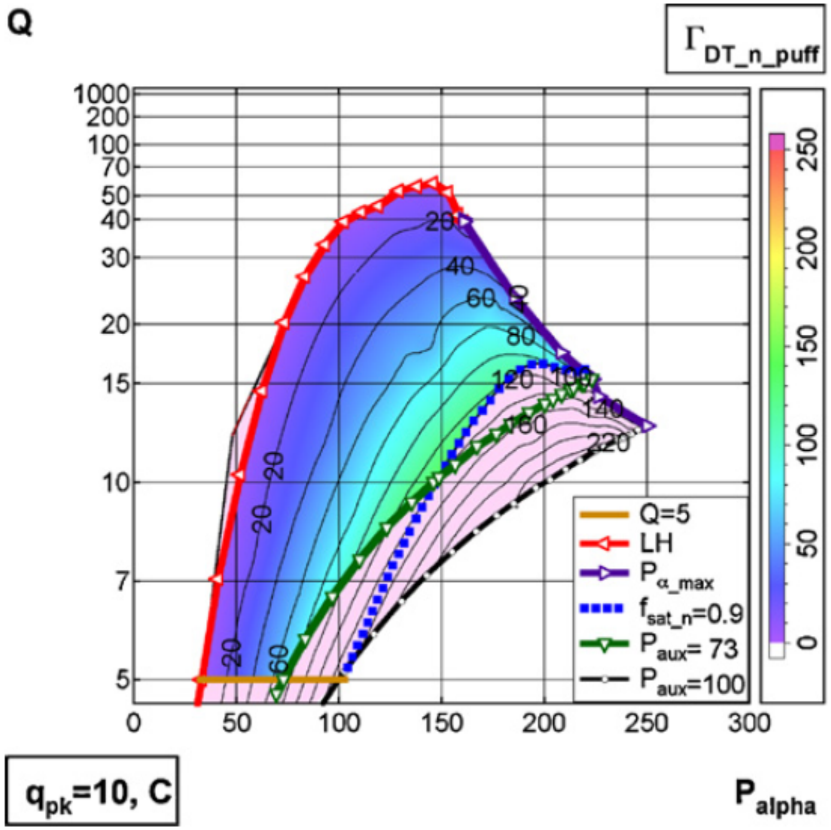
Uncertainty quantification

- Forward Uncertainy Quantification
  - output uncertainty based on uncertainty propagation (sensitivity analysis)

- Inverse Uncertainty Quantification
  - parameter estimation/calibration with experiments

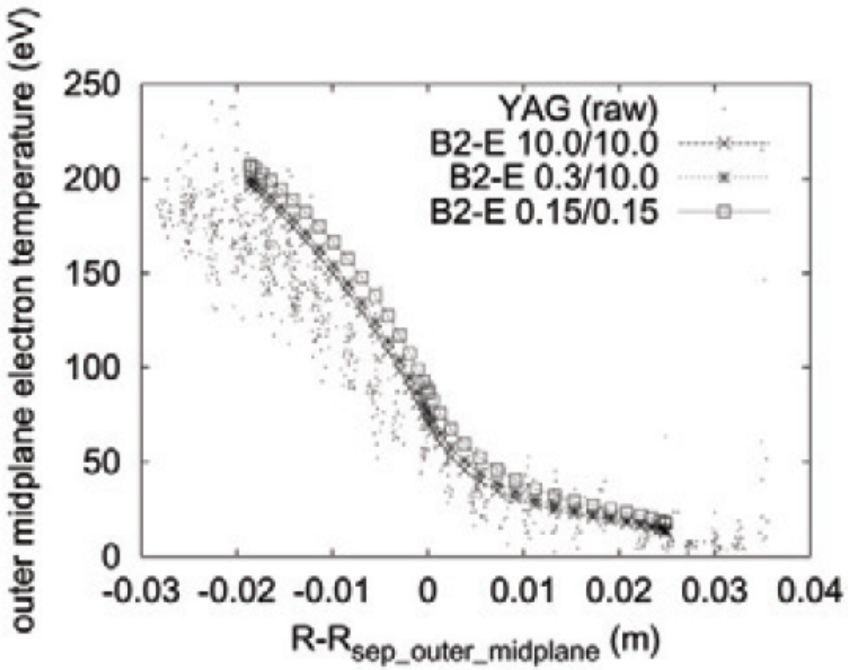
Uncertainty quantification: ITER

– <u>Forward UQ:</u>
operational flexibility



Finalizing the ITER divertor design: The key role of SOLPS modeling – Kukushkin et al.

– <u>Inverse UQ:</u>
parameter scan



Simulation of the Edge Plasma in Tokamaks – Coster et al.

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Objectives

- Develop efficient **forward** *and* **inverse** UQ method for uncertainties in optimal divertor design

  1) how sensitive and robust is the optimal design?
  2) Which input parameter values match the simulation with experimental data?

- Focus on adjoint-gradient-based UQ methods

  – efficient in dealing with large number of uncertainties
  – (cfr. shape-optimization in aerodynamics/divertor design)

- 'Proof of Method': simplified computational domain (MATLAB)

- High-level uncertainties and operational parameters
  – slightly simplified edge plasma model

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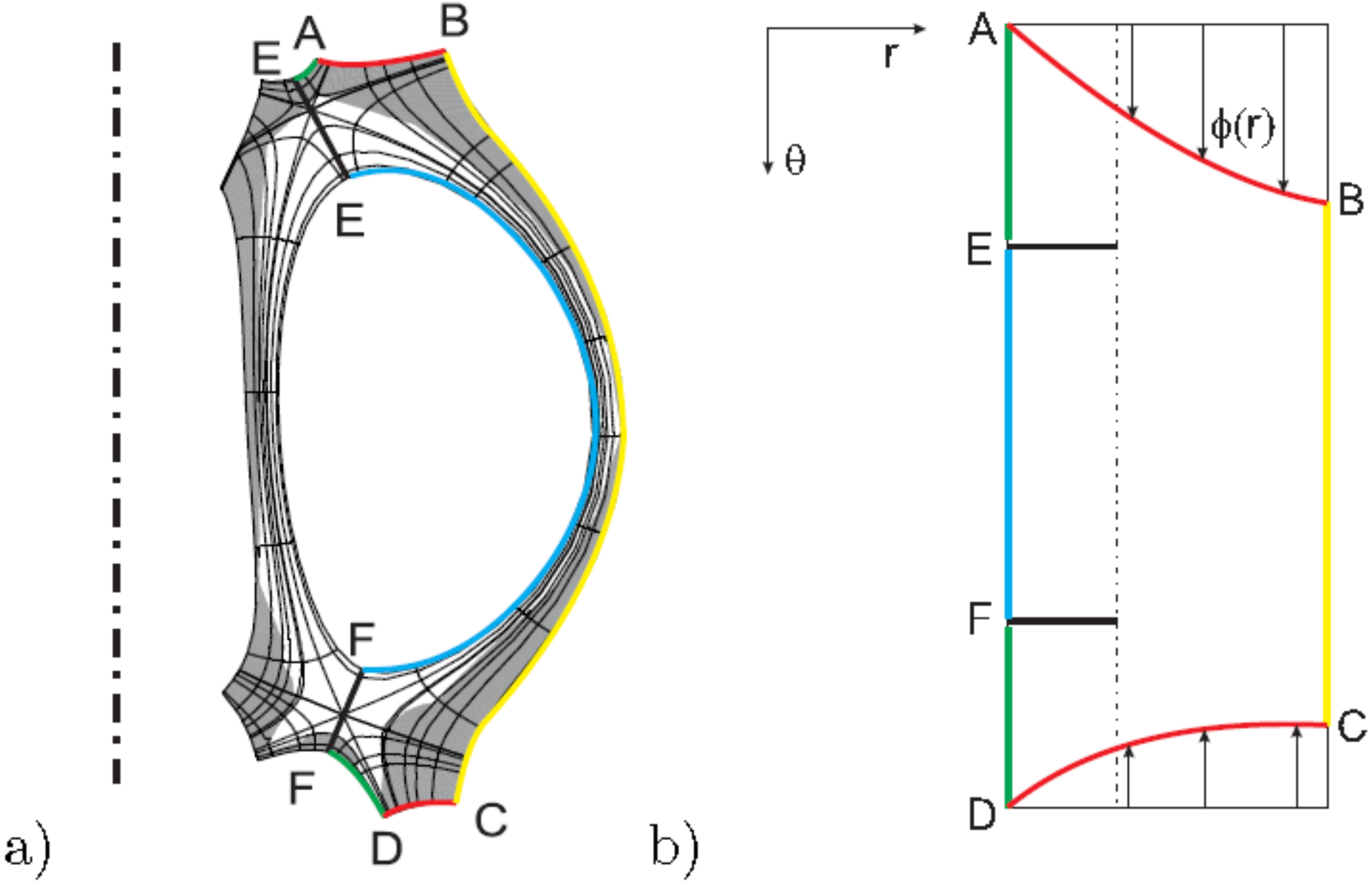
Computational domain



Choice of UQ methods

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- **Forward:** double constrained optimization problem
  - ‘best’ and ‘worst case scenario’
  - output is measure for divertor performance
  - compute max. and min. of output
  - efficient for high-dimensional UQ-problem if adjoint-gradient-based optimizer is applied
- **Inverse:** non-linear regression
  - find ‘best-fit’-values for uncertain parameters
  - adjoint-gradient-based optimizer

Adjoint differentiation

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- Cost functional

$$I(\mathbf{q}(\phi), \phi) = \int_S I_S(\mathbf{q}(\phi), \phi) d\sigma.$$

- Lagrangian

$$L(\mathbf{q}, \phi, \mathbf{q}^*, \mathbf{q}_S^*) = I(\mathbf{q}, \phi) - \int_V (\mathbf{q}^*)^T B(\mathbf{q}, \phi) d\Omega - \int_S (\mathbf{q}_S^*)^T B_S(\mathbf{q}, \phi) d\sigma.$$

cost function — state equations — BC

- Optimality conditions

$$\begin{cases} \nabla_{q^*} L(\mathbf{q}, \phi, \mathbf{q}^*, \mathbf{q}_S^*) = 0 & \text{state equations} \\ \nabla_{q_S^*} L(\mathbf{q}, \phi, \mathbf{q}^*, \mathbf{q}_S^*) = 0 & \text{boundary conditions} \\ \nabla_q L(\mathbf{q}, \phi, \mathbf{q}^*, \mathbf{q}_S^*) = 0 & \text{adjoint equations} \\ \nabla_\phi L(\mathbf{q}, \phi, \mathbf{q}^*, \mathbf{q}_S^*) \geq 0 & \text{design equation} \end{cases}$$

Adjoint differentiation

- When state and adjoint equations satisfied
  → cost function gradient = design equation

$$\frac{dI}{d\phi}\phi' = L_\phi \phi' = I_\phi \phi' - \int_V (\mathbf{q}^*)^T B_\phi \phi' d\Omega - \int_S (\mathbf{q}_\mathbf{S}^*)^T B_{S,\phi} \phi' d\sigma.$$

- Motivation
  - computational cost is roughly independent of #variables!
  - higher accuracy than FD
  - more information of spatial distribution of sensitivity
  - justifies the computational cost of the adjoint state (≈ computational cost of forward solution)
  - justifies extra theoretical and implementation work

Optimization algorithms

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- BFGS method with line-search (quasi-Newton)
  - every iteration, hessian matrix is updated
  - line-search along step direction → approximative minimum
  - Wolfe conditions
  - convergence guaranteed
- One-shot method with steepest-descent method
  - forward, adjoint and optimization problem converge simultaneously
  - choice of relaxation factor crucial for convergence
  - proved to be the fastest in divertor shape optimization

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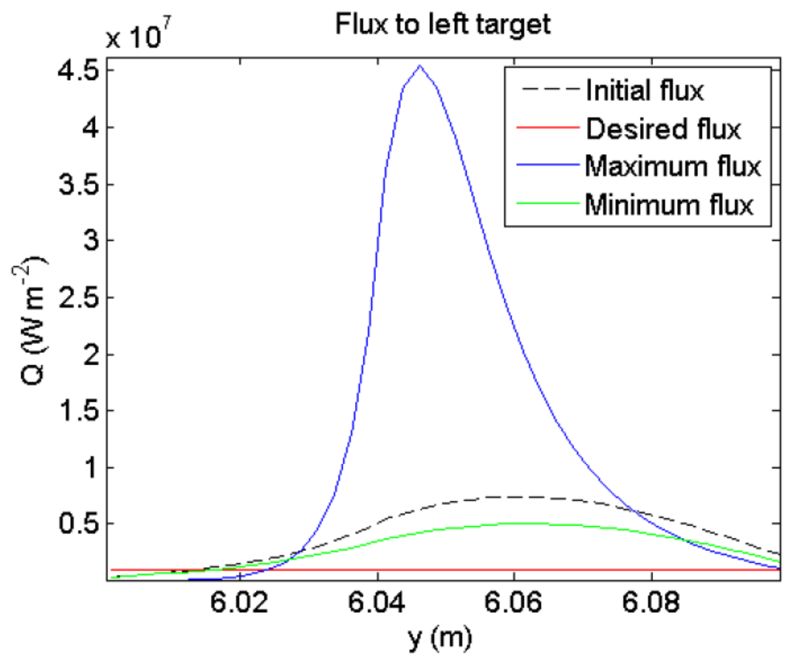
Forward UQ: conclusion

- Speed:
  - BFGS → 36 f
  - one-shot → 57 f (due to low relaxation factor)
    → dependent on initial state!

- Accuracy:
  - same for both methods
  - cost functional probably monotonous in design space

- Interpretation:
  - input interval too wide
  - allow spatial variation to include more information
  - insight in diffusion coefficient crucial!



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Inverse UQ: conclusion

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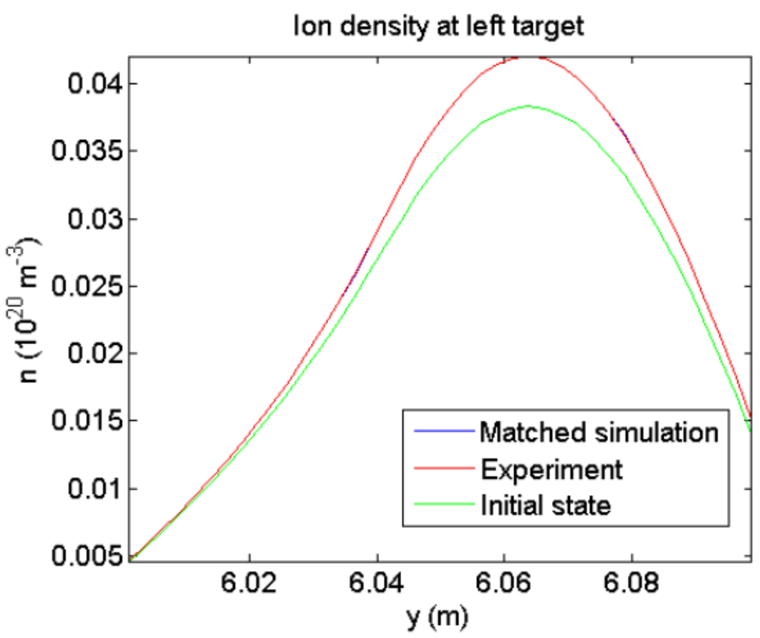
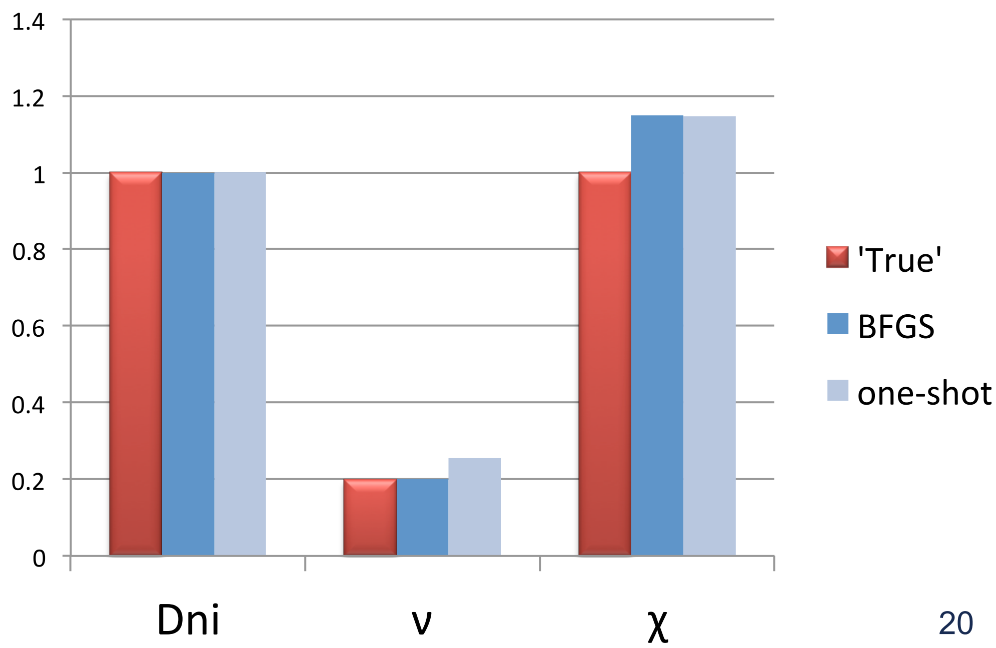
- Speed:
  - BFGS → 40 f
  - one-shot → 120 f (due to low relaxation factor!)

- Accuracy:





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General conclusions

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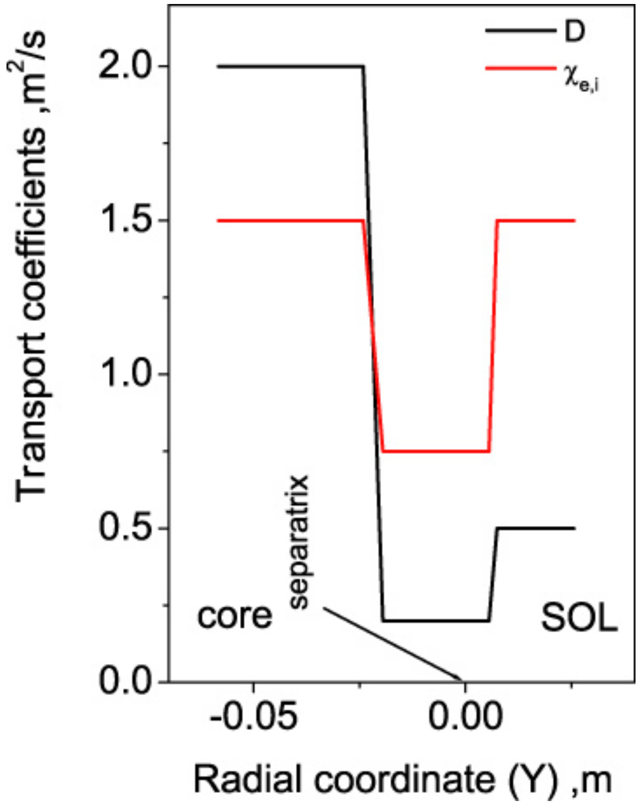
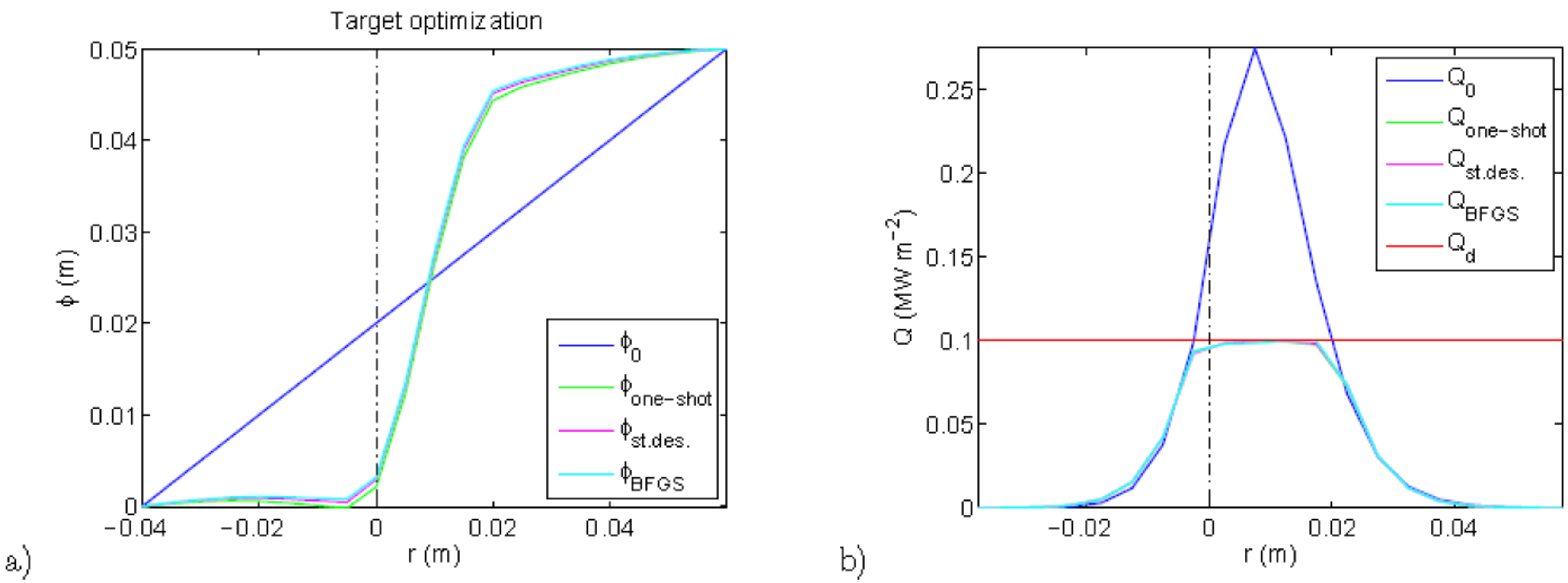
- Forward UQ technique
  - computes the output uncertainty interval
  - specific idea of the robustness of a divertor design
  - computational cost of roughly 36 f
  - roughly independent of # uncertainties
- Inverse UQ technique
  - ameliorates simulation predicting capability
  - calibrates most influental parameters very accurately
  - matches simulation and experiment precisely
  - computational cost of roughly 40 f
  - roughly independent of # calibration parameters

Future work

- Include spatial variation of uncertain parameters
- Expand edge plasma model
  - add presence of neutrals
  - up to complete edge codes (e.g. B2-Eirene)
- Parameter estimation with real experiments
- ‘Robust design’
  - include ‘cheap’ adjoint sensitivities as a penalty term in cost functional
  - optimizer will favour insensitive minima over lower but sensitive minima

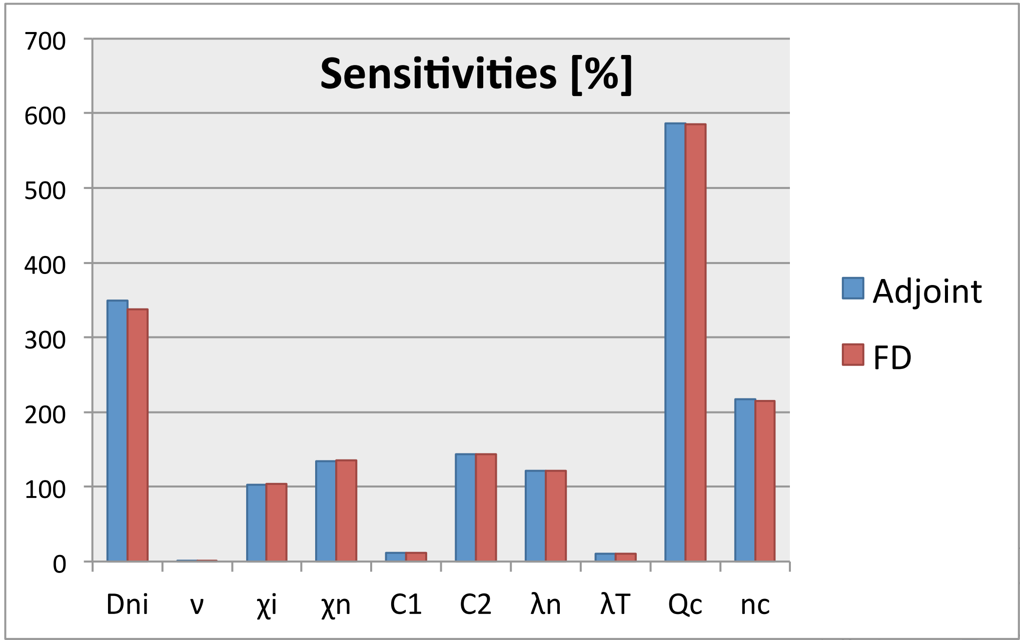


Optimal design: heat spreading



Optimal Shape Design for Divertors - W. Dekeyser et al.

Sensitivity verification



$$S_{\phi_i} = \frac{\phi_i}{I}\frac{dI}{d\phi_i}.$$